Chapter 2. Flow Characteristics

2.1 Isotropic Turbulence

For a viscous fluid there are two distinct states of motion: laminar and turbulent. For example, a fluid passes through a pipe of diameter $l$ with an average velocity $v$, through flow visualization using coloring dye we can observe that at low velocities, the streamline is smooth and clearly defined. These conditions are characteristic of a laminar flow. However, at increased velocities, the streamline is no longer smooth and the fluid undergoes irregular and random motion (Ishimaru, 1978). The latter is indicative of a turbulent flow. Under turbulent fluid motion various quantities show a random variation with time and space coordinates. Thus, it is an irregular condition of flow such that statistically distinct average values can be discerned (Hinze, 1959). In cases where the turbulence has the same structure in all parts of the flow field, quantitatively, the turbulence can be defined as homogeneous. Furthermore, the turbulence is classified as isotropic if its statistical features do not change with position throughout the flow, so that perfect disorder resigns (Hinze, 1959).

We use isotropic turbulence because it is the most easily characterized and is well defined in the literature. A grid is a convenient way to make isotropic turbulence. Thus, for our purposes we are primarily concerned with the underlying physical characteristics of grid-generated turbulence. Comprehensive reviews of homogeneous and isotropic turbulence are presented by Batchelor (1953), Hinze (1959) and Monin & Yaglom (1975) (Mohamed, LaRue, 1990). Additionally, information regarding specification of areas of
Paradoxically, the unique characteristics of grid turbulence violate the definition of the media because it is not self-sustaining. For example, to generate grid turbulence, a grid of (say) circular rods is placed perpendicular to a uniform flow stream. After a certain distance, a homogenous, isotropic field of turbulence is achieved as the vortices generated by the cylinder interact (Panton, 1996). Thus, to a large extent the turbulent flow within the test section can be classified as isotropic. The streamwise evolution of a temporally stationary turbulence field produced by a grid placed in stream of a steady uniform duct flow holds a strong resemblance to the time evolution of the mathematical ideal of isotropic turbulence.
This was first observed by Simmons & Salter (1934) (Comte-Bellot, Corrsin, 1971). Thus, from these observations, we know that a practically isotropic turbulence can be produced by means of grids placed in stream of a uniform flow (Hinze, 1959). However, as the turbulence decays with increasing distance moving away from the grid, the Reynolds number will decrease, and so the character of the turbulence will change as well (Hinze, 1959). Although the characteristics and behavior of isotropic turbulence are well documented and understood, true isotropy is never realized by any turbulent flow. Rather, turbulent flow conditions can be controlled to facilitate a flow that more or less approaches isotropy with a high degree of similarity.

The characteristics of turbulent flows, usually irregular fluctuations in velocity, are observed in all three spatial dimensions (Panton, 1996). While tracking changes in
velocity at a fixed point in a flow, the time history of the fluctuations at first glance resembles a random signal. However, there is structure to these fluctuations; thus, it is not accurate to classify them as random (Panton, 1996).

Irregularities that are observed in the velocity field are frequently envisioned as eddies. These eddies are the building blocks of turbulence, so to speak, and are generated in large and small sizes, often one on top of another and even one inside of the other (Ishimaru, 1978).

By Kolomogorov (1941), variations in the average velocity cause energy to be introduced into turbulence (Ishimaru, 1978). Following Ishimaru (1978), we can estimate

\begin{center}
\begin{tikzpicture}
\draw[->, dashed] (0,0) -- (4,0) node[midway, below] {\(L_0\)} node[below] {outer scale of turbulence};
\draw[->, dashed] (4,0) -- (8,0) node[midway, below] {\(\lambda_0\)} node[below] {inner scale of turbulence};
\end{tikzpicture}
\end{center}

**Figure 2-2 - Turbulent length scales**
eddy sizes as illustrated in Fig. 2.2. For example, the turbulent eddies that are produced in the atmosphere differ from those produced closer the earths surface due to the varying horizontal wind velocities dependence on altitude. Thus, the resulting turbulence will be of a size approximately equal to the determined height. This size is called the outer scale of turbulence and is designated by $L_o$, corresponding to the size at which the energy enters into the turbulence (Ishimaru, 1978). Eddies that are of the order greater than $L_o$ generally are anisotropic. Naturally, eddies of smaller sizes compared to $L_o$ are generally isotropic. From this relationship, we find that the kinetic energy per unit mass per unit time and the energy dissipation per unit mass per unit time is approximately on the order of:

$$\frac{V_o^3}{L_o} \quad (2.1.1)$$

$$\nu \frac{V_o^2}{L_o^2} \quad (2.1.2)$$

where $V_o$ denotes the velocity associated with eddies of the order of $L_o$, and $\nu$ the kinematic viscosity (Ishimaru, 1978). In these cases since the Reynolds number is considerably large, the kinetic energy is sufficiently larger than the dissipative. Physically, the bulk of this kinetic energy may be transferred to smaller eddies. Thus, given velocities $V_1, V_2, \ldots, V_n$ corresponding to eddies of sizes $L_1, L_2, \ldots, L_n$ the kinetic energies per unit mass per unit time for eddies of all sizes can be expressed as:

$$\frac{V_o^3}{L_o} \approx \frac{V_1^3}{L_1} \approx \frac{V_2^3}{L_2} \approx \frac{V_3^3}{L_3} \approx \ldots \approx \frac{V_n^3}{L_n} \quad (2.1.3)$$

