A Novel Radio Frequency Coil Design for Breast Cancer Screening in a Magnetic Resonance Imaging System

by

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Abstract

Magnetic Resonance Imaging (MRI) is a widely used soft tissue imaging technique that has gained considerable success because of its sensitivity to several tissue parameters. However, commercially available whole-body imaging systems with large encircling radio frequency (RF) and gradient coils are less efficient when the goal is to obtain detailed, high-resolution images with high specificity and sensitivity from localized regions of the body such as the female breast. This research addresses these problems by proposing a new design in RF coil development for breast cancer screening in a conventional 1.5T MRI system. The new design provides two resonant receiving modes that operate in a quadrature configuration, and a region of interest (ROI) that closely conforms to the shape of the female breast. We adopted an optimum design strategy that combined the analytic Biot-Savart integral equation with the Method of Moment formulation in the development of electromagnetic models and simulation tools. These models were used to analyze the magnetic field distribution and the spatial field coverage, as well as the magnetic field uniformity in the ROI. Results from our analysis were employed in the construction of a highly scalable prototype. The validation of our design strategy is confirmed by comparisons with the commercial Ansoft HFSS v8.5 finite element package.
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Chapter 1

INTRODUCTION

1.1 Medical Imaging and MRI

Modern medical imaging systems rely on the principles and characteristics that govern the interaction between energy, in one form or another and biological tissues. The interaction depends largely on the spatial and material properties of the associated tissues: it can be used to discriminate between the numerous tissues that constitute animal and human anatomies, as well as between healthy and pathological tissues. The form of energy employed, acoustic waves or electromagnetic radiation is a characteristic of the particular imaging modality. For example, acoustic radiation employs ultrasonic sound waves while microwave radiation uses scattered high frequency electromagnetic waves as the intervening form of energy [26].

Magnetic Resonance Imaging (MRI) is one imaging modality that has evolved over the past several years into a powerful and versatile technique capable of providing in-vivo diagnostic images of human and animal anatomies [1]-[4]. MRI belongs to a larger group of imaging techniques that are based on the phenomenon of nuclear magnetic resonance (NMR) [5]. Unlike other image modalities that utilize ionizing radiation as X-ray based Computer Tomography (CT) Scanning, MRI is a noninvasive imaging
technique that does not employ the use of potentially harmful radiation. Furthermore, the only tissue-specific parameter that can interact with X-rays is the electron density, and this does not vary significantly from one soft tissue to another, hence necessitating the need of contrasting agents \[6\], \[7\]. On the other hand, there exists a multitude of tissue-specific properties that affect and interact with MRI signals. The two most significant of these properties are the longitudinal (spin-lattice) relaxation time and the transverse (spin-spin) relaxation time. They cover a broad range of values in normal and pathological tissues \[7\]. In addition, signal acquisition parameters can be manipulated in several ways in order to enhance the contrast of the image generated. Thus, MRI possesses considerably greater diagnostic capabilities when compared with X-ray CT in specific applications, and complements all other in-vivo medical imaging modalities.

### 1.2 Motivation

Breast cancer is a major cause of death among women all over the world and a true emergency for the national healthcare systems. In the United States alone, breast cancer is the second leading cause of female cancer mortality \[8\] and the most frequent cause of death in the age group ranging between 40 and 49 \[9\]. A potentially important strategy for the reduction of breast cancer mortality is the detection of early stage tumors \[10\]. The American Cancer Society reports that 96% of all women survive breast cancer when it is detected in its early stages. Unfortunately, one in four women is unlikely to survive with late detection. Currently, X-ray CT mammography is the most effective and widely used screening modality for detecting clinically occult breast cancers. However, approximately 10-30% of women, who have breast cancer and have undergone mammography, have had negative mammograms \[11\], \[12\]. The significant number of false negatives obtained maybe attributed to the limitations of
mammography in accessing dense glandular tissue and regions located close to the chest wall or underarm, and imaging very early-stage tumors that do not yet exhibit micro calcifications [13]. Another serious concern is the high rate of false positives associated with screening mammograms [12], [14]. These limitations clearly indicate the need for complementary imaging modalities with high sensitivity and specificity for effective early diagnosis.

As reported by a number of researchers, MRI has been proven to be a valuable imaging modality in the detection of breast tumors [15], [16]. However, commercially available whole-body imaging systems with large encircling gradient and radio frequency (RF) coils are less efficient when the goal is to obtain detailed, high-resolution images with high specificity and sensitivity from localized regions of the body such as the female breast. These limitations have been recognized, and have led to the development of a number of highly specialized RF coils [17]-[22], including smaller surface coils in a multi-array configuration which provide better signal-to-noise (S/N) ratio [23], [24]. Particularly, the array coil concept attempts to exploit the use of multiple receiver channels to improve the signal-to-noise ratio. Typical sensitivity ranges between 95% and 98%, while specificity varies widely from 38% to 97% [53].

Unfortunately, these array coil configurations are less efficient due to their fixed wire arrangement that typically cover only a small region of interest (ROI) in the center of the breast, leaving the critical diagnostic areas such as the chest wall and axillary regions uncovered. Therefore, there is a critical need to provide an anatomically shaped RF coil that has a ROI that extends over the entire volume of the breast, and a high signal-to-noise (S/N) to provide sufficient sensitivity and specificity that yields detectable responses such as localized increases in blood volume present during angiogenic activity observed in early tumor growth. Clearly, an RF coil that is able to acquire the MR signals uniformly over the entire breast volume would be desirable. This thesis addresses this need by the design and development of an anatomically
shaped cup-coil configuration featuring microstrip conductors that can be attached to a pendant female breast and interfaced through standard coaxial cables to existing clinical MRI instruments.

1.3 Objective

The objective of this thesis is to design a prototype RF coil for breast cancer screening in a conventional 1.5T MRI system. The intention of the design is to overcome the limitations of spatial field coverage found in most conventional RF coils thereby increasing sensitivity and specificity. The unique design will be made up of cylindrical microstrip structures arranged in such a way as to closely conform to the shape of the breast. Our approach is to develop electromagnetic models that will be used in the analysis of the spatial field coverage and the magnetic field uniformity, as well as the determination of the important self inductance and mutual inductance parameters. Based on the results obtained from these models, we will develop a highly scalable prototype coil that will provide sufficient magnetic field coverage for different breast sizes and different resonance frequencies.

1.4 Thesis Outline

The thesis is divided into seven chapters. Following the introductory chapter, Chapter 2 discusses the basic principles of MRI and MRI systems, including a brief discussion on the role of the RF coil in MRI systems. In Chapter 3, we present the basic design structure and its two modes of operation, followed by a discussion of the design of the receiving interface. Also, we discuss the development of procedures for tuning and matching using L-section networks, and the derivation of expressions for evaluating the quality factor of RF coils. Chapter 4 briefly reviews modeling method used in RF
coil design and investigates the theoretical formulation of various models that were developed. These include the Method of Moments (MoM) model and an analytic Biot-Savart based integral equation model used in the determination of the self and mutual inductances. Chapter 5 outlines the analysis of the different models and the implementation of the design. We present simulation results and compare the results of the various models. We also discuss procedures for the determination of tuning and matching capacitors. In Chapter 6, we outline the construction of the coil prototype and the testing procedure adopted to verify its operation. Results obtained from the testing procedure are also presented. Finally, Chapter 7 concludes with a summary of achievements and future works. Additional details of the RF coil and modeling methods together with procedures developed for tuning and matching appear in the attached cd-rom disc.
Chapter 2

BACKGROUND RESEARCH

2.1 Basic Principles of MRI

Modern quantum mechanics revealed the existence of a property of atomic nuclei known as spin angular momentum. Spin angular momentum is the basis of the magnetic resonance phenomenon and hence magnetic resonance imaging. Magnetic resonance imaging utilizes variations in the spin angular momentum of certain atomic nuclei that constitute biological structures to derive images that contain valuable information concerning the condition of the associated tissue. The variations in spin angular momentum result from interactions with an applied static magnetic field and electromagnetic radiation.

From a classical mechanics point of view, spin angular momentum can be thought of as originating from the motion of elementary charged particles that make up the nucleus of the atoms as they spin around their axis. Positive and negative charged particles can be regarded as spheres of distributed positive or negative charge, while neutral electrical particles such as the neutron can be thought of as a combination of distributed positive and negative charges. Since the particles that constitute the atom have mass, their rotation generates angular momentum. Moreover the motion
of the distributed charge circulating around the axis of the particle will generate a small magnetic field. This magnetic field is called the magnetic moment. As a case in point, let us consider the positively charged particle, the proton. There exists a relationship between the angular momentum and the magnetic moment of the particle [25]. This relationship is given by:

$$\mu = \gamma J$$

(2.1)

where

$\mu =$ magnetic moment in $\text{A} \cdot \text{m}^2$

$J =$ angular momentum in $\text{Kg} \cdot \text{m}^2/\text{s}$

$\gamma =$ gyromagnetic ratio (a characteristic constant of the given nucleus) in MHz/T

The strength of the magnetic moment is a property of the given nucleus and it determines the sensitivity of magnetic resonance. Hydrogen nuclei, containing a single proton, possess the largest magnetic moment, which together with its high concentration in biological tissue make it the nucleus of choice in MRI.

Consider the case of a single hydrogen nucleus in the presence of an applied static magnetic field. Because of the interaction between the magnetic moment of the nucleus and the applied magnetic field, the nucleus will align itself with the applied field in one of two possible states: either with the field (the more probable low energy state also known as the parallel or spin up state) or against the field (the anti-parallel or spin down) state. This is shown in Figure 2.1. The energy difference between the two states is directly proportional to the strength of the applied magnetic field and is given by:

$$\Delta E = 2\mu_z B_0$$

(2.2)
where

\[ \mu_z = z\text{-component of the magnetic moment } \mu \]

\[ B_0 = \text{magnetic flux density of the applied static field} \]

The direction of \( B_0 \) is assumed to be aligned along the \( z \)-axis in this and all other discussions throughout this chapter. It is observed that \( \mu \) does not align completely with, but at an angle \( \theta \).

![Figure 2.1: Orientation of a proton in a static magnetic field.](image)

It should be noted that is a property of the nucleus and its value may be calculated using quantum mechanical considerations. Nuclei can change from one state to another by absorbing or emitting photons with energy equal to the energy difference. From Planck's quantum theory, the frequency of these photons can be found using:

\[ \Delta E = hf \tag{2.3} \]

where \( h \) is Planck’s constant. By substituting 2.3 in 2.2, the frequency of the photons can be determined using:

\[ f = \left[ \frac{2\mu_z}{h} \right] B_0 \tag{2.4} \]

Hence, for a given nucleus, the frequency is directly proportional to the flux density of the applied field. The effect of the applied field \( B_0 \) is the formation of a net
magnetic moment along the $z$-axis and the precession of the nucleus about the $z$-axis. The frequency of precession is called the Larmor frequency which is equal to the frequency of the emitted or absorbed photons as calculated in 2.4. This frequency can be expressed in terms of the gyromagnetic ratio $\gamma$ and is given by:

$$f = \frac{\gamma B_0}{2\pi}$$

(2.5)

The detection of an NMR signal is facilitated by the establishment of a resonance condition [7]. The resonance condition represents a state of alternating absorption and dissipation of energy. Energy absorption is achieved through the application of RF pulses, while energy dissipation is caused by relaxation processes; both transverse and longitudinal relaxation. Consider the application of RF radiation at the Larmor frequency to a bulk sample of non-magnetic material in an applied static magnetic field. This is depicted in Figure 2.3 below. The applied RF radiation is composed of coupled electric and magnetic field components. The magnetic field component is denoted as $B_1$. In Figure 2.3 and all MRI discussion, $B_1$ resides in a plane perpendicular to $B_0$ and precesses about $B_0$ at the Larmor frequency. Upon application of the RF radiation, the net magnetization $M$ starts to rotate about the axis of $B_1$. Since $B_1$ and $M$ are rotating about $B_0$ at the Larmor frequency, they appear stationary relative to each other. This is also depicted in Figure 2.3. The effect of the application
Figure 2.3: Precession of net magnetization $\mathbf{M}$ resulting from the application of RF radiation denoted by vector $\mathbf{B}_1$.

of $\mathbf{B}_1$ is to rotate $\mathbf{M}$ by a certain angle away from the $\mathbf{B}_0$ axis. This angle is called the flip angle. In brief, the flip angle is directly proportional to the duration of the applied RF radiation. Hence, if $\mathbf{B}_1$ persists for the right duration of time, $\mathbf{M}$ can be made to rotate onto the transverse plane. This is shown in Figure 2.4. While in the transverse plane, rotating at the Larmor frequency, $\mathbf{M}$ will induce an NMR signal in the RF receiver coil which is oriented in the transverse plane. This current can be used to observe the characteristics of $\mathbf{M}$ in the transverse plane. The RF radiation that brings $\mathbf{M}$ unto the transverse plane is usually referred to as the 90°C pulse.

In MRI, 90°C and 180°C RF pulses are commonly used. The 90°C and 180°C refers to the resulting flip angle after the application of the RF radiation. The 90°C flip angle is very important because the strongest NMR signal is obtained when $\mathbf{M}$ is on the transverse

Figure 2.4: Helical rotation of $\mathbf{M}$ on the transverse plane due to the application of a 90°C RF pulse.
plane. The $180^\circ$ flip angle is primary important in spin-echo imaging techniques where it is used to reverse the direction of $\mathbf{M}$. Once in the transverse plane, $\mathbf{M}$ exponentially decays to zero. This gradual decay is termed transverse or $T_2$ relaxation, where $T_2$ refers to the time constant of the decay process and is dependent on the characteristics of the sample. As $\mathbf{M}$ decays to zero in the transverse plane, there is an exponential build up of the net magnetization along the $\mathbf{B}_0$ axis. This is termed longitudinal or $T_1$ relaxation and it is also material dependent. Hence, after the application of a $90^\circ$ RF pulse, there is a steady decay of transverse magnetization and a build up of longitudinal magnetization. Eventually, the transverse magnetization would decay to zero while the longitudinal magnetization would return to its maximum value of $\mathbf{M}$ aligned with $\mathbf{B}_0$. The decay of transverse magnetization is due to the lost in synchronization of the precessing nuclei that make up the sample. This is known as dephasing in the transverse plane [7]. Since the signal observed is the sum of all the transverse components, sufficient dephasing will eventually lead to complete signal cancellation. Dephasing is mainly due to the inhomogeneity of the $\mathbf{B}_0$ field and the mutual interactions between the magnetic moments of atoms and molecules that constitute the sample. Dephasing due to $\mathbf{B}_0$ field inhomogeneity can be reversed by the application of $180^\circ$ RF pulses. Transverse relaxation that is reversible by the application of $180^\circ$ RF pulses is assigned the time constant $T_2^*$, and the re-appearance of the NMR signal after the $180^\circ$ pulse is called a spin echo. The build up of longitudinal magnetization is inevitable and it is due to the persistent $\mathbf{B}_0$ field.

In MRI image acquisition, there is a need to introduce spatial variations in the received NMR signal. This is necessary in order to distinguish between signals from several regions of the sample. Spatial localization is achieved by the application of a linearly varying gradient magnetic field that modifies the main $\mathbf{B}_0$ magnetic field in the ROI containing the sample. According to 2.4 and 2.5, this variation of $\mathbf{B}_0$ would cause a variation in the precession frequencies of the different nuclei that constitute the
sample. Then, the detected NMR signal would be an interference combination of different precession frequencies from different spatial locations within the sample. These frequencies can then be separated in the frequency domain using the Fourier Transform. Hence by using a set of three orthogonal gradient magnetic fields along the three physical spatial axes (x, y and z axes) the signal from a slice of the sample in the ROI can be acquired as a two-dimensional image. Important slice characteristics are its thickness, field of view or ROI and its resolution. The image contrast can be determined by $T_1$, $T_2$, $T_2^*$, proton density, diffusion or a combination of these factors. Other contrasting schemes are available and most of them are obtained by processing images obtained using slightly different parameters or different times. Indeed, MRI has the most diverse selection of contrasting schemes of all medical imaging modalities. One such contrasting scheme relies on the element Gadolinium (Gd) to exert a strong influence on the magnetic moment of the hydrogen nuclei in the vicinity of a tumor [26].

2.2 The MRI System

The basic hardware components of an MRI System are illustrated in Figure 2.5. These components include a main magnet, a set of gradient coils, RF coils (both transmitter and receiver) and a computer system. The main magnet produces the primary magnetic field, the $B_0$ field, over the ROI. $B_0$ fields of 1.5T are common in the medical imaging field, while fields as high as 19T exist in research systems. It is desirable that the primary magnetic field be uniform throughout the desired ROI.

The main magnet is usually a solenoid-type electromagnet with a cylindrical bore. Such high-field magnets are almost exclusively superconducting. The superconductors are cooled to a temperature near absolute zero by using liquid helium and liquid nitrogen combination. This ensures that the superconductor retains its superconduct-
Figure 2.5: Block diagram of a generic MRI System [5].
ing properties. The gradient coil system consists of a set of three orthogonal coils that provide the orthogonal field gradients along the $x$, $y$ and $z$ axes. By superimposing these field gradients on the main $B_0$ field, layered selection and spatial encoding are realized.

Strong field gradients that are highly uniform in the ROI are desirable in order to minimize image distortions. Besides producing strong uniform field gradients, the gradient coils must be able to switch on and off rapidly in order to avoid image obscurity due to motion effects that occur in real-time imaging such as the cardiac cycle. Hence, gradient coils are optimized for low inductance. The RF coil produces the high-frequency homogeneous electromagnetic field, the $B_1$ field, necessary to excite the nuclei of the sample being imaged into coherent precession. By reciprocity, the coil can be used to couple emitted RF energy from the nuclei into an external circuitry. A single RF coil can be employed as a transmitter and a receiver, or separate coils can be provided for transmitting and receiving. The $B_1$ field generated by the RF transmit coil must be uniform across the entire ROI. The sensitivity of the RF receiver coil is important to obtain a high image signal-to-noise ratio, but it does not necessarily have to be uniform in the ROI. The gradient amplifier, gradient pulse generator, RF amplifier, RF pulse generator, RF receiver and digitizer constitute the drive electronics for the MRI system. The various amplifiers are typically housed separately, and drive all the coils in the MRI system. The computer system sets up the pulse sequences, controls all the coils drivers, and reconstructs the image for display.

### 2.3 RF Coils

Almost all RF coils are designed using the concept of resonance. Resonance ensures selectivity and rudimentary signal amplification. RF coils possess an intrinsic inductance due mainly to the spatial arrangement of the conductors that make up the coil,
and an inherent parasitic capacitance. Additional capacitive elements are required to establish the resonance condition at the desired operating frequency. When the coil resonates at the desired frequency, the Larmor frequency, large voltage and current oscillations are developed from the application of a small input signal. The strength of the resonance is described by its quality factor $Q$, and is defined as

$$Q = 2\pi f \cdot \frac{\text{Maximum stored energy}}{\text{Energy dissipated per cycle}}$$

(2.6)

The quality factor is affected by such parameters as the strength of the $B_1$ field produced, image signal-to-noise ratio, and the sensitivity of the RF coil. Although the main magnet plays a crucial role in the determination of MRI image quality, improved RF coil design techniques may provide greater image quality enhancements than an improvement in the field strength of the main magnet. RF coils are generally categorized as volume coils or surface coils. Volume coils can be used for transmitting or receiving RF signals since they provide a relatively homogeneous $B_1$ field in the ROI. They generally enclose the entire ROI in order to achieve high field uniformity. Examples of volumes coils are the Bird Cage coil, the Saddle coil [5], and the TEM resonator [38]. Surface coils on the other hand are primarily used as receive-only coils because of their poor field uniformity [5]. They only cover a small region of the imaging volume and hence, possess a high signal-to-noise ratio compared with volume coils. The field strength is very high in the region closest to the coil, but it drops off very rapidly as the distance from the coil increases. Examples of MRI surface RF coil designs include: single-loop coils, multi-loop coils [5] and phased array coils [27]. In some MRI systems, a dual-coil configuration may exist where the volume coil is used as the RF transmit coil and the surface coil is used as the RF receive coil. Such a configuration benefits from the uniformity of the $B_1$ field produced by the volume coils, and the high sensitivity of the surface coils resulting in
improved image quality. The main design considerations in the development of RF coils include desired operating frequency, volume of the ROI, uniformity of the B_1 field produced, filling factor, and Q or coil loss [28]. According to 2.5, the desired operating frequency is determined by the strength of the B_0 field and the gyromagnetic ratio of the given nucleus. Hydrogen nuclei are of primary importance in MRI because of its abundance in biological tissue. From 2.5, \( \gamma/2\pi = 42.58 \text{ MHz/T} \) for the Hydrogen nuclei. For most RF coils, as the desired ROI increases, the dimensions of the RF coil increase resulting in an increase in the inductance of the coil. The upper bound of the operating frequency is determined primarily by the inherent inductance and stray or parasitic capacitance of the coil. Hence, there exists a strong relationship between the desired operating frequency and the size of the ROI [29]. The filling factor defines how well the RF coil encloses the sample in the region of interest. For optimum signal quality, it is imperative that the RF coil covers as much volume of the sample as necessary. Hence, most coil designs incorporate a ROI that is anatomically shaped and closely encloses the biological sample. The RF field homogeneity is determined primarily by the spatial arrangement of the conductors that constitute the RF coil. The field homogeneity can also be improved upon by enlarging the dimensions of the RF coil. This technique has the disadvantage of increased coil losses and decreased filling factor.
Chapter 3

THEORETICAL
CONSIDERATION

3.1 Basic Coil Structure

The basic structure of the proposed RF coil is shown below in Figure 3.1. The structure features an anatomically correct cup-coil configuration that can be realized using microstrip lines. The coil can be operated in a single or dual coil system configuration. However, in order to provide the highest signal-to-noise ratio, and hence the highest spatial resolution, the design will focus primarily on the development of a single receiver coil system configuration.

The coil system configuration is composed of the base ring and an interconnecting upper strap as shown in Figure 3.1. The base ring and the upper strap are essentially cylindrical structures although the upper strap can also be defined as a frustum of a hemisphere. Together, the base ring and the upper strap are characterized by a radius $r$ and a width $w$ that define a particular volume that closely matches the anatomy of the female breast. The unique design of the RF coil allows for an optimum filling
factor, adequate B$_1$ field coverage, and an enhanced signal-to-noise ratio. Figure 3.2 shows an artistic view of the coil operating in a single coil system configuration and attached to a pendant female breast.

The cylindrical design of the base ring guarantees optimal contact with the chest wall and thus ensures good horizontal magnetic field penetration. This enables optimum sensitivity in the diagnostically important lymph node region of the breast in the area of the axilla. Lymph nodes play an important role in the spread of breast cancer. The axillary lymph nodes are particularly important, as they are among the first places that cancer is likely to be found if it metastasizes from the breast [30]. This lymph node cluster is often referred to as the Tail or level I nodes [30]. Level II nodes are
located underneath the pectoralis minor muscle, and level III nodes are found near the center of the collarbone [30]. Ideally, breast screening coils should exhibit good coverage into the axillary lymph nodes chain. Figure 3.3 shows the anatomy of the female breast with the lymph node chains.

3.2 Basic Modes of Operation

The RF coil will provide two basic resonant receiving modes that can be operated in a quadrature configuration. The modes are defined as a result of the segmented design of the coil: they yield two resonant current paths defined by the base ring and the upper strap. This is shown in Figure 3.4 and Figure 3.5. The first resonant mode, Mode 0, is defined by a cylindrical current flow in the base ring only, while the second resonant mode, Mode 1, is defined by splitting the current flowing in the upper strap evenly between the two halves of the base ring. Mode 0 and Mode 1 are configured such that they operate in quadrature resulting in a 90° phase shift between the current flows in the individual paths. The superposition of both modes establishes a rotating magnetic field phasor whose direction is orthogonal to the main magnetic field, $B_0$, which is oriented along the $z$-axis as shown in Figure 3.6.
Figure 3.4: Mode 0 is defined by a circular current flow on the base ring only.

Figure 3.5: Mode 1 is defined by splitting the strap current evenly between the two halves of the base ring.

Figure 3.6: Quadrature superposition of Mode 0 and Mode 1.


3.3 Receiver Interface and Setup

The receiver interface system is composed of tuning and matching components, a \(180^\circ\) hybrid, two optional preamplifiers, and an optional \(90^\circ\) quadrature hybrid. The tuning and matching components constitute an essential part in any RF coil system: they enable the RF coil to resonate at the Larmor frequency and they match the impedance of the coil to the impedance of the external coupling network that interfaces to the receiver system. The \(180^\circ\) hybrid is a reciprocal four-port network device \([32]\) that provides two equal amplitude in-phase output signals when fed from its sum port \(\Sigma\) and two equal amplitude \(180^\circ\) out-of-phase output signals when fed from its difference port \(\Delta\). The scattering matrix of the \(180^\circ\) hybrid is shown in Figure 3.7(b). Since the \(180^\circ\) hybrid is a reciprocal device, feeding it at the two output ports will produce a sum and a difference signal at the \(\Sigma\) and the \(\Delta\) ports. The signals at the \(\Sigma\) and the \(\Delta\) ports will thus provide information about the two modes of operation. The optional preamplifier is a low noise amplifier (LNA) that prevents the weak received signal from being dominated by noise as it travels down the transmission line to the receiver system. The \(90^\circ\) quadrature hybrid is a 3dB directional coupler with a \(90^\circ\) phase shift in the output of the through and coupled arms. This can be seen by examining its scattering matrix shown in Figure 3.7(a). It is often made in microstrip or stripline form, and is also known as a branch-line hybrid. It is an optional component of the receiver interface. It is required when there is only one receiving channel in the receiver system. In that case, its function would be to combine the signals from the two preamplifiers in quadrature. Figure 3.7 shows a block schematic of the receiving system. There are two ports on the base ring with a common signal ground. The ports serve to couple the RF signal to an external circuit. It will be shown later that the two modes of operation can be achieved simultaneously if the voltages at both ports are in quadrature.
3.4 Tuning and Matching

Tuning and matching of the coil is achieved using lumped element capacitors in a two-element or so-called L-section network configuration. All RF coils possess an inherent inductance that is due to the spatial distribution of their conductors around the ROI, and also because of the size of the conductors. RF coils are highly efficient when they operate at their resonance frequencies. The resonance phenomenon guarantees some form of rudimentary signal amplification. Tuning enables the RF coil to resonate at the desired Larmor frequency. The result is that the typically weak NMR signal will equate to large signal changes in the RF coil. This large signal, although still not strong enough, can further be amplified and processed in order to reveal important information about the properties of the sample.
Matching is an important requirement for the design of RF circuits. It guarantees the transfer of maximum power from the signal source to the load. According to the maximum power transfer or conjugate matching theorem, maximum power is transferred from the signal source if, and only if, the input impedance presented to the signal source is equal to the complex conjugate of the source impedance. A proof of the conjugate matching theorem is given in [32] and [33]. It should be noted that conjugate matching does not necessarily yield a system with the best efficiency. As an example, consider the case where the source impedance is real and the input impedance presented to the source is also real. In this case, the load and the generator are matched and there are no reflections on the transmission line. But only half the power generated is delivered to the load yielding an efficiency of 50%. Hence, the efficiency of the system can only be improved by making the source impedance as small as possible. The L-section network is the simplest type of narrow band matching network. It uses two lumped reactive elements to match arbitrary load impedances to a transmission line. There exist two possible configurations for L-section networks. They are shown in Figure 3.8 and Figure 3.9, while Figure 3.10 shows a typical ZY Smith Chart. The characteristic impedance of the transmission line is denoted by $Z_0$ while the load impedance is denoted by $Z_L$. The normalized impedance $z_L$ is given as

$$z_L = \frac{Z_L}{Z_0} \quad (3.1)$$

The two lumped reactive elements are denoted by $jB$ and $jX$. In each configuration, the lumped reactive elements could be either inductors or capacitors, depending on the load impedance $Z_L$. Thus, there exist eight distinct possibilities for L-section matching using a combination of two lumped element capacitors or inductors.

Let us consider the case of matching the RF coil with an impedance of $Z_L$ to the source transmission line with a real characteristic impedance $Z_0$ such that $Z_0 = Z_0^*$. 


Figure 3.8: L-section network used when $z_L$ lies inside the $r = 1$ constant resistance circle in the Smith Chart.

Figure 3.9: L-section network used when $z_L$ lies outside the $r = 1$ constant resistance circle in the Smith Chart.

The coil impedance is given as $Z_L = R_L + jX_L$. Here, $R_L$ denotes the resistance of the coil and $X_L$ denotes the reactance of the coil. The normalized coil impedance is given by

$$z_L = \frac{Z_L}{Z_0} = \frac{R_L}{Z_0} + \frac{jX_L}{Z_0} = r + jx_L$$  \hspace{1cm} (3.2)

Let the normalized impedance $z_L$ reside inside the $r = 1$ circle as seen in Figure 3.10. This situation is depicted in Figure 3.11 based on the topology of the L-section.

Figure 3.10: A typical ZY Smith Chart with circles of constant resistance (right) and constant conductance (left).
matching network shown in Figure 3.8. It should be noted that $z_L$ lies within the $r = 1$ circle if and only if $r > 1$. As can be seen from Figure 3.11, a constant conductance circle will always pass through $z_L$ as long it resides within the $r = 1$ circle. The constant conductance circle will intersect the $r = 1$ circle at two points with normalized impedances of $z_A$ and $z_B$ as depicted in Figure 3.11. These normalized impedances are complex conjugates of each other and are of the form $z_{A,B} = 1 \pm j\Delta$, where the + sign refers to $z_A$ and the - sign refers to $z_B$, and $\Delta$ is a normalized reactance.

![Constant conductance circle through $z_L$](image)

*Figure 3.11:* Graphical approach for designing an L-section matching network when $z_L$ lies within the $r = 1$ circle.

The normalized coil admittance $y_L$ is given as

$$y_L = \frac{1}{z_L} = \frac{1}{r + jx_L} = \frac{r - jx_L}{r^2 + x_L^2} = \alpha - j\beta$$

while the normalized admittances of the intersection points on the $r = 1$ circle can be written as

$$y_{A,B} = \frac{1}{1 \pm j\Delta} = \frac{1 \pm j\Delta}{1 + \Delta^2}$$

Since $y_{A,B}$ and $y_L$ reside on the same conductance circle, they must have equal real
components, or equal conductance components. Thus, we have that

\[ \alpha = \frac{1}{1 + \Delta^2} \]  

(3.5)

and solving for \( \Delta \) gives

\[ \Delta = \sqrt{\frac{1}{\alpha} - 1} \]  

(3.6)

Hence, the addition of an admittance component in parallel with the RF coil of value \( \frac{\pm j\Delta}{1 + \Delta^2} + j\beta \) will move the impedance of the coil onto the \( r = 1 \) circle. This value of reactance is the difference between the reactive components of \( y_A, y_B \) and \( y_L \). Once residing on the \( r = 1 \) circle, a series component of value \( \pm j\Delta \) will move the intersection points to the center of the Smith Chart. Thus, matching is achieved using two lumped components. With reference to Figure 3.8, we have that

\[ jB = \frac{-jZ_0}{\left(\frac{\pm \Delta}{1 + \Delta^2} + \beta\right)} \]  

(3.7)

and

\[ jX = \pm jZ_0 \cdot \Delta \]  

(3.8)

For the case where the normalized coil impedance does not lie in the \( r = 1 \) circle, a constant conductance or constant resistance circle or both can pass through \( z_L \) depending on whether \( z_L \) is in the \( g = 1 \) circle or not. If \( z_L \) lies within the \( g = 1 \) circle, only a constant resistance circle will pass through it. On the other hand, if \( z_L \) does not lie within the \( g = 1 \) circle, then a constant conductance or constant resistance circle can be drawn to pass through it. This is shown in Figure 3.12 and Figure 3.13. We now consider the case where a constant resistance circle passes through \( z_L \) and intersects the \( g = 1 \) circle. The admittance \( y_A \) and \( y_B \) at the points of intersection are a conjugate pair of the form \( 1 \pm jK \). The corresponding impedances
Figure 3.12: Graphical approach for designing an L-section matching network when $z_L$ lies outside the $r = 1$ and $g = 1$ circles.

Figure 3.13: Graphical approach for designing an L-section matching network when $z_L$ lies within the $g = 1$ circle.

$z_{A,B}$ are given by

$$z_{A,B} = \frac{1}{1 \pm jK} = \frac{1 \pm jK}{1 + K^2}$$ (3.9)

Since $z_L$ and $z_{A,B}$ reside on the same resistance circle, they must have equal real components. Thus,

$$r = \frac{1}{1 + K^2}$$ (3.10)

yielding

$$K = \sqrt{\frac{1}{r} - 1}$$ (3.11)

Hence, the addition of an impedance component of value $\frac{\pm jK}{1 + K^2} - jx_L$, which is the
difference between the reactive components of $z_{A,B}$ and $z_L$, will move the impedance of the coil onto the $g = 1$ circle. Once $z_L$ is on the circle, the addition of an admittance component of value $\pm jK$ will move the intersection points $y_A$ and $y_B$ to the center of the Smith Chart. Hence, with reference to Figure 3.9, we have

$$jX = jZ_0 \cdot \left( \frac{\pm K}{1 + K^2} - x_L \right) \quad (3.12)$$

$$jB = \frac{\pm jZ_0}{K} \quad (3.13)$$

The sign of $jB$ and $jX$ determine if a capacitor or an inductor is required in the matching network. A negative value indicates a capacitive component while a positive value indicates an inductive component. We will use a capacitive only solution for the implementation of the network because capacitors have lower losses and smaller physical dimensions when compared with inductors. A Matlab procedure was developed specifically to determine the capacitive only solutions and the values of the capacitors for arbitrary values of resonance frequency, coil impedance and external characteristic impedance. The procedure is available in the attached cd-rom.

### 3.5 Quality Factor Determination

We will now show that the quality factor $Q$ of an RF coil can be determined from its half-power fractional bandwidth. Let us consider the series RLC resonant circuit model of the RF coil system shown in Figure 3.14. The coil system is made up of a resistance $R$, an inductance $L$ and a tuning capacitance $C$. The input impedance $Z_{in}$ of the system is

$$Z_{in} = R + j\omega L + \frac{1}{j\omega C} = R + j\omega L - j \frac{1}{\omega C} \quad (3.14)$$
while the power $P_{in}$ delivered to the system from the external source is

$$P_{in} = \frac{V}{\sqrt{2}} \cdot \frac{I^*}{\sqrt{2}} = \frac{1}{2} |I|^2 |Z_{in}| \quad (3.15)$$

Where $V$ and $I$ represent the sinusoidal voltage and current amplitudes associated with the external source. The power loss $P_{loss}$ resulting from power dissipation in the resistance $R$ is given as

$$P_{loss} = \frac{I}{\sqrt{2}} \cdot \frac{I^*}{\sqrt{2}} \cdot R = \frac{1}{2} |I|^2 |R| \quad (3.16)$$

The magnetic energy $E_m$ stored in the inductance $L$ and the electric energy $E_e$ stored in the capacitance $C$ can be written as

$$E_m = \frac{1}{2} \cdot \frac{I}{\sqrt{2}} \cdot \frac{I^*}{\sqrt{2}} \cdot L = \frac{1}{4} |I|^2 |L| \quad (3.17)$$

$$E_e = \frac{1}{2} \cdot C \cdot \frac{V_c}{\sqrt{2}} \cdot \frac{V_c^*}{\sqrt{2}} = \frac{1}{4} C |V_c|^2 = \frac{1}{4} C \frac{|I|^2}{\omega^2 C} = \frac{1}{4} |I|^2 \frac{1}{\omega^2 C} \quad (3.18)$$

where $V_c$ represents the voltage phasor across the capacitance $C$. Substituting 3.14 in 3.15 gives

$$P_{in} = \frac{1}{2} |I|^2 \left( R + j \omega L - j \frac{1}{\omega C} \right) = \frac{1}{2} |I|^2 |R| + 2 j \omega \left( \frac{1}{4} |I|^2 |L| - \frac{1}{4} |I|^2 \frac{1}{\omega^2 C} \right)$$

$$= P_{loss} + 2 j \omega (E_m - E_e) \quad (3.19)$$
The RF coil exhibits resonance when the stored magnetic energy is equal to the stored electric energy, i.e. when $E_m = E_e$. Equating 3.17 with 3.18, we obtain the resonance frequency $\omega_r$ as

$$\omega_r = \frac{1}{\sqrt{LC}}$$  \hspace{1cm} (3.20)

Substituting $\omega_r$ in 3.20 for $\omega$ in 3.14 gives the impedance at resonance as

$$Z_{in} = R$$

The quality factor $Q$ of the coil can be determined using 2.6 as

$$Q = 2\pi f \frac{\text{Maximum stored energy}}{\text{Energy dissipated per cycle}} = \omega \frac{E_m + E_e}{P_{loss}}$$  \hspace{1cm} (3.21)

Since $E_m = E_e$ at resonance, we have that

$$Q = \omega_r \frac{2E_m}{P_{loss}} = \omega_r \frac{2E_e}{P_{loss}} = \omega_r \frac{L}{R} = \omega_r \frac{1}{RC}$$  \hspace{1cm} (3.22)

3.22 indicates that $Q$ increases as $R$ decreases. We now investigate the behavior of $Z_{in}$ near the resonance frequency $\omega_r$. We define $\omega = \omega_r + \Delta \omega$, where $\Delta \omega$ is a small change in frequency near $\omega_r$. $Z_{in}$ can be rewritten using 3.14 as

$$Z_{in} = R + j\omega L - j \frac{1}{\omega C} = R + j\omega L \left(1 - \frac{1}{\omega^2 LC}\right)$$

$$= R + j\omega L \left(\frac{\omega^2 - \omega_r^2}{\omega^2}\right)$$  \hspace{1cm} (3.23)

But $\omega^2 - \omega_r^2 = (\omega - \omega_r)(\omega + \omega_r) = (2\omega - \omega_r)\Delta \omega \approx 2\omega_0 \Delta \omega$ for a small change $\Delta \omega$ in frequency. Thus, using 3.23 and the fact that $L = QR/\omega_r$ from 3.22, we have

$$Z_{in} \approx R + 2jL\Delta \omega \approx R + j \frac{2QR\Delta \omega}{\omega_r}$$  \hspace{1cm} (3.24)
Now consider Figure 3.15 which shows the variation of the magnitude of the input impedance $Z_{in}$ versus the normalized frequency. Let $|Z_{in}^2 = 2R^2|$ at frequency $\omega_1$ and $\omega_2$. The quantity $(\omega_2 - \omega_1)/\omega_r$ is referred to as the half-power fractional bandwidth. If $\omega = \omega_2$ then

$$\frac{\Delta \omega}{\omega_r} = \frac{\omega_2 - \omega_r}{\omega_r} = \frac{1}{2} \frac{\omega_2 - \omega_1}{\omega_r}$$

This can be deduced from Figure 3.15 since $\omega_1$ and $\omega_2$ are symmetrical about $\omega_r$.

Using (3.24) gives

$$|R + jRQ \cdot \frac{\omega_2 - \omega_1}{\omega_r}| \approx 2R^2$$

or

$$\frac{\omega_2 - \omega_1}{\omega_r} \approx \frac{1}{Q}$$

(3.25)

Thus, the $Q$ of the coil system can be express as the inverse of its half-power fractional bandwidth giving

$$Q \approx \frac{\omega_r}{\omega_2 - \omega_1} \approx \frac{f_r}{f_2 - f_1}$$

(3.26)

A similar derivation of $Q$ can also be made for a parallel $RLC$ resonance circuit model of the RF coil system. Consider the parallel $RLC$ resonance circuit model shown in Figure 3.16. The variation of the magnitude of its input impedance with normalized frequency is shown in Figure 3.17. The input impedance $Z_{in}$ of the coil is now given

![Figure 3.15: Input impedance magnitude versus normalized frequency of the series $RLC$ resonance circuit model.](image-url)
Figure 3.16: A simple parallel RLC resonance circuit model.

Figure 3.17: Input impedance magnitude versus normalized frequency of the parallel RLC resonance circuit model.

by

\[ Z_{in} = \left( \frac{1}{R} + \frac{1}{j\omega L} + j\omega C \right)^{-1} \]  

(3.27)

and the power conveyed to the coil system from the extern source follows from

\[
P_{in} = \frac{V}{\sqrt{2}} \cdot \frac{I^*}{\sqrt{2}} = \frac{1}{2} |V^2| \frac{1}{Z_{in}}
\]

\[
= \frac{1}{2} |V^2| \left( \frac{1}{R} + \frac{1}{j\omega L} + j\omega C \right)
\]  

(3.28)

The power loss \( P_{loss} \) resulting from power dissipation in the resistance \( R \) is

\[
P_{loss} = \frac{1}{2} |V^2| \frac{1}{R}
\]  

(3.29)

The magnetic energy \( E_m \) stored in the inductance \( L \) and the electric energy \( E_e \) stored
in the capacitance $C$ can be written as

\[
E_m = \frac{1}{2} \cdot \frac{I_L}{\sqrt{2}} \cdot \frac{I_L^*}{\sqrt{2}} \cdot L = \frac{1}{4} |I_L^2| L = \frac{1}{4} |V^2| \frac{1}{\omega^2 L^2} L
\]

and

\[
E_e = \frac{1}{2} C \cdot \frac{V}{\sqrt{2}} \cdot \frac{V^*}{\sqrt{2}} = \frac{1}{4} C |V^2|
\]

where $I_L$ denotes the current flowing through the inductance $L$. Equating 3.30 and 3.31, we obtain the resonance frequency $\omega_r$ as

\[
\omega_r = \frac{1}{\sqrt{LC}}
\]

Substituting 3.32 for $\omega$ in 3.27 gives the impedance at resonance as

\[
Z_{in} = R
\]

Thus, the resonance frequency $\omega_r$ and the input impedance at resonance $Z_{in}$ are identical for both the series and parallel $RLC$ resonance circuit models. From the definition of $Q$ in 3.21 and the fact that $E_m = E_e$ at resonance, we can express $Q$ at resonance as

\[
Q = \omega_r \frac{2E_m}{P_{loss}} = \omega_r \frac{2E_e}{P_{loss}} = \omega_r \frac{R}{L} = \omega_r RC
\]

Thus, $Q$ is directly proportional to $R$. We now investigate the behavior of $Z_{in}$ near $\omega_r$. As before, we define $\omega = \omega_r + \Delta \omega$, where $\Delta \omega$ is a small change in frequency near $\omega_r$. Substituting $\omega_r + \Delta \omega$ for $\omega$ in 3.27 gives $Z_{in}$ as

\[
Z_{in} = \left( \frac{1}{R} + \frac{1}{j(\omega_r + \Delta \omega)L + j(\omega_r + \Delta \omega)C} \right)^{-1}
\]
\[
Z_{\text{in}} \approx \left( \frac{1}{R} + \frac{1}{j(1+\Delta \omega/\omega_r)\omega_r L} + j\omega_r C + j\Delta \omega C \right)^{-1}
\]  \hspace{1cm} (3.34)

If $\Delta \omega \ll \omega_r$, then we can simplify $(1 + \Delta \omega/\omega_r)^{-1}$ using a binomial series expansion as

\[
(1 + \Delta \omega/\omega_r)^{-1} \approx 1 - \Delta \omega/\omega_r + \cdots
\]  \hspace{1cm} (3.35)

Hence, 3.34 reduces to

\[
\begin{align*}
Z_{\text{in}} & \approx \left( \frac{1}{R} + \frac{1 - \Delta \omega/\omega_r}{j\omega_r L} + j\omega_r C + j\Delta \omega C \right)^{-1} \\
& \approx \left( \frac{1}{R} + \frac{\Delta \omega}{j\omega_r^2 L} + j\omega_r C + j\Delta \omega C \right)^{-1} \\
& \approx \left( \frac{1}{R} + 2j\Delta \omega C \right)^{-1} \approx \frac{R}{1 + 2j\Delta \omega RC} \\
& \approx \frac{R}{1 + 2jQ \Delta \omega/\omega_r}
\end{align*}
\]  \hspace{1cm} (3.36)

Finally, let us consider the half-power fractional bandwidth $(\omega_2 - \omega_1)/\omega_r$ of the coil system. If $\omega = \omega_2$ then

\[
\Delta \omega/\omega_r = \frac{\omega_2 - \omega_r}{\omega_r} = \frac{1}{2} \frac{\omega_2 - \omega_1}{\omega_r}
\]

Also, let $|Z_{\text{in}}|^2 = R^2/2$ when $\omega$ is equal to $\omega_1$ or $\omega_2$. Using this in 3.36 gives

\[
\left| \frac{R}{1 + jQ \frac{\omega_2 - \omega_1}{\omega_r}} \right|^2 \approx \frac{R^2}{2}
\]

or

\[
\frac{\omega_2 - \omega_1}{\omega_r} \approx \frac{1}{Q}
\]  \hspace{1cm} (3.37)

Thus, the $Q$ of the system can be expressed as

\[
Q \approx \frac{\omega_r}{\omega_2 - \omega_1} \approx \frac{f_r}{f_2 - f_1}
\]  \hspace{1cm} (3.38)
The quality factor $Q$ defined in 2.6 and 3.38 are referred to as the unloaded $Q$ of the system and it is a characteristic of the particular RF coil system. It is termed unloaded because it does not take into account any loading effects caused by coupling to an external network. Invariably, RF coils are always coupled to some external circuit and this has the effect of lowering the value of the unloaded $Q$. When loading effects are taken into consideration, the resulting $Q$ is referred to as the loaded $Q$, $Q_{LD}$. Figure 3.18 and Figure 3.19 are examples of loaded series and parallel $RLC$ resonance circuit models. The resistance $R_L$ is the resistance introduced into the system by the external coupling network.

As can be seen from Figure 3.18, the resistance $R_L$ adds in series with the coil resistance $R$ resulting in an effective resistance of $R + R_L$ in 3.22. Figure 3.19 reveals that $R_L$ adds in parallel with $R$ resulting in an effective resistance of $RR_L/(R + R_L)$ in 3.33.
Now, if we define an external $Q, Q_E$ [32] as

$$Q_E = \begin{cases} \frac{\omega R}{L} & \text{for a series RLC resonance model} \\ \frac{R L}{\omega L} & \text{for a parallel RLC resonance model} \end{cases}$$

then $Q_{LD}$, can be expressed as

$$\frac{1}{Q_{LD}} = \frac{1}{Q} + \frac{1}{Q_E} \quad (3.39)$$
Chapter 4

THEORETICAL MODEL FORMULATION

4.1 Modeling Methods for RF Coils

Several numerical and analytical methods are available for modeling and analyzing the electrical and magnetic characteristics of RF coils. Each method offers a variety of capabilities with various degrees of accuracy. The lumped-element circuit model [37], [51] is one of several modeling methods that can be used to analyze a variety of RF coils at relatively low frequencies. The conductive elements that make up the coil are modeled as lumped inductances in combination with mutual inductances arising from interactions between the elements. Any capacitive element added to the coil configuration is treated as a lumped capacitance. Thus, the entire coil configuration is modeled as a lumped circuit that can be analyzed using well-established techniques that offer very fast computational evaluation. Hence, the lumped-element circuit model is the most established simulation technique for RF coil analysis [34] - [37]. It is very well suited for RF coil modeling when the dimensions of coil are small compared to the free space wavelength of the electromagnetic waves. In high-field
MRI studies when the dimensions of the coil become comparable to the free space wavelength of the RF signal, the lumped-element circuit model becomes increasingly inaccurate. In addition, the model cannot be used to analyze the complex field behavior that exists inside biological samples at high frequencies. At high frequencies when electromagnetic wave phenomena become more dominant, other modeling methods that satisfy the complete set of Maxwell's field equations can be applied. Examples of such modeling methods include: Finite Difference Time Domain (FDTD) methods, Method of Moments (MoM), Multi-conductor Transmission Line (MTL) model [38] and the Finite Element Method (FEM). In general, full-wave modeling techniques are based on the principles of finite element, finite difference, and the method of moments.

### 4.2 Inductive Behavior

The primary challenge when employing the lumped-element circuit model for RF coil analysis lies in the computation of the self and mutual inductances of the conductive elements that constitute the coil. Once the total inductance has been determined, capacitive elements can be deployed in order to tune the RF coil to the desired resonance frequency. The total inductance of the RF coil is directly proportional to its stored magnetic energy, which generally depends on the geometry, position and orientation of the conductive elements of the coil. At frequencies of interest where the wavelength of the electromagnetic wave is greater than ten times the largest dimension of the coil structure, the inductance of the coil can be calculated by a numerical evaluation of the Biot-Savart integral equation. Let us now consider a current carrying conductive element in free space with a current density $\mathbf{J} \ \text{A/m}^2$. Associated with the current density $\mathbf{J}$ is the stored magnetic energy $E_m$ and the self
inductance \( L \). \( L \) is proportional to \( E_m \) and is defined as

\[
E_m = \frac{1}{2} LI^2 \tag{4.1}
\]

where \( I \) denotes the total current flow (in A) through the conductor and is defined as

\[
I = \int_s \mathbf{J} \cdot d\mathbf{S} \tag{4.2}
\]

Here \( S \) represents any surface element intersecting the path of \( \mathbf{J} \). The magnetic energy \( E_m \) can also be written as

\[
E_m = \frac{1}{2} \int_V \mathbf{B} \cdot \mathbf{H} dV \tag{4.3}
\]

where \( \mathbf{B} \) and \( \mathbf{H} \) are the magnetic flux density and the magnetic field intensity produced by \( \mathbf{J} \), and \( V \) is the total volume of the conductive element. Since \( \mathbf{B} \) can be defined in terms of the magnetic vector potential \( \mathbf{A} \) as \( \mathbf{B} = \nabla \times \mathbf{A} \) and \( \mathbf{J} = \nabla \times \mathbf{H} \), we can simplify

\[
\mathbf{B} \cdot \mathbf{H} = (\nabla \times \mathbf{A}) \cdot \mathbf{H} = \mathbf{A} \cdot (\nabla \times \mathbf{H}) + \nabla \cdot (\mathbf{A} \times \mathbf{H})
\]

\[
= \mathbf{A} \cdot \mathbf{J} + \nabla \cdot (\mathbf{A} \times \mathbf{H}) \tag{4.4}
\]

Using 4.4 in 4.3, we have

\[
E_m = \frac{1}{2} \int_v \mathbf{A} \cdot \mathbf{J} dV + \frac{1}{2} \int_v \nabla \cdot (\mathbf{A} \times \mathbf{H}) dV
\]

\[
= \frac{1}{2} \int_v \mathbf{A} \cdot \mathbf{J} dV + \frac{1}{2} \oint_s (\mathbf{A} \times \mathbf{H}) \cdot d\mathbf{S} \tag{4.5}
\]

where we have applied the divergence theorem to simplify the \( \int_v \nabla \cdot (\mathbf{A} \times \mathbf{H}) dV \) term. It should be noted that the vector fields \( \mathbf{A} \) and \( \mathbf{H} \) produced by \( \mathbf{J} \) extend infinitely into space. A spherical surface \( S \) that encloses these fields would have a radius \( r \) that approaches infinity. On this spherical surface, in the limit as \( r \to \infty \), the fields drop
off $A \approx 1/r$ as and $B \approx 1/r^2$. Thus, $rac{1}{2} \int_A (A \times B) \cdot dV = 0$ and 4.5 becomes

$$E_m = \frac{1}{2} \int_V A \cdot J dV \quad (4.6)$$

Equating 4.1 to 4.6 and solving for $L$, we obtain the expression of self-inductance in terms of $A$ and $J$ as

$$L = \frac{1}{I^2} \int_V A \cdot J dV \quad (4.7)$$

From Biot-Savart Law, the magnetic induction, or magnetic flux density $B(r)$ is given by the expression

$$B(r) = \frac{\mu_0}{4\pi} \int_V \frac{J(r') \times u_R}{R^2} dV \quad (4.8)$$

where $\mu_0$ is the permeability of free space, $R = |r' - r|$, $r'$ is a vector from the reference origin to the source point, $r$ is a vector from the reference point to the observation point, and $u_R$ is the unit vector along $r' - r$. Now using the fact that $[39] \nabla \frac{1}{R} = -\frac{u_R}{R^2}$ we have that

$$\nabla \times \frac{1}{R} = \nabla \frac{1}{R} \times J(r') + \frac{1}{R} \nabla \times J(r') = \nabla \frac{1}{R} \times J(r')$$

since $\frac{1}{R} \nabla \times J(r') = 0$ [39]. As a result of this, we can write 4.8 as

$$B(r) = \frac{\mu_0}{4\pi} \int_V \nabla \times \frac{J(r')}{R} dV \quad (4.9)$$

Furthermore, we can write 4.9 as

$$B(r) = \frac{\mu_0}{4\pi} \int_V \nabla \times \frac{J(r')}{R} dV = \nabla \times \frac{\mu_0}{4\pi} \int_V \frac{J(r')}{R} dV \quad (4.10)$$

Because $B(r) = \nabla \times A(r)$, we have

$$A(r) = \frac{\mu_0}{4\pi} \int_V \frac{J(r')}{R} dV \quad (4.11)$$
Using 4.11 in 4.7, the self-inductance of the conductive element can be expressed as

\[
L = \frac{\mu_0}{4\pi I^2} \int_V \int_{V'} \frac{J(r') \cdot J(r)}{R} dV' dV
\]  

(4.12)

For the case of two current densities \( J_1 \) and \( J_2 \) occupying volumes \( V_1 \) and \( V_2 \) in a closed system, the magnetic energy is given by [39] as

\[
E_m = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M I_1 I_2
\]  

(4.13)

where we have used the fact that the system is linear and isotropic with \( M_{12} = M_{21} = M \). \( M_{12} \) is the mutual inductance of subsystem one with respect to subsystem two while \( M_{21} \) is the mutual inductance of subsystem two with respect to subsystem one. From 4.13, the magnetic energy of the system as a result of mutual interaction is \( M I_1 I_2 \). Using 4.6, this can also be written as [45]

\[
M I_1 I_2 = \frac{1}{2} \int_{V_2} A_1 \cdot J_2 dV_2 = \frac{1}{2} \int_{V_1} A_2 \cdot J_1 dV_1
\]  

(4.14)

Thus, using the definition of \( A(r) \) in 4.11, the final expression for the mutual inductance is given by

\[
M = \frac{1}{2I_1 I_2} \int_{V_2} A_1 \cdot J_2 dV_2 = \frac{1}{2I_1 I_2} \int_{V_1} A_2 \cdot J_1 dV_1
\]

\[
= \frac{\mu_0}{8\pi I_1 I_2} \int_{V_1} \int_{V_2} \frac{J_1(r) \cdot J_2(r')}{R} dV_1 dV_2
\]  

(4.15)
4.3 Lumped-Element Circuit Model

The basis of developing a theoretical model of the RF coil hinged on the simultaneous operation of the coil in both modes and on the symmetry of the coil spatial configuration. The lumped-element circuit model in T-network configuration is shown in Figure 4.1. There are two ports associated with the T-Network model; port 1 between points (1-3) and port 2 between points (2-3), with terminal 3 being the common ground. The coil can be considered to be made up of 3 straps: (1-4), (3-4) and (2-4), with all the straps connected at junction 4. Associated with each strap is a resistance, a self-inductance and 2 mutual inductances resulting from magnetic field coupling from the other straps. For example, strap (1-4) has resistance $R_1$, self-inductance $L_1$ and mutual inductances $M_{12}$ resulting from interactions between strap (1-4) and (2-4), and $M_{13}$ resulting from interactions between strap (1-4) and (3-4). It should be noted that because of the inherent symmetry of the coil configuration, resistance $R_1$ is equal to $R_2$, and inductance $L_1$ is equal to $L_2$.

![Figure 4.1: Lumped-element T-network model of the RF coil.](image)
We now consider the application of 4.12 in the evaluation of the self-inductance of a single strap of conductive element. Each element is characterized by a width $w$ and a radius $r$, with the elements treated as surface elements of negligible thickness. The strap is oriented in space as shown in Figure 4.2, with the axes of reference located on the plane that divides the strap into two equal parts along its width, and centered on the center of the cylinder that constitute the strap. We consider two points on the surface of the strap identified by position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively. As shown in Figure 4.3, the projection of vector $\mathbf{r}_2$ on the $x$-$y$ plane makes an angle of $\Omega_2$ with the $x$ axis, while the projection of vector $\mathbf{r}_1$ makes an angle of $\Omega_2$ with the $x$ axis. Associated with the projections of vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ are unit vectors $\mathbf{u}_1$ and $\mathbf{u}_2$. These unit vectors are perpendicular to the projections of $\mathbf{r}_1$ and $\mathbf{r}_2$ on the $x$-$y$ plane, and their direction is an indication of the direction of current flow along the surface of the strap element. The current density on the surface of the strap element is assumed to be uniform. This assumption is certainly valid at low frequencies where the transmission line effect does not play a dominant role. The directions of $\mathbf{u}_1$ and $\mathbf{u}_2$ indicate the direction of current flow. The magnitude of the surface current density

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure4_2.png}
\caption{Geometry for the determination of the self-inductance of a conductive strap element of the RF coil.}
\end{figure}
Figure 4.3: Projection of the conductive strap element onto the x-y plane.

\( J_s \) on the strap element is given by

\[ J_s = \frac{I}{w} \]  

(4.16)

where \( I \) is the current flowing over the element. Since the current density is confined to the surface of the cylindrical strap element, it depends only on the direction of the unit normal vector. Hence, the current densities \( J_1 \) and \( J_2 \) at the two differential element points on the surface are given by

\[ J_1 = J_s u_1 A/m \]  

(4.17)

\[ J_2 = J_s u_2 A/m \]  

(4.18)

and

\[ u_1 = -\sin(\Omega_1)u_x + \cos(\Omega_1)u_y \]  

(4.19)

\[ u_2 = -\sin(\Omega_2)u_x + \cos(\Omega_2)u_y \]  

(4.20)

The vectors \( u_x \) and \( u_y \) are unit vectors along the \( x \) and \( y \) axis. Since the RF coil system has been reduced to a system of surface elements, 4.12 reduces to

\[ L = \frac{\mu_0}{4\pi I^2} \int_S \int_S \frac{J_1 \cdot J_2}{R} dS' dS \]  

(4.21)
The vectors \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \) are given in rectangular coordinates as

\[
\mathbf{r}_1 = r \cos(\Omega_1) \mathbf{u}_x + r \sin(\Omega_1) \mathbf{u}_y + z_1 \mathbf{u}_z \tag{4.22}
\]

\[
\mathbf{r}_2 = r \cos(\Omega_2) \mathbf{u}_x + r \sin(\Omega_2) \mathbf{u}_y + z_2 \mathbf{u}_z \tag{4.23}
\]

Now, we calculate the vector \( \mathbf{r} \) from

\[
\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 \tag{4.24}
\]

Thus,

\[
\mathbf{r} = [r \cos(\Omega_2) - r \cos(\Omega_1)] \mathbf{u}_x + [r \sin(\Omega_2) - r \sin(\Omega_1)] \mathbf{u}_y + (z_2 - z_1) \mathbf{u}_z \tag{4.25}
\]

Since \( R = |\mathbf{r}| \), we have

\[
R = \sqrt{[r \cos(\Omega_2) - r \cos(\Omega_1)]^2 + [r \sin(\Omega_2) - r \sin(\Omega_1)]^2 + (z_2 - z_1)^2}
\]

\[
= \sqrt{2r^2 - 2r^2 \cos(\Omega_2 - \Omega_1) + (z_2 - z_1)^2}
\]

\[
= \sqrt{2r^2 - 2r^2 \cos(\Omega_2 - \Omega_1) + (z_2 - z_1)^2} \tag{4.26}
\]

where \( \cos(\Omega_2 - \Omega_1) = \cos(\Omega_2) \cos(\Omega_1) + \sin(\Omega_2) \sin(\Omega_1) \). The surfaces \( S \) and \( S' \) are identical surfaces since they represent the surface of the cylindrical strap element.

The differential surfaces \( dS \) and \( dS' \) can be identified with vectors \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \). Hence, they can be written in cylindrical coordinates as

\[
dS = rd\Omega_2 dz_2 \tag{4.27}
\]

\[
dS' = rd\Omega_2 dz_1 \tag{4.28}
\]
Thus, 4.21 becomes

\[
L = \frac{\mu_0 r^2}{4\pi w^2} \int_0^\pi \int_{-\frac{w}{2}}^{\frac{w}{2}} \int_{-\frac{w}{2}}^{\frac{w}{2}} \frac{\mathbf{u}_1 \cdot \mathbf{u}_2}{R} \, dz_1 \, dz_2 \, d\Omega_1 \, d\Omega_2
\]  

(4.29)

From 4.19 and 4.20, we had that

\[
\mathbf{u}_1 \cdot \mathbf{u}_2 = \cos(\Omega_2) \cos(\Omega_1) + \sin(\Omega_2) \sin(\Omega_1) = \cos(\Omega_2 - \Omega_1)
\]

Hence, 4.29 can be simplified as

\[
L = \frac{\mu_0 r^2}{4\pi w^2} \int_0^\pi \int_{-\frac{w}{2}}^{\frac{w}{2}} \int_{-\frac{w}{2}}^{\frac{w}{2}} \frac{\cos(\Omega_2 - \Omega_1)}{R} \, dz_1 \, dz_2 \, d\Omega_1 \, d\Omega_2
\]  

(4.30)

Evaluating the multiple integral in 4.30 with respect to \(dz_2\) and \(dz_1\), we have

\[
L = \frac{\mu_0 r^2}{2\pi w^2} \int_0^\pi \int_{-\frac{w}{2}}^{\frac{w}{2}} w \cos(\Omega_2 - \Omega_1) \ln(K_1) \, d\Omega_1 \, d\Omega_2
\]

\[
- \frac{\mu_0 r^2}{2\pi w^2} \int_0^\pi \int_{-\frac{w}{2}}^{\frac{w}{2}} w \cos(\Omega_2 - \Omega_1) \ln(-w + \sqrt{w^2 + K_1^2}) \, d\Omega_1 \, d\Omega_2
\]

\[
- \frac{\mu_0 r^2}{2\pi w^2} \int_0^\pi \int_{-\frac{w}{2}}^{\frac{w}{2}} \cos(\Omega_2 - \Omega_1) \sqrt{w^2 + K_1^2} \, d\Omega_1 \, d\Omega_2
\]

\[+ \frac{\mu_0 r^2}{2\pi w^2} \int_0^\pi \int_{-\frac{w}{2}}^{\frac{w}{2}} \cos(\Omega_2 - \Omega_1) K_1 \, d\Omega_1 \, d\Omega_2
\]  

(4.31)

where \(K_1^2 = 2r^2 - 2r^2 \cos(\Omega_2 - \Omega_1)\). 4.31 applies to all three strap elements depicted in Figure 4.1 with \(L = L_1 = L_2 = L_3\). From 4.31, it can be seen that the inductance \(L\) of a strap element is dependent only on the values of \(r\) and \(w\) that characterize the strap.

It is worth mentioning that in the integral formulation of 4.31, we assumed that the RF coil is immersed in free space. If the coil is immersed in some other environment with an effective permeability different from that of free space permeability, then \(\mu_0\) in all equations should be replaced by \(\mu\), where \(\mu\) is the effective permeability of the surrounding environment. The integrals in 4.31 can be evaluated numerically.
using any mathematical package. We prefer Mathcad, since it handles integrals with singularities on the integrand at one or both integration limits. Also, a general purpose routine can be written to evaluate 4.31 using Simpsons Rule or Gaussian Quadrature in multiple dimensions. Figure 4.4 provides inductance predictions for various widths as a function of the radius of the coil.

Next, we consider the determination of the mutual inductances between the RF coil elements using the model of Figure 4.1. Due to the inherent symmetry of the RF coil configuration, we have that $M_{12} = M_{21}$ and $M_{13} = M_{23} = M_{31}M_{32}$. For the determination of $M_{12}$ or $M_{21}$ we refer to Figure 4.5. It shows the spatial orientation of the two conductive elements that constitute the base ring. We define $M_{\text{base}}$ such that $M_{\text{base}} = M_{12} = M_{21}$. As with the derivation of the self-inductance integral formulation, the axes of reference are located on the plane that divides the base ring into two equal parts along its width, and centered with respect to the base ring. We next consider two points on the surface of the base ring with position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively. Figure 4.6 shows the projection of the base ring onto the $x$-$y$ plane. Assuming a uniform current density over the surface of the ring, $\mathbf{J}_1$ and $\mathbf{J}_2$ at the two
Figure 4.5: Determination of the mutual-inductance $M_{\text{base}} = M_{12} = M_{21}$ between the two conductive elements of the base ring of the RF coil.

Figure 4.6: Projection of the base ring onto the $x$-$y$ plane.

differential element points are given by 4.17 and 4.18, and their direction is along the unit vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ as indicated in Figure 4.6. Based on Figure 4.6, unit vector $\mathbf{u}_1$ is given as

$$\mathbf{u}_1 = -\sin(\Omega_1)\mathbf{u}_x - \cos(\Omega_1)\mathbf{u}_y$$

while $\mathbf{u}_2$ is given by 4.20. Thus we can now reduce 4.15 to

$$M_{\text{base}} = \frac{\mu_0}{8\pi I_1 I_2} \int_{S_1} \int_{S_2} \frac{\mathbf{J}_1 \cdot \mathbf{J}_2}{R} dS_1 dS_2$$

(4.33)

Furthermore $R = |\mathbf{r}_2 - \mathbf{r}_1|$ where $\mathbf{r}_2$ is given by 4.23 and $\mathbf{r}_1$ is given by

$$\mathbf{r}_1 = r \cos(\Omega_1)\mathbf{u}_x - r \sin(\Omega_1)\mathbf{u}_y + z_1 \mathbf{u}_z$$

(4.34)
Hence,

\[
R = \sqrt{[r \cos(\Omega_2) - r \cos(\Omega_1)]^2 + [r \sin(\Omega_2) + r \sin(\Omega_1)]^2 + (z_2 - z_1)^2}
\]

\[
= \sqrt{2r^2 - 2r^2 \cos(\Omega_2) \cos(\Omega_1) - \sin(\Omega_2) \sin(\Omega_1)] + (z_2 - z_1)^2}
\]

\[= \sqrt{2r^2 - 2r^2 \cos(\Omega_2 + \Omega_1) + (z_2 - z_1)^2} \quad (4.35)
\]

Using 4.17, 4.18, 4.20 and 4.32 we can simplify \( \mathbf{J}_1 \cdot \mathbf{J}_2 \) as

\[
\mathbf{J}_1 \cdot \mathbf{J}_2 = \frac{I_1}{w} \mathbf{u}_1 \cdot \frac{I_2}{w} \mathbf{u}_2 = -\frac{I_1 I_2}{w^2} \left[ \cos(\Omega_2) \cos(\Omega_1) - \sin(\Omega_2) \sin(\Omega_1) \right]
\]

\[= -\frac{I_1 I_2}{w^2} \cos(\Omega_2 + \Omega_1) \quad (4.36)
\]

Hence, 4.33 can be simplified as

\[
M_{\text{base}} = \frac{\mu_0 r^2}{8 \pi w^2} \int_0^\pi \int_0^\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{-\cos(\Omega_2 + \Omega_1)}{R} d z_1 d z_2 d \Omega_1 d \Omega_2 
\]

\[= \frac{\mu_0 r^2}{4 \pi w^2} \int_0^\pi \int_0^\pi w \cos(\Omega_2 + \Omega_1) \ln(K_2) d \Omega_1 d \Omega_2
\]

\[
+ \frac{\mu_0 r^2}{4 \pi w^2} \int_0^\pi \int_0^\pi \cos(\Omega_2 + \Omega_1) \sqrt{w^2 + K_2^2} d \Omega_1 d \Omega_2
\]

\[
+ \frac{\mu_0 r^2}{4 \pi w^2} \int_0^\pi \int_0^\pi \cos(\Omega_2 + \Omega_1) \sqrt{w^2 + K_2^2} d \Omega_1 d \Omega_2
\]

\[- \frac{\mu_0 r^2}{4 \pi w^2} \int_0^\pi \int_0^\pi \cos(\Omega_2 + \Omega_1) K_2 d \Omega_1 d \Omega_2 \quad (4.37)
\]

Thus, evaluating 4.37 with respect to \( d z_1 \) and \( d z_2 \), we have

\[
M_{\text{base}} = \frac{\mu_0 r^2}{4 \pi w^2} \int_0^\pi \int_0^\pi \int_0^\pi \int_0^\pi w \cos(\Omega_2 + \Omega_1) \ln(K_2) d \Omega_1 d \Omega_2
\]

\[
+ \frac{\mu_0 r^2}{4 \pi w^2} \int_0^\pi \int_0^\pi \cos(\Omega_2 + \Omega_1) \sqrt{w^2 + K_2^2} d \Omega_1 d \Omega_2
\]

\[
+ \frac{\mu_0 r^2}{4 \pi w^2} \int_0^\pi \int_0^\pi \cos(\Omega_2 + \Omega_1) \sqrt{w^2 + K_2^2} d \Omega_1 d \Omega_2
\]

\[- \frac{\mu_0 r^2}{4 \pi w^2} \int_0^\pi \int_0^\pi \cos(\Omega_2 + \Omega_1) K_2 d \Omega_1 d \Omega_2 \quad (4.38)
\]

where \( K_2^2 = 2r^2 - 2r^2 \cos(\Omega_2 + \Omega_1) \). As with the integral formulation of the self-inductance \( L \), 4.38 can be evaluated numerically using a mathematical package. Figure 4.7 shows the variation of the mutual inductance \( M_{\text{base}} \) for a fixed \( w \) as a function of \( r \).
Figure 4.7: Variation of the mutual-inductance $M_{\text{base}}$ with width $w$ and radius $r$ of the two conductive elements of the base ring.

Finally, we consider the evaluation of the mutual inductance between the base ring and the upper strap element. We denote this inductance as $M_{\text{ring}}$. The system is oriented as shown in Figure 4.8 with the strap element facing downward towards the negative $z$ axis. The current densities $J_1$ and $J_2$ at the two differential element points on the surface of the coil are given by 4.17 and 4.18, and their direction is along the unit vectors $u_1$ and $u_2$. Now, unit vector $u_1$ is given as

$$u_1 = \sin(\Omega_1)u_x - \cos(\Omega_1)u_y$$ (4.39)

Figure 4.8: The determination of the mutual-inductance $M_{\text{ring}}$ between the strap element and one element of the base ring.
while \( u_2 \) is given by

\[
    u_2 = \sin(\Omega_2)u_x - \cos(\Omega_2)u_z
\]  

(4.40)

Thus, 4.15 gives

\[
    M_{ring} = \frac{\mu_0}{8\pi I_1 I_2} \int_{S_1} \int_{S_2} \frac{J_1 \cdot J_2}{R} dS_1 dS_2
\]

\[
    = \frac{\mu_0}{8\pi w^2} \int_{S_1} \int_{S_2} \frac{u_1 \cdot u_2}{R} dS_1 dS_2
\]  

(4.41)

The angle \( \Omega_1 \) in 4.39 is the angle between the \( x-y \) projection of vector \( r_1 \) and the \( x \) axis, while angle \( \Omega_2 \) in 4.40 denotes the angle between the \( x-z \) projection of vector \( r_2 \) and the \( x \) axis. We note that \( R = |r_2 - r_1| \), but \( r_1 \) and \( r_2 \) are given by

\[
    r_1 = r \cos(\Omega_1)u_x + r \sin(\Omega_1)u_y + zu_z
\]  

(4.42)

\[
    r_2 = r \cos(\Omega_2)u_x + yu_y + r \sin(\Omega_2)u_z
\]  

(4.43)

As a result,

\[
    R = \left| r \cos(\Omega_2) - r \cos(\Omega_1) \right| u_x + \left| y - r \sin(\Omega_1) \right| u_y + \left| r \sin(\Omega_2) - z \right| u_z
\]

\[
    = \sqrt{2r^2 + y^2 + z^2 - 2r y \sin(\Omega_1) - 2r z \sin(\Omega_2) - 2r^2 \cos(\Omega_1) \cos(\Omega_2)}
\]  

(4.44)

From 4.39 and 4.40, the dot product \( u_1 \cdot u_2 \) can be evaluated as

\[
    u_1 \cdot u_2 = \sin(\Omega_1) \sin(\Omega_2)
\]

Hence, the integral formulation becomes

\[
    M_{ring} = \frac{\mu_0 r^2}{8\pi w^2} \int_0^\pi \int_0^\pi \int_{-w}^w \frac{\sin(\Omega_1) \sin(\Omega_2)}{R} dy dz d\Omega_1 d\Omega_2
\]  

(4.45)
The numerical evaluation of 4.45 as a function of coil radius for a fixed coil width results in Figure 4.9.

![Figure 4.9: Variation of the mutual inductance $M_{\text{ring}}$ with width $w$ and radius $r$ of a base element and a strap element.](image)

### 4.5 Method of Moments Formulation

All macroscopic electromagnetic phenomena are governed by Maxwells equations. These equations are given in differential form by

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (4.46)
\]

\[
\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad (4.47)
\]

\[
\nabla \cdot \mathbf{B} = 0 \quad (4.48)
\]

\[
\nabla \cdot \mathbf{D} = \rho \quad (4.49)
\]

where

- $\mathbf{H} =$ magnetic field intensity, in A/m
- $\mathbf{B} =$ magnetic flux density, in T or Wb/m²
\( D = \) electric flux density, in \( \text{Coul/m}^2 \)

\( E = \) electric field intensity, in \( \text{V/m} \)

\( J = \) total volume electric current density, in \( \text{A/m}^2 \)

\( \rho = \) electric charge density, in \( \text{Coul/m}^3 \)

The interaction of electromagnetic fields with various material media are described by the constitutive relations

\[
J = \sigma E \tag{4.50}
\]

\[
D = \varepsilon E \tag{4.51}
\]

\[
B = \mu H \tag{4.52}
\]

where \( \sigma \) is the electrical conductivity, \( \varepsilon \) is the electric permittivity, and \( \mu \) is the magnetic permeability of the respective medium. The magnetic vector potential \( A \) is defined by \( B = \nabla \times A \). Substituting this expression in \( 4.46 \) yields

\[
\nabla \times E = -\frac{\partial (\nabla \times A)}{\partial t} \tag{4.53}
\]

In a fixed reference system with respect to time, \( 4.53 \) reduces to

\[
\nabla \times E = -\nabla \times \frac{\partial A}{\partial t}
\]

\[
\nabla \times (E + \frac{\partial A}{\partial t}) = 0 \tag{4.54}
\]

The quantity within the parentheses must be equal to a quantity whose curl is zero, namely a gradient \([39]\). This quantity is the electric scalar potential \( V \). Thus, we can set

\[
E = -\nabla V - \frac{\partial A}{\partial t} \tag{4.55}
\]
Under steady state conditions, 4.55 reduces to

\[ \mathbf{E} = -\nabla V \]  

(4.56)

4.55 is the general expression for \( \mathbf{E} \). It states that electric field intensity can arise from both accumulations of charge, through the \(-\nabla V\) term, and from changing magnetic fields, through the \(-\frac{\partial \mathbf{A}}{\partial t}\) term [39]. Assuming a linear isotropic medium and using the constitutive relationships defined in 4.50 - 4.52, 4.47 simplifies to

\[ \nabla \times \mathbf{B} = \mu \mathbf{J} + \mu \varepsilon \frac{\partial \mathbf{E}}{\partial t} \]  

(4.57)

Using the definition of the vector potential and 4.55, 4.57 becomes

\[ \nabla \times (\nabla \times \mathbf{A}) = \mu \mathbf{J} + \mu \varepsilon \frac{\partial (-\nabla V - \frac{\partial \mathbf{A}}{\partial t})}{\partial t} \]  

(4.58)

which reduces to

\[ \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} = \mu \mathbf{J} - \mu \varepsilon \frac{\partial (\nabla V)}{\partial t} - \mu \varepsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} \]  

(4.59)

and finally to

\[ \nabla^2 \mathbf{A} - \mu \varepsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{J} + \nabla (\nabla \cdot \mathbf{A}) + \mu \varepsilon \frac{\partial (\nabla V)}{\partial t} \]  

(4.60)

where \( \nabla \times (\nabla \times \mathbf{A}) \) was simplified using the definition of the vector triple product. Equation 4.60 is one of the free space wave equations for the potential fields derived from Maxwell equations [40]. The other wave equation can be obtained by substituting 4.55 in 4.49. Thus,

\[ \nabla \cdot \varepsilon (-\nabla V - \frac{\partial \mathbf{A}}{\partial t}) = \rho \]  

(4.61)

yielding

\[ \nabla^2 V + \frac{\partial (\nabla \cdot \mathbf{A})}{\partial t} = -\frac{\rho}{\varepsilon} \]  

(4.62)
Equations 4.60 and 4.62 are the so-called coupled potential wave equations. In order to decouple these wave equations, a gauge must be chosen. The choice of a gauge is arbitrary and is purely for mathematical convenience. The gauge condition as it is also known results from the fact that the divergence of the magnetic vector potential \( A \) is not specified in its definition [40]. According to Helmholtz's theorem, if \( A \) is to be determined to within an additive constant its divergence and its curl must be specified. The most common choice is the Lorentz gauge

\[
\nabla \cdot A = -\mu \varepsilon \frac{\partial V}{\partial t}
\]

(4.63)

This leads to a complete separation of the wave equations for \( A \) and \( V \). Using the Lorentz gauge, 4.60 and 4.62 become the familiar

\[
\nabla^2 V - \mu \varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon}
\]

(4.64)

\[
\nabla^2 A - \mu \varepsilon \frac{\partial^2 A}{\partial t^2} = -\mu J
\]

(4.65)

Equation 4.64 and 4.65 reveal that the source of the scalar potential \( V \) is the charge density, while the source of the vector potential \( A \) is the current density. In order to solve 4.64 and 4.65, it is convenient to convert these equations into their phasor form in order to eliminate the time dependence. Thus, the equations simplify into

\[
\nabla^2 V + \omega^2 \mu \varepsilon V = -\frac{\rho}{\varepsilon}
\]

(4.66)

and

\[
\nabla^2 A + \omega^2 \mu \varepsilon A = -\mu J
\]

(4.67)

where the implied \( e^{j\omega t} \) time dependence transformed the \( \frac{\partial^2}{\partial t^2} \) operator into \( -\omega^2 \). Well
known physical solutions to 4.66 and 4.67 are given as [41]

\[
A(r) = \frac{\mu}{4\pi} \int_V \frac{\mathbf{J}(r')e^{-jk_0|r-r'|}}{|r-r'|} dV' \tag{4.68}
\]

\[
V(r) = \frac{1}{4\pi\varepsilon} \int_V \frac{\mathbf{\rho}(r')e^{-jk_0|r-r'|}}{|r-r'|} dV' \tag{4.69}
\]

where

\[
k_0 = \omega \sqrt{\frac{\mu}{\varepsilon}} = \frac{2\pi}{\lambda}
\]

The continuity equation relates \(\mathbf{J}(r')\) and \(\mathbf{\rho}(r')\). This relationship can be obtained by taking the divergence of 4.47 yielding

\[
\nabla \cdot (\nabla \times \mathbf{H}) = \nabla \cdot (\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}) = 0
\]

\[
= \nabla \cdot \mathbf{J} + \frac{\partial (\nabla \cdot \mathbf{D})}{\partial t}
\]

\[
= \nabla \cdot \mathbf{J} + \frac{\partial \mathbf{\rho}}{\partial t} \tag{4.70}
\]

Equation 4.70 is the continuity equation and it is a statement of the conservation of charge. It can be written in phasor form as

\[
\nabla \cdot \mathbf{J} + j\omega \mathbf{\rho} = 0 \tag{4.71}
\]

Using 4.71 in 4.69 gives

\[
V(r) = \frac{j}{4\pi\varepsilon\omega} \int_V \frac{\nabla' \cdot \mathbf{J}(r')e^{-jk_0|r-r'|}}{|r-r'|} dV' \tag{4.72}
\]

Substituting 4.68 and 4.72 in 4.55 gives the electric field as

\[
\mathbf{E}(r) = -\frac{j}{4\pi\varepsilon\omega} \int_V \nabla' \cdot \mathbf{J}(r') \frac{e^{-jk_0|r-r'|}}{|r-r'|} dV' - \frac{j\omega\mu}{4\pi} \int_V \frac{\mathbf{J}(r')e^{-jk_0|r-r'|}}{|r-r'|} dV'
\]

\[
= -\frac{j}{4\pi\varepsilon\omega} \int_V \nabla' \cdot \mathbf{J}(r') \nabla G(r, r') dV' - \frac{j\omega\mu}{4\pi} \int_V \mathbf{J}(r')G(r, r') dV' \tag{4.73}
\]
where

\[
G(r, r') = e^{-jk_0 |r-r'|} \frac{|r-r'|}{|r-r'|}
\] (4.74)

is the free space Green function of the Helmholtz equation. The Method of Moments can be applied to 4.73 in order to solve for the electric field \( \mathbf{E}(r) \). The solution of \( \mathbf{E}(r) \) can then be substituted into Maxwell's equations or the constitutive relations in order to determine the other electromagnetic parameters. We now transform the linear functional equation in 4.73 into a linear matrix equation via the MoM. This methodology can be considered as an error-minimizing process in a linear vector space. The application of the Method of Moments begins with the discretization of the system and the definition of a basis function as shown in Figure 4.10 and Figure 4.11.

The basis function is a modification of the triangular Rao-Wilton-Glisson (RWG) basis function [42]. Planar triangular patched elements are ideal for modeling bodies of arbitrary shaped surfaces because they are capable of accurately conforming to any geometric surface or boundary. It is also easily specified for computer processing and varying patched element density can be used depending on the resolution required [42]. Figure 4.10 defines a triangular patched element as a pair of triangles sharing a common edge. The triangles are categorized as left triangle \( T_n^+ \) and right triangle \( T_n^- \).

![Figure 4.10: Geometry of triangular patched element used in the definition of the basis function.](image)
Figure 4.11: Discretization of the RF coil into triangular patched elements.

Associated with each triangular patched element is a vector basis function defined as

\[
f_n(r) = \begin{cases} 
\frac{1}{2A_n^+} \rho_n^+ & \text{for } r \in T_n^+ \\
\frac{1}{2A_n^-} \rho_n^- & \text{for } r \in T_n^- \\
0 & \text{otherwise}
\end{cases}
\]  

(4.75)

where \( A_n^+ \) and \( A_n^- \) are the areas of triangle \( T_n^+ \) and \( T_n^- \), \( \rho_n^+ \) is a vector drawn from the free vertex point of triangle \( T_n^+ \) to the observation point defined by vector \( r \), and \( \rho_n^- \) is a vector drawn from the observation point \( r \) to the free vertex point of triangle \( T_n^- \). The subscript \( n \) denotes the \( n \)th triangular patched element. On triangle \( T_n^+ \), \( f_n(r) \) is a vector field that is radially diverging from the free vertex point of \( T_n^+ \), while on triangle \( T_n^- \), it is radially converging to the free vertex point of \( T_n^- \). The basis function \( f_n(r) \) is used to approximate the surface current density of the RF coil. In other words, the current flow is from \( T_n^+ \) to \( T_n^- \). The surface divergence of \( f_n(r) \) is given by

\[
\nabla \cdot f_n(r) = \begin{cases} 
\frac{1}{A_n^+} & \text{for } r \in T_n^+ \\
-\frac{1}{A_n^-} & \text{for } r \in T_n^- \\
0 & \text{otherwise}
\end{cases}
\]  

(4.76)

and the surface current density \( J(r') \) can be approximated using

\[
J(r') = \sum_{n=1}^{N} I_n f_n(r)
\]  

(4.77)
where $I_n$ is the net current flowing through the connected edge of $T_n^+$ and $T_n^-$ of the $n$th patched element, and $N$ is the total number of triangular patched elements. The next step in the application of the MoM is the generation of $N$ independent equations for the unknown expansion coefficients $I_n$ from the linear functional equation of 4.73. This is achieved by using the Galerkin formulation of weighted residuals [43]. The idea behind the use of the formulation is to minimize the residual or error of the assumed solution by projecting it unto the subspace spanned by the weighted or testing functions, where the weighted or testing functions are identical to the basis functions. Upon substituting 4.77 into 4.73, we have

$$\sum_{n=1}^{N} E_n^e(r) = \sum_{n=1}^{N} I_n \left[ \frac{j}{4\pi\omega\varepsilon} \int_V \nabla' \cdot f_n(r) \nabla G(r, r') dV' - \frac{j\omega\mu}{4\pi} \int_V f_n(r) G(r, r') dV' \right]$$

and

$$R = \sum_{n=1}^{N} \{E_n(r) - E_n^e(r)\} \quad (4.78)$$

where $R$ is the residual or error resulting from the use of the basis function $f_n(r)$ in $J(r')$, and $E_n^e(r)$ is the approximated electric field. We now seek to minimize this residual by projecting it unto the subspace spanned by our weighted or testing function $W_m$. The Galerkin formulation stipulates that we define $W_m$ using the same basis function as used in $J(r')$ i.e.

$$W_m = \sum_{m=1}^{N} f_m(r) \quad (4.79)$$

Projecting $R$ unto the subspace spanned by $W_m$, we have

$$\langle R \cdot W_m \rangle = \sum_{m=1}^{N} \sum_{n=1}^{N} \left\{ \int_V \langle f_m(r) \cdot E_n(r) - f_m(r) \cdot E_n^e(r) \rangle dV \right\} \quad (4.80)$$

where $\langle R \cdot W_m \rangle$ is an inner product. We desire expansion coefficients $I_n$ that will force
\((R \cdot W_m)\) to zero. Equation 4.80 is a linear matrix equation with \(N\) unknowns. We will assume that the surface of the RF coil is a perfect conductor. Thus, the term

\[
f_m(r) \cdot E_n(r) = 0
\]  

since the electric field on the surface of a perfect conductor is along the normal to the surface, and the basis function \(f_m(r)\) is defined along the surface tangent. Thus, 4.80 reduces to

\[
- \sum_{m=1}^{N} \sum_{n=1}^{N} \left\{ \int_V f_m(r) \cdot E_n^* (r) dV \right\} = 0
\]

or

\[
\sum_{m=1}^{N} \sum_{n=1}^{N} I_n \left[ \frac{j}{4\pi \omega \epsilon} V_{mn}(r) + \frac{j \omega \mu}{4\pi} A_{mn}(r) \right] = 0
\]  

(4.82)

where

\[
V_{mn}(r) = \int_V \int_V \nabla' \cdot f_n(r) f_m(r) \cdot \nabla G(r, r') dV dV'
\]

\[
A_{mn}(r) = \int_V \int_V f_m(r) \cdot f_n(r) G(r, r') dV dV'
\]

The \(f_m(r) \cdot \nabla G(r, r')\) term of \(V_{mn}(r)\) can be simplified as

\[
f_m(r) \cdot \nabla G(r, r') = \nabla \left[ G(r, r') f_m(r) \right] - G(r, r') \nabla \cdot f_m(r)
\]  

(4.83)

But,

\[
\int_V \nabla \cdot \left[ G(r, r') f_m(r) \right] dV = \int_S \left[ G(r, r') f_m(r) \right] \cdot \mathbf{n} dS = 0
\]  

(4.84)

because the basis function \(f_m(r)\) is defined along the surface tangent and hence \(f_m(r) \cdot \).
Thus, \( V_{mn}(\mathbf{r}) \) in 4.82 becomes

\[
V_{mn}(\mathbf{r}) = \int_V \int_{V'} \nabla \cdot \mathbf{f}_m(\mathbf{r}) \nabla' \cdot \mathbf{f}_n(\mathbf{r}) G(\mathbf{r}, \mathbf{r'}) dV dV'
\] (4.85)

Since the RF coil will be implemented using microstrip conductors, the thickness of the coil can be neglected. Consequently, the volume integrals in \( V_{mn}(\mathbf{r}) \) and \( A_{mn}(\mathbf{r}) \) become surface integrals with

\[
V_{mn}(\mathbf{r}) = \int_S \int_S \nabla' \cdot \mathbf{f}_n(\mathbf{r}) \mathbf{f}_m(\mathbf{r}) \cdot \nabla G(\mathbf{r}, \mathbf{r'}) dS dS'
\]
\[
A_{mn}(\mathbf{r}) = \int_S \int_S \mathbf{f}_m(\mathbf{r}) \cdot \mathbf{f}_n(\mathbf{r}) G(\mathbf{r}, \mathbf{r'}) dS dS'
\] (4.86)

4.82 reduces to a linear matrix equation of the form

\[
\sum_{m=1}^{N} \sum_{n=1}^{N} Z_{mn} I_n = 0
\] (4.87)

where

\[
Z_{mn} = j \frac{4\pi \omega \varepsilon}{4\pi} V_{mn}(\mathbf{r}) + j \frac{\omega \mu}{4\pi} A_{mn}(\mathbf{r})
\] (4.88)

Equation 4.87 can be written in matrix form as

\[
Z \cdot \mathbf{I} = 0
\] (4.89)

and \( Z \) is referred to as the impedance matrix with \( Z = [Z_{mn}], \mathbf{I} = [I_n] \). For the case of the RF coil system, there are voltage excitations on the input ports. Thus, 4.81 will not be zero at the ports of the coil, but

\[
Z \cdot \mathbf{I} = [V_n]
\] (4.90)

where \( V_n \) is the voltage applied across the \( n \)th triangular patched element. The final
step in the formulation with voltage excitation at the two ports is the evaluation of the matrix $\mathbf{Z}$. In order to evaluate $\mathbf{Z}$, we consider the evaluation of $V_{mn}(\mathbf{r})$ and $\mathbf{A}_{mn}(\mathbf{r})$ in 4.88. Simplifying $V_{mn}(\mathbf{r})$ using 4.76, we obtain

$$V_{mn}(\mathbf{r}) = \frac{1}{A_m A_n^+} \int_{T_m} \int_{T_n^+} G(\mathbf{r}, \mathbf{r}') dS' - \frac{1}{A_m A_n^-} \int_{T_m} \int_{T_n^-} G(\mathbf{r}, \mathbf{r}') dS'$$

$$- \frac{1}{A_m A_n^+} \int_{T_m} \int_{T_n^+} G(\mathbf{r}, \mathbf{r}') dS' + \frac{1}{A_m A_n^-} \int_{T_m} \int_{T_n^-} G(\mathbf{r}, \mathbf{r}') dS'$$

(4.91)

and using 4.75 in $\mathbf{A}_{mn}(\mathbf{r})$ gives as

$$\mathbf{A}_{mn}(\mathbf{r}) = \frac{1}{4A_m A_n^+} \int_{T_m} \int_{T_n^+} \left( \mathbf{r}_m^+ \cdot \mathbf{r}_n^+ \right) G(\mathbf{r}, \mathbf{r}') dS'$$

$$+ \frac{1}{4A_m A_n^-} \int_{T_m} \int_{T_n^-} \left( \mathbf{r}_m^- \cdot \mathbf{r}_n^- \right) G(\mathbf{r}, \mathbf{r}') dS'$$

$$+ \frac{1}{4A_m A_n^+} \int_{T_m} \int_{T_n^+} \left( \mathbf{r}_m^- \cdot \mathbf{r}_n^+ \right) G(\mathbf{r}, \mathbf{r}') dS'$$

$$+ \frac{1}{4A_m A_n^-} \int_{T_m} \int_{T_n^-} \left( \mathbf{r}_m^+ \cdot \mathbf{r}_n^- \right) G(\mathbf{r}, \mathbf{r}') dS'$$

(4.92)

In order to evaluate the integrals presented in 4.91 and 4.92, we approximated $G(\mathbf{r}, \mathbf{r}')$, $\mathbf{r}_m^+, \mathbf{r}_m^-, \mathbf{r}_n^+, \mathbf{r}_n^-$ by their values at the centroid of the base triangle. Thus, 4.91 becomes

$$V_{mn}(\mathbf{r}) \approx G(\mathbf{r}_m^+, \mathbf{r}_n^+) - G(\mathbf{r}_m^+, \mathbf{r}_n^-) - G(\mathbf{r}_m^-, \mathbf{r}_n^+) + G(\mathbf{r}_m^-, \mathbf{r}_n^-)$$

(4.93)

while 4.92 becomes

$$\mathbf{A}_{mn}(\mathbf{r}) \approx \frac{1}{4} (\mathbf{r}_m^+ \cdot \mathbf{r}_n^+) G(\mathbf{r}_m^+, \mathbf{r}_n^+) + \frac{1}{4} (\mathbf{r}_m^- \cdot \mathbf{r}_n^-) G(\mathbf{r}_m^+, \mathbf{r}_n^-)$$

$$+ \frac{1}{4} (\mathbf{r}_m^- \cdot \mathbf{r}_n^+) G(\mathbf{r}_m^-, \mathbf{r}_n^+) + \frac{1}{4} (\mathbf{r}_m^+ \cdot \mathbf{r}_n^-) G(\mathbf{r}_m^-, \mathbf{r}_n^-)$$

(4.94)

$\mathbf{r}_m^\pm$ and $\mathbf{r}_n^\pm$ are the position vectors of the centroid of triangles $T_m^\pm$ and $T_n^\pm$ with respect to the vertex reference, while $\mathbf{r}_m^\pm$ and $\mathbf{r}_n^\pm$ are position vectors of the centroids.
Z_{mn} \approx \frac{j}{4\pi\omega}\left[ G(r_m^+, r_n^+) - G(r_m^-, r_n^-) - G(r_m^c, r_n^c) + G(r_m^c, r_n^c) \right] 
+ \frac{j\omega\mu}{16\pi} \left[ (\rho_m^c \cdot \rho_n^c)G(r_m^c, r_n^c) + (\rho_m^c \cdot \rho_n^c)G(r_m^c, r_n^c) \right] 
+ \frac{j\omega\mu}{16\pi} \left[ (\rho_m^c \cdot \rho_n^c)G(r_m^c, r_n^c) + (\rho_m^c \cdot \rho_n^c)G(r_m^c, r_n^c) \right] \quad (4.95)

The calculation of $Z_{mn}$ formed a huge part in the determination of $Z$. Because of the presence of the Green function in 4.95, there will be singularities in the evaluation of the impedance matrix when $r_m^c = r_n^c$. These singularities were avoided by using a number of analytical base integrals [44] to evaluate 4.91 and 4.92 at the singular points.
Chapter 5

RF COIL IMPLEMENTATION

5.1 Design Specifications

The RF coil was developed specifically for screening women with a C cup bra size [46], although it can also be used for screening women with lower sizes. The spatial configuration and orientation of the RF coil closely conforms to the shape of the female breast, thus increasing its filling factor and spatial resolution. The symmetric nature of the design guarantees extendability, scalability to various breast sizes, and flexibility of use with existing MRI systems with $B_0$ field strengths in the range from 0.5T to 2.0T. The design explicitly targeted conventional 1.5T MRI breast screening systems with a resonance frequency of 63.87MHz. Table 5.1 shows a list of the important design parameters.

Table 5.1: Design parameters of the RF coil for a 1.5T MRI system.

<table>
<thead>
<tr>
<th>Element</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element Radius</td>
<td>$r = 0.06$</td>
</tr>
<tr>
<td>Element Width</td>
<td>$w = 0.03$</td>
</tr>
<tr>
<td>Resonance Frequency</td>
<td>$f = 63.87$MHz</td>
</tr>
</tbody>
</table>
5.2 Determination of the Magnetic Field Distribution

The determination of the magnetic field distribution in the ROI is first realized using the Biot-Savat integral equation as presented in 4.8. We begin by considering coil operation in Mode 0 and later consider operation in Mode 1. Operation in Mode 0 is characterized by a circular ring current as shown in Figure 3.4. Thus, the coil reduces to a cylinder of radius \( r_2 \) and height \( w \) as shown in Figure 5.1. Figure 5.2 shows the projection of the coil on the \( x-y \) plane. From 4.8, the magnetic induction at point \( B \) is defined as

\[
B(r) = \frac{\mu_0}{4\pi} \int_S \frac{J(r') \times r}{r^3} dS
\]  

(5.1)

We will assume a uniform current distribution on the surface of the coil, and that there are no magnetic materials in the vicinity of the coil. At point \( B \), we introduce a cylindrical reference frame with unit vectors \( u_r, u_\Omega \) and \( u_z \). Vectors \( u_r \) and \( u_\Omega \) are defined as shown in Figure 5.2, and \( u_z \) is defined to point along the \( z \) axis. The vector \( r \) from the differential current element at point \( A \) to the field point at \( B \) is given by

\[
r = r_1 - r_2
\]  

(5.2)

Using the reference frame at point \( B \), we can express \( r_1 \) and \( r_2 \) as

\[
r_1 = r_1 u_r + z_1 u_z
\]  

(5.3)

\[
r_2 = r_2 \cos(\Omega_1 - \Omega_2) u_r - r_2 \sin(\Omega_1 - \Omega_2) u_\Omega + z_2 u_z
\]  

(5.4)

By substituting 5.3 and 5.4 into 5.2, we can obtain \( r \) as

\[
r = [r_1 - r_2 \cos(\Omega_1 - \Omega_2)] u_r + r_2 \sin(\Omega_1 - \Omega_2) u_\Omega + (z_1 - z_2) u_z
\]  

(5.5)
Figure 5.1: Determination of the magnetic field strength at observation point $B$ due to the differential current element at point $A$.

The cylindrical current on the surface of the base ring is confined to its surface and is uniformly distributed on the surface. Thus, the surface current density at point $A$ is given by

$$ J = \frac{I}{w} \left[ \sin(\Omega_1 - \Omega_2)u_r + \cos(\Omega_1 - \Omega_2)u_\Omega \right] \quad (5.6) $$

and the differential surface area is given as

$$ dS = r_2 dz_2 d\Omega_2 \quad (5.7) $$

The term $\sin(\Omega_1 - \Omega_2)u_r + \cos(\Omega_1 - \Omega_2)u_\Omega$ is the unit vector along the tangent to the

Figure 5.2: Projection of the RF coil on the $x$-$y$ plane for Mode 0 operation.
surface of the base ring at point $A$ when referenced from point $B$. 5.1 now becomes

$$
\mathbf{B}(r) = \frac{\mu_0}{4\pi} \int_0^{2\pi} \int_{-\frac{w}{2}}^{\frac{w}{2}} \frac{\mathbf{J}(r') \times r}{\sqrt{r_1'^2 + r_2'^2 + (z_1 - z_2)^2 - 2r_1 r_2 \cos(\Omega_1 - \Omega_2)}} dz_2 d\Omega_2 (5.8)
$$

From 5.5 and 5.6, we can obtain $\mathbf{J}(r') \times r$ as

$$
\mathbf{J}(r') \times r = \frac{I}{w} (z_1 - z_2) [\cos(\Omega_1 - \Omega_2) \mathbf{u}_r - \sin(\Omega_1 - \Omega_2) \mathbf{u}_\Omega] \\
+ \frac{I}{w} [r_1 - r_2 \cos(\Omega_1 - \Omega_2)] \mathbf{u}_z
$$

Thus 5.8 can be expanded into its components along unit vectors $\mathbf{u}_r$, $\mathbf{u}_\Omega$ and $\mathbf{u}_z$ giving

$$
B_r = \frac{\mu_0 r_2 I}{4\pi w} \int_0^{2\pi} \int_{-\frac{w}{2}}^{\frac{w}{2}} \frac{(z_1 - z_2) \cos(\Omega_1 - \Omega_2)}{\sqrt{r_1^2 + r_2^2 + (z_1 - z_2)^2 - 2r_1 r_2 \cos(\Omega_1 - \Omega_2)}} dz_2 d\Omega_2 (5.9)
$$

$$
B_\Omega = \frac{\mu_0 I}{4\pi w} \int_0^{2\pi} \int_{-\frac{w}{2}}^{\frac{w}{2}} \frac{-(z_1 - z_2) \sin(\Omega_1 - \Omega_2)}{\sqrt{r_1^2 + r_2^2 + (z_1 - z_2)^2 - 2r_1 r_2 \cos(\Omega_1 - \Omega_2)}} dz_2 d\Omega_2 (5.10)
$$

$$
B_z = \frac{\mu_0 I}{4\pi w} \int_0^{2\pi} \int_{-\frac{w}{2}}^{\frac{w}{2}} \frac{r_1 - r_2 \cos(\Omega_1 - \Omega_2)}{\sqrt{r_1^2 + r_2^2 + (z_1 - z_2)^2 - 2r_1 r_2 \cos(\Omega_1 - \Omega_2)}} dz_2 d\Omega_2 (5.11)
$$

Because of the inherent symmetry of the system around $\mathbf{u}_\Omega$ and the fact that $\mathbf{J}(r')$ is defined along $\mathbf{u}_\Omega$, $B_\Omega$ in 5.10 is equal to zero. Also, the symmetry of the coil allows the $xyz$ reference system at the center of the base ring to be oriented in such a way that the $x$ axis is always aligned with the unit vector $\mathbf{u}_r$ at point $B$. This eliminates the $\Omega_1$ dependence of 5.9 and 5.11, and $\Omega_1$ will now equal zero. Hence, the magnetic field distribution at any point in space is given by

$$
\mathbf{B} = B_r \mathbf{u}_r + B_z \mathbf{u}_z (5.12)
$$
with

\[ B_r = \frac{\mu_0 r_2 I}{4\pi w} \int_0^{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{(z_1 - z_2) \cos(\Omega_2)}{\sqrt{r_1^2 + r_2^2 + (z_1 - z_2)^2 - 2r_1 r_2 \cos(\Omega_2)}} \frac{dz_2 d\Omega_2}{3} \]  

(5.13)

\[ B_z = \frac{\mu_0 I}{4\pi w} \int_0^{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{r_1 - r_2 \cos(\Omega_2)}{\sqrt{r_1^2 + r_2^2 + (z_1 - z_2)^2 - 2r_1 r_2 \cos(\Omega_2)}} \frac{dz_2 d\Omega_2}{3} \]  

(5.14)

The magnetic field distribution over several section planes parallel to the \( x-y \) plane are shown in Figure 5.4 to Figure 5.6. They were obtained using 5.12 with configuration parameters \( I = 1 \text{A}, r_2 = 0.06 \text{m} \) and \( w = 0.03 \text{m} \). The planes are at \( z = 0 \text{m}, z = 0.03 \text{m} \) and \( z = 0.06 \text{m} \) above the \( x-y \) plane. Figure 5.3 shows a cross section through the coil on the \( z = 0 \) section plane. The \( z = 0 \) section plane coincides with the \( x-y \) plane since it is defined by \( z = 0 \).

![Figure 5.3: A cross section through the RF coil with the \( z = 0 \) section plane.](image-url)
Figure 5.4: Magnitude plot of the magnetic field distribution in $0.1\mu$T over the $z = 0$ section plane for Mode 0 operation, (a) 2D cross section, (b) 3D surface plot. All spatial dimensions are given in m.
Figure 5.5: Magnitude plot of the magnetic field distribution in 0.1µT over the $z = 0.03$ section plane for Mode 0 operation, (a) 2D cross section, (b) 3D surface plot. All spatial dimensions are given in m.
Figure 5.6: Magnitude plot of the magnetic field distribution in $0.1\mu T$ over the $z = 0.06$ section plane for Mode 0 operation, (a) 2D cross section, (b) 3D surface plot. All spatial dimensions are given in m.
Figure 5.7: Magnitude of the magnetic field distribution for Mode 0 operation showing discontinuities at the boundary where the section plane intersects the base ring.

The plots shown in Figure 5.4 through Figure 5.6 reveal symmetry about the $z$ axis. It was observed that the plots of the magnetic field distribution with had discontinuities at $r = 0.06$m on section planes that intersect with the base ring. This is a direct consequence of the magnetic boundary condition at the conductive surface of the base ring where an air-metallic boundary exists. It introduces a discontinuity in the tangential component of the magnetic field at the air-metallic interface as a result of the surface current density on the ring. This can be seen in Figure 5.7. For very large radial distances from the axis of the coil as compared with its radius, the magnitude of the magnetic field decreases with an inverse square law relationship. Also, as the section plane moves further away from the $z$ axis, the magnitude of the magnetic field distribution on the plane becomes narrower and narrower, approaching a point distribution. This is consistent with the idea of a magnetic dipole since the coil approximates a magnetic dipole at very large distances while operating in Mode 0.

Next, we now consider the variation of the magnitude of the magnetic field distribution along section planes that are parallel to the $x$-$z$ plane. A typical cross section through
the coil over the \( y = 0 \) plane is shown in Figure 5.8. We consider section planes at \( y = 0 \) m, \( y = 0.03 \) m and \( y = 0.06 \) m. The coil is operating in Mode 0 with configuration parameters \( I = 1 \) A, \( r_2 = 0.06 \) m and \( w = 0.03 \) m. The corresponding 2D and 3D plots are shown in Fig. 5.10 through Fig. 5.12.

*Figure 5.8:* A cross section through the RF coil with the \( y = 0 \) section plane.
Figure 5.9: Magnitude plot of the magnetic field distribution in 0.1\( \mu \text{T} \) over the \( y = 0 \) section plane for Mode 0 operation, (a) 2D cross section, (b) 3D surface plot. All spatial dimensions are given in m.
Figure 5.10: Magnitude plot of the magnetic field distribution in 0.1µT over the $y = 0.03$ section plane for Mode 0 operation, (a) 2D cross section, (b) 3D surface plot. All spatial dimensions are given in m.
Figure 5.11: Magnitude plot of the magnetic field distribution in 0.1µT over the y = 0.06 section plane for Mode 0 operation, (a) 2D cross section, (b) 3D surface plot. All spatial dimensions are given in m.
Figure 5.12: Magnitude plot of the magnetic field distribution in 0.1 \( \mu \)T over the \( z = 0 \) section plane for Mode 1 operation, (a) 2D cross section, (b) 3D surface plot. All spatial dimensions are given in m.
Figure 5.13: Magnitude plot of the magnetic field distribution in 0.1µT over the y = 0 section plane for Mode 1 operation, (a) 2D cross section, (b) 3D surface plot. All spatial dimensions are given in m.
In order to determine the magnetic field distribution of the RF coil in Mode 1, we applied 5.12, 5.9 and 5.11 to each of the three straps that made up the coil, see (Figure 4.1) and summed the resulting vector fields. There would be no contributions from 5.10 because the direction of the surface current is along $u_{\Omega}$. In evaluating the field integrals presented in 5.9 and 5.11 for Mode 1 operation, the upper limits of the outer integrals were changed from $2\pi$ to $\pi$ because the strap elements are no longer circular, but possess a semi circular orientation. Plots of the magnitude of the magnetic field distribution on the $z = 0$ and $y = 0$ section planes are shown in Figure 5.12 and Figure 5.13. The configuration parameters remain unchanged except for the introduction of a surface current of magnitude $I = 2$A on the top strap element. Here again, the discontinuities in the magnetic field at the metal-air boundaries are clearly visible in both plots.

5.3 Method of Moments Software

Implementation

The software implementation of the Method of Moments formulation discussed in chapter 4 was developed using the Microsoft Visual Studio.Net C++ development environment and the OpenGL application programming interface (API) graphics library. The Microsoft Visual Studio.Net C++ development environment provides the Microsoft Foundation Class and associated Template library (MFC&T) that provide the tools needed to create full-featured Windows based applications. OpenGL is the leading industry standard application programming interface for interactive 3D graphics rendering and 2D imaging. It provides device-independent support for common low-level 3D graphics drawing operations such as polygon specification, basic lighting control, transformation specification, and frame buffer operations like blending and depth-buffering [47]. The OpenGL API graphics library also comes bundled with the
The software implementation of the Method of Moments showing the main application window and a loaded surface mesh structure.


The main application window of the software implementation of the MoM, showing the main graphical user interface, is shown in Figure 5.14. The application was developed specifically to solve simulation problems that are modeled using only triangular surface mesh elements. It determines the electromagnetic characteristics of the system by solving the MoM formulation for the current distribution, and then uses Maxwell's equations to obtain solutions for the other electromagnetic parameters.

The main graphical user interface of the application is made up of a top level menu system, a toolbar, a docked dialog bar and a status bar. The top level menu system allows the users to specify and configure the system to be solved. It also includes menu items for pre-processing and post-processing of the solution. The dialog bar on the left of the graphical user interface contains display and control options. The toolbar, located just under the top level menu system, provides a subset of the functionality of the top level menu system. Its main purpose is to serve as a shortcut for frequently used menu items.
The user is expected to provide a mesh of the structure, identify the voltage ports and specify solution parameters such as voltage magnitude and operating frequency. The mesh should be made up of triangular surface elements with zero-based indices. The application then generates the necessary field solutions.

5.4 Method of Moments Simulation

We now consider the field solutions of the RF coil system when it is operating in Mode 0. The configuration parameters are identical to those used in the Biot-Savart simulation except for the introduction of voltage excitations at the two ports of the base ring instead of using current excitations. Operation in Mode 0 is characterized by the flow of a cylindrical ring current in the base ring and no current in the upper strap element. Hence, the voltage excitations at the two ports must be 180° out of phase. We will incorporate two voltage sources of -1V and 1V at port 1 and 2 respectively. We begin the solution process by creating a triangular surface mesh of the RF coil using EasyMesh [48]. EasyMesh generates two dimensional, unstructured, Delaunay, and constrained Delaunay triangulations in general domains. As such, another application was developed to transform the 2D generated mesh into a 3D surface mesh representation of the coil. The generated 2D and 3D mesh of the system are shown in Figure 5.15.
Figure 5.15: Triangular surface mesh representation of the RF coil generated by EasyMesh: (a) 2D mesh before transformation, (b) 3D mesh after transformation.

After the 3D surface mesh was generated, it was imported into the application from the top level menu. Next, we define the voltage elements and the operating frequency, and apply the voltage excitations. This was done using the Solution dialog window. The Solution dialog window is shown in Figure 5.16 with the Element and Voltage Element tabs selected. The required frequency of operation is 63.87MHz in a 1.5T MRI system [2.5].
Figure 5.16: The Solution dialog window showing: (a) the Element tab, (b) the Voltage Element tab.
On the *Element* tab we define the voltage elements at the two ports, while we specify the operating frequency and apply the voltage excitations on the *Voltage Element* tab. We now solve for the current distribution on the surface of the RF coil. Figure 5.17 displays the current distribution on the surface of the coil. The current distribution is relatively uniform across the surface of a strip, but it increases sharply as we move towards the edges of the strip. This agrees very well with our initial assumption that the surface current density is uniform across the surface of a strap element.

*Figure 5.17:* Magnitude plot of the surface current distribution over the RF coil when operating in Mode 0.

For obtaining field solutions in the Mode 1 configuration, we used the same application parameters as in Mode 0, but with a different voltage excitation of 1V at each of the two ports. The voltages at the ports must be in phase in order to operate in the Mode 1 configuration where the current in the upper strap element is twice that in each of the base ring elements. A solution was obtained and the current distribution over the surface of the coil is as shown in Figure 5.18. Again, we observed that the surface current density is relatively uniform across the surface. The solution to the field problem is complete and the results can now be used to determine other...
relevant electromagnetic properties of the system. We consider the determination of the magnitude of the magnetic field distribution in section planes that are parallel to the $x$-$y$ and $x$-$z$ planes. The 2D surface meshes of the planes were created using EasyMesh and loaded from the top level menu of the application. The magnetic field variations in the planes for Mode 0 and Mode 1 operation were plotted and are displayed in Figure 5.19 and Figure 5.20. A comparison of Figure 5.19 and Figure 5.20 with the 2D field plots obtained using Biot-Savart Law in the $x$-$y$ and $x$-$z$ planes reveal marked similarities in the spatial distribution of the magnetic field across the 2D planes.

![Figure 5.18](image.png)

*Figure 5.18:* Magnitude plot of the surface current distribution over the RF coil when operating in Mode 1.
Figure 5.19: 2D magnitude plot of the magnetic field distribution in (a) the $x$-$y$ plane and (b) the $x$-$z$ plane for Mode 0 operation.
Figure 5.20: 2D magnitude plot of the magnetic field distribution in (a) the $x$-$y$ plane and (b) the $x$-$z$ plane for Mode 1 operation.
Finally, we consider the determination of the magnitude of the magnetic field distribution over the external surface of the female breast for Mode 0 and Mode 1 operation. For this, we acquired a 3D triangular mesh of the female torso from 3D Café [49] and converted it into the appropriate format used by the application. We then loaded the mesh using the appropriate menu item from the top level menu of the application and plotted the magnetic field distribution over the surface of the mesh. Figure 5.21 and Figure 5.22 show the frontal and lateral views of the magnetic field distribution for Mode 0 and Mode 1 operation.

\[ E \]

\[ B \]

\[ H \]

\[ D \]

\[ I \]

\[ J \]

\[ K \]

\[ L \]

\[ M \]

\[ N \]

\[ O \]

\[ P \]

\[ Q \]

\[ R \]

\[ S \]

\[ T \]

\[ U \]

\[ V \]

\[ W \]

\[ X \]

\[ Y \]

\[ Z \]

\[ \alpha \]

\[ \beta \]

\[ \gamma \]

\[ \delta \]

\[ \epsilon \]

\[ \zeta \]

\[ \eta \]

\[ \theta \]

\[ \iota \]

\[ \kappa \]

\[ \lambda \]

\[ \mu \]

\[ \nu \]

\[ \xi \]

\[ \omicron \]

\[ \pi \]

\[ \rho \]

\[ \sigma \]

\[ \tau \]

\[ \upsilon \]

\[ \phi \]

\[ \chi \]

\[ \psi \]

\[ \omega \]

\[ \alpha \]

\[ \beta \]

\[ \gamma \]

\[ \delta \]

\[ \epsilon \]

\[ \zeta \]

\[ \eta \]

\[ \theta \]

\[ \iota \]

\[ \kappa \]

\[ \lambda \]

\[ \mu \]

\[ \nu \]

\[ \xi \]

\[ \omicron \]

\[ \pi \]

\[ \rho \]

\[ \sigma \]

\[ \tau \]

\[ \upsilon \]

\[ \phi \]

\[ \chi \]

\[ \psi \]

\[ \omega \]

\[ \alpha \]

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\[ \gamma \]

\[ \delta \]

\[ \epsilon \]

\[ \zeta \]

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\[ \theta \]

\[ \iota \]

\[ \kappa \]

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\[ \alpha \]

\[ \beta \]

\[ \gamma \]

\[ \delta \]

\[ \epsilon \]

\[ \zeta \]
Figure 5.22: Frontal views of the magnetic field distribution over the surface of the female breast for (a) Mode 0 operation, and (b) Mode 1 operation.
around the region of the axillary lymph nodes chain [Figure 3.3]. Also, the magnetic field distribution is fairly uniform across this region ensuring optimum sensitivity and specificity. This agrees very much with the analytical results obtained for the magnetic field distribution as presented in Figure 5.7 when the coil is operating in Mode 0. Figure 5.7 reveals the existence of a uniform magnetic field gradient at distances between 0.015m and 0.06m from the base ring. We also observed the existence of a fairly uniform magnetic field around the region of the internal mammary lymph nodes chain [Figure 3.3] when the coil is operating in Mode 1. This magnetic field also extends over much of the volume around the breast as shown in Figure 5.21(b) and Figure 5.22(b). We recognize that if both modes are operated simultaneously in a quadrature configuration, then a fairly constant magnetic field distribution would exist around the breast and the chest wall. Thus, based on the results obtained using the MoM, we conclude that the unique orientation of the RF coil conductors relative to each other guarantee ample magnetic field uniformity and magnetic field coverage around the ROI.
5.5 Finite Element Simulation with Ansoft HFSS v8.5

In this section, we present simulation results in the form of 2D magnitude plots obtained using the Ansoft HFSS v8.5 finite element package. Ansoft HFSS v8.5 is a commercially available interactive software package for calculating the electromagnetic characteristics of a system [50]. It is one of the market's leading software solutions available for the modeling of electromagnetic phenomena. It models the system using finite element analysis and provides solutions for the basic electromagnetic field quantities including radiated near and far fields, characteristic port impedances and propagation constants, generalized S-parameters and S-parameters renormalized to specific port impedances, and the eigen or resonance modes of the system.

We now present 2D magnitude plots of the magnetic field distribution in the $x$-$y$ and $x$-$z$ planes, and 3D surface plots of the magnitude of the current density on the surface of the coil for Mode 0 and Mode 1 operation. The Ansoft HFSS v8.5 finite element model used in creating these plots had the same parameter configuration as the model used in the Method of Moment simulation, including the voltage excitation that distinguishes the modes of operation. The model was created using the 3D Modeler package that came bundled with the Ansoft package. The results of the simulation are shown in Figure 5.23 - Figure 5.25. It was observed that these plots had the same uniformity in the magnetic field distribution, and exhibited the same degree of coverage as those obtained using the Method of Moments and Biot-Savarts Law. Thus, this asserts the validity of the simulations obtained using the Method of Moments and Biot-Savarts law.
Figure 5.23: 2D Magnitude plot of the magnetic field distribution in (a) the $x$-$y$ plane and (b) the $x$-$z$ plane for Mode 0 operation.
Figure 5.24: 2D Magnitude plot of the magnetic field distribution in (a) the x-y plane and (b) the x-z plane for Mode 1 operation.
Figure 5.25: Magnitude plot of the surface current distribution of the RF coil for (a) Mode 0 operation, and (b) Mode 1.
5.6 System Model Analysis and Results

Figure 5.26 shows a simplified T-Network model of the RF coil. The inductances $L_1$, $L_2$ and $L_3$ are total inductances and are made up of a self-inductance and mutual-inductances due to neighboring circuit elements. Resistances $R_1$, $R_2$ and $R_3$ are the electrical resistance of the circuit elements respectively. The circuit path 1-3-2 represents the base ring element and the circuit path 3-4 represent the upper strap element. The simplified T-Network model of the coil results from the fact that the total inductance of the RF coil is dependent only on its spatial configuration and material properties.

![Simplified T-network circuit model of the RF coil.](image)

*Figure 5.26: Simplified T-network circuit model of the RF coil.*

The impedance matrix of the T-Network model [32] is given as

$$Z = \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix} \tag{5.15}$$

where

$$Z_1 = R_1 + j\omega L_1$$

$$Z_2 = R_2 + j\omega L_2$$

$$Z_3 = R_3 + j\omega L_3$$
The symmetry of the passive linear system dictates that $Z_1 = Z_2$. If a voltage $V_1$ is applied to port 1 and a voltage $V_2$ is applied to port 2, then the currents $I_{13}$, $I_{23}$ and $I_{34}$ in the three circuit paths 1-3, 2-3, 3-4 can be found using

$$
\begin{bmatrix}
V_1 \\
V_2 
\end{bmatrix} = \begin{bmatrix}
Z_1 + Z_3 & Z_3 \\
Z_3 & Z_2 + Z_3 
\end{bmatrix} \cdot
\begin{bmatrix}
I_{13} \\
I_{23} 
\end{bmatrix}
$$

(5.16)

and

$$I_{34} = I_{13} + I_{23}
$$

(5.17)

Since operation in the Mode 0 configuration is defined by a cylindrical current flow in the base ring only (see Figure 3.4), $I_{34}$ in 5.17 is ideally zero. Thus,

$$I_{13} = -I_{23}
$$

(5.18)

Using 5.18 in 5.17 yields

$$
\begin{bmatrix}
V_1 \\
V_2 
\end{bmatrix} = \begin{bmatrix}
Z_1 + Z_3 & Z_3 \\
Z_3 & Z_2 + Z_3 
\end{bmatrix} \cdot
\begin{bmatrix}
-I_{23} \\
I_{23} 
\end{bmatrix} = \begin{bmatrix}
-(Z_1 + Z_3)I_{23} + Z_3I_{23} \\
-Z_3I_{23} + (Z_2 + Z_3)I_{23} 
\end{bmatrix}
$$

(5.19)

which implies that

$$V_1 = -V_2
$$

(5.20)

Thus, the voltage at port 1 and port 2 must be $180^\circ$ out of phase when the RF coil is operating in Mode 0. For operation in Mode 1, the symmetry of the RF coil ensures that $I_{13} = I_{23}$. Hence, 5.16 reduces to

$$V_1 = V_2
$$

(5.21)

which indicates that the voltages at both ports must be in phase.
Next, we consider the quadrature superposition of Mode 0 and Mode 1 resulting in the simultaneous operation of both modes. Let the voltage at port 1, $V_1$, and that at port 2, $V_2$, be given by

$$V_1 = V - jV \quad (5.22)$$

$$V_2 = -V - jV \quad (5.23)$$

Using the basic definition of $Z$ for a two port network, 5.16 can be rewritten as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_{13} \\ I_{23} \end{bmatrix} = \begin{bmatrix} Z_{11}I_{13} + Z_{12}I_{23} \\ Z_{21}I_{13} + Z_{22}I_{23} \end{bmatrix} \quad (5.24)$$

where $Z_{11} = Z_1 + Z_3$, $Z_{12} = Z_{21} = Z_3$, and $Z_{22} = Z_2 + Z_3$. Dividing 5.22 by 5.23 and using this relationship in 5.24 yields

$$\frac{V_1}{V_2} = \frac{Z_{11}I_{13} + Z_{12}I_{23}}{Z_{21}I_{13} + Z_{22}I_{23}} = j \quad (5.25)$$

Simplifying 5.25 using $K$ such that $K = \frac{I_{23}}{I_{13}}$ gives

$$j = \frac{Z_{11} + Z_{12}K}{Z_{21} + Z_{22}K} \quad (5.26)$$

Solving for $K$ yields

$$K = \frac{-Z_{11} + jZ_{21}}{Z_{12} - jZ_{22}} \quad (5.27)$$

From 5.24, the input impedance $Z@1$ seen looking into port 1 is given by

$$Z@1 = \frac{V_1}{I_{13}} = Z_{11} + KZ_{12} \quad (5.28)$$

while the input impedance $Z@2$ seen looking into port 2 is given by

$$Z@2 = \frac{V_2}{I_{23}} = Z_{22} + \frac{Z_{21}}{K} \quad (5.29)$$
Thus, knowledge of the input impedance at port 1 and port 2 would facilitate the
design of tuning and matching networks that would tune and match both operating
modes simultaneously.

With reference to Figure 5.26, Table 5.2 compares the network parameters $L_1$, $L_2$ and
$L_3$ obtained using the three simulation method. In the integral formulation technique,
4.31, 4.38 and 4.45 were applied to the model shown in Figure 4.1 using configuration
parameters obtained from Table 5.1. It should be noted that the model in Figure
5.26 represents a simplified version of the model depicted in Figure 4.1. The results
obtained using the MoM differ by about 4nH from those obtained using the other
techniques. This can be attributed to the number of triangles in the mesh model
used. As a comparison, the Ansoft HFSS v8.5 mesh model contained 64351 tetra-
hedral versus the 2996 triangles that make up the MoM mesh model. The complete
impedance matrix $Z$ of the RF coil was obtained using Ansoft HFSS v8.5 and it is
given below as

$$
Z = \begin{bmatrix}
0.1571 + j64.941\Omega & 0.1115 + j30.551\Omega \\
0.1115 + j30.551\Omega & 0.1571 + j64.941\Omega 
\end{bmatrix}
$$

(5.30)

<table>
<thead>
<tr>
<th>Simulation Technique</th>
<th>$L_1$</th>
<th>$L_2$</th>
<th>$L_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integral Formulation</td>
<td>86.97nH</td>
<td>86.97nH</td>
<td>76.156nH</td>
</tr>
<tr>
<td>Method of Moments</td>
<td>79.36nH</td>
<td>79.36nH</td>
<td>72.57nH</td>
</tr>
<tr>
<td>Ansoft HFSS v8.5</td>
<td>85.696nH</td>
<td>85.696nH</td>
<td>76.1275nH</td>
</tr>
</tbody>
</table>

Table 5.2: Comparison of inductive network parameters using the three different simulation
techniques.
5.7 Determination of Tuning and Matching Capacitors

In order to tune and match the coil system for simultaneous operation in Mode 0 and Mode 1, the input impedances presented at port 1 and port 2 must be tuned and matched to the external system. The input impedance at port 1 and port 2 can be found using 5.27, 5.28 and 5.29 once the impedance matrix $Z$ of the coil system has been determined. The tuning and matching network chosen was the lumped-element L-section network discussed in Chapter 3. The lumped-element L-section network was chosen because of its simplicity in design and implementation. The network was implemented using only capacitive elements because of size constraints and reduced parasitic effects.

We now consider the determination of the tuning and matching capacitors that constitute the lumped-element L-section network. We start by computing the input impedance at port 1 and port 2. This is obtained using 5.27, 5.28 and 5.29 as

$$Z_{@1} = 19.5314 + j41.3567 \Omega$$

$$Z_{@2} = -19.4259 + j41.4542 \Omega$$

Since the real part of $Z_{@2}$ is less than zero, the coil system cannot be tuned and matched to the external network using a lumped-element L-section network. In order to overcome this problem, capacitors must be introduced into the structure of the RF