As the size of the eddies get smaller and approach the lower limit $l_o$, the dissipation
\( \nu V_o^2 / L_o^2 \) increases until it is on the same order as the kinetic energy dissipation \( \varepsilon \), which can be expressed as:

\[
\frac{V_o^3}{L_o} \cong \frac{V_1^3}{L_1} \cong \cdots \cong \frac{V_{l_i}^3}{l_i} \cong \frac{\nu V_{l_i}^2}{l_i^2} \cong \varepsilon \tag{2.1.4}
\]

At this size \( l_o \), all the energy is dissipated into heat and practically no energy is left for eddies of size smaller than \( l_o \). Thus, this size \( l_o \) is called the inner scale, or Kolmogorov scale, of turbulence (Ishimaru, 1978). In our experiment, the inner turbulent micro scales for the \( \frac{1}{4} \) and \( \frac{1}{2} \) inch grids were approximately 6mm and 12mm respectively. The parameter \( \varepsilon \) present in equation (2.1.4) above denotes the energy dissipation rate. Thus, we determine that for eddies falling within the upper and lower limits \( L_o \) and \( l_o \) respectively, the velocity \( V \) can be related to the energy dissipation rate \( \varepsilon \) by:

\[
V \sim (\varepsilon L)^{1/3} \tag{2.1.5}
\]

From equation (2.1.5), the form of the structure function for the velocity fluctuation can be derived and is expressed as:

\[
D_v(r) = C(\varepsilon r)^{2/3} \tag{2.1.6}
\]

for,

\[
l_o << r << L_o \tag{2.1.7}
\]

where the quantity \( C \) is a dimensionless constant (Andreeva, 2003). Equation (2.1.6) is known as the “two-thirds law” which was first formulated by Kolmogorov and Obukhov.
2.2 Flow Development and Determination of Local Isotropy

As described by Monin & Yaglom (1975), the flow downstream of a grid can be divided into three regions. The first region is nearest the grid, and is the developing region of the flow where the rod wakes merge. The flow is this region is inhomogeneous and anisotropic, consequently producing turbulent kinetic energy. In the second region that follows, the flow is nearly homogenous, isotropic and locally isotropic where there is appreciable energy transfer from one wave number to another. The third and last region of decay encompasses the area furthest downstream from the grid where viscous effects act directly on the largest energy containing scales. These three regions of turbulent generation and decay are important is assessing the applicability of the decay power law.

The decay power law, only valid in the second region as outlined above, and often referred to as the power-law region, is applied to determine quantitatively the expected areas of isotropy downstream of the grid. In an isotropic flow, the skewness of the velocity, as described by:

\[ S(u) = \frac{u^3}{u^2} \]  

has a zero value. Furthermore, Batchelor (1953) showed that the skewness of the velocity derivative, as described by:

\[ S(\partial u/\partial x) = \left( \frac{\partial u/\partial x}{((\partial u/\partial x)^2)^{3/2}} \right)^{3/2} \]  

(2.2.2)
should be a constant in the isotropic region. Thus, the point where the skewness of the velocity derivative becomes constant can be estimated as the position where the flow is locally isotropic.

Perhaps a third indicator of isotropic positions downstream of the grid is achieved through a comparison of the dissipation rate of turbulent kinetic energy (Mohamed, LaRue, 1990). In the region downstream of the grid that is nearly homogeneous and isotropic, the turbulent energy equation is expressed as:

\[
\varepsilon_u^* = -\frac{1}{2} \frac{dq^2}{dt} \tag{2.2.3}
\]

where,

\[
q^2 = (u^2 + v^2 + \bar{w}^2) \tag{2.2.4}
\]

From Taylor’s (1935) hypothesis, and assuming the decay power law region is indeed homogeneous and isotropic we satisfy the following condition, and thus the turbulent energy equation as presented above in Eq. (2.2.3) is rewritten as:

\[
\bar{u}^2 \approx \bar{v}^2 \approx \bar{w}^2 \tag{2.2.5}
\]

\[
\varepsilon_u^* = -\frac{3}{2} U \frac{d\bar{u}^2}{dx} \tag{2.2.6}
\]

where \( d\bar{u}^2/dx \) is computed directly from the plot relating \( \bar{u}^2 \) and \( x/M_u \). However, near the grid this form of the turbulent kinetic energy equation is not realistic introducing considerable error, and thus a third variation of the equation is written as expressed in Eq. (2.2.7) below.
This equation is an independent estimate of the dissipation rate, $\varepsilon_u$, as obtain from the assumption of local isotropy, the measured time derivative of the velocity downstream of the grid, and Taylor’s hypothesis. However, the expression for $\varepsilon_u^*$ as presented in Eq. (2.2.6) is based both on the assumptions of isotropy and homogeneity. Thus, at downstream distances far enough from the grid where the flow is taken to be nearly homogeneous, isotropic and locally isotropic, the ratio of the two dissipation rates as expressed in Eq. (2.2.8), should be nearly unity (Mohamed, LaRue, 1990).

$$\frac{\varepsilon_u}{\varepsilon_u^*} \approx 1$$

Experimentally, regions of isotropy can be determined through hot wire anemometry. A hot-wire probe, can be used to measure the fluctuating velocity, as conducted by Mohamed, et al (1990). The electronic circuitry supplies the applied voltage needed to sustain a constant probe temperature while immersed in the mean flow and subject to cooling by convection. Analytically, there is a relationship between the fluctuation of these bridge voltages over time and the corresponding velocity fluctuations in the flow stream.
The actual hot film sensor that measures the flow velocity is a small cylindrical element, on the order of a few microns, mounted between two stainless steel fingers as illustrated in Fig. 2.3. The sensor becomes one of the resistive elements in a four-corner Wheatstone bridge circuit as shown in Fig. 2.4. The circuit heats the sensor to a constant temperature by adjusting the power dissipated in the sensor such that its resistance, and hence temperature, remains constant.
When the bridge circuit is activated, the applied voltage, called the bridge voltage, is imposed on the top node. As a result, a current flows down both legs of the bridge. If the resistance $R_1$ is higher than the sensor resistance $R_s$, the voltage input into the D-C differential amplifier will be non zero. The amplifier communicates this to the power supply and regulates the bridge voltage until the current passing through the sensor heats it to the exact temperature, and hence resistance, needed to balance the bridge. These adjustments happen very fast, usually on the order of a few microseconds. When a fluid having a temperature lower than the sensor flows over it, the bridge voltage fluctuates to regulate the sensor temperature until it is again constant. The ambient temperature, $T_a$ and the sensor temperature $T_s$, is related to the sensor resistance by Eq. (2.2.9), where $\alpha$ denotes the temperature coefficient of resistance (specific to each sensor).

$$\frac{R_s}{R_a} = 1 + \alpha(T_s - T_a) \quad (2.2.9)$$

The power dissipated in the heated hot film sensor is given by Eq. (2.2.10).

$$P = I^2R_s \quad (2.2.10)$$

Thus, the turbulent intensity, corresponding to the axial orientation of the hotwire probe, can be calculated by Eq. (2.2.11), where the numerator is the RMS (Root Mean Square) value of the velocity fluctuation $u'$, and the denominator is the mean velocity as measured with the anemometer.

$$I_s \equiv \frac{\left(\frac{u_i u_i}{2}\right)^{1/2}}{U_o} \quad (2.2.11)$$
The electrical resistance of the sensor, $R_s$, can be approximated using a calibration curve that demonstrates a monotonically increasing voltage with flow. These curves are generated from the fundamental measurements of bridge voltage and velocity. Figure 2.5, below, shows a typical calibration curve where the bridge voltage is plotted against mass flux (lb/ft$^2$-s).

![Typical calibration curve, bridge voltage vs. mass flux](image)

**Figure 2-5 - Typical calibration curve, bridge voltage vs. mass flux**

Turbulent intensity is determined from a calibration curve, similar to Fig. 2.5, which relates bridge voltage $e_b$ to velocity. From the calibration curve, one would observe that
small fluctuations in flow velocity result in small changes in bridge voltage. Thus, turbulent intensity is usually a small number on the order of a few percent, and the slope of the calibration curve varies very little at operating points where the mean or time average velocity is constant. As such, fluctuations in the instantaneous bridge voltage are related to changes in flow velocity by Eq. (2.2.12), where B denotes the slope of the calibration curve.

\[ e'_b = Bu' \]  

(2.2.12)

Thus, the bridge voltage fluctuations can be related to turbulent intensity by Eq. (2.2.13) below.

\[ I \equiv \frac{\sqrt{e'_b^2}}{B U_o} \]  

(2.2.13)

The numerator is the RMS value of the bridge voltage divided by the slope of the calibration curve at the operating point, and the denominator is the measured steady mean velocity. Thus by Panton (1996), the overall turbulent intensity is defined by Eq. (2.2.14) as the average of the three respective axial turbulent intensities, \( I_x, I_y, \) and \( I_z \) corresponding to probe orientation.

\[ I \equiv \frac{\sqrt{\frac{1}{3} \left( \frac{1}{u_i} \right)^{3/2}}}{U_o} \]  

(2.2.14)
For isotropic turbulence:

\[
\overline{u_1 u_1} = \overline{u_2 u_2} = \overline{u_3 u_3} \tag{2.2.15}
\]

and the turbulence intensity is equal to the axial component turbulent intensities.

\[
I_x = I_y = I_z \equiv I \tag{2.2.16}
\]

Thus, the data obtained through hotwire anemometry for the three probed Cartesian coordinate axes in the flow can be used implicitly to determine regions of isotropy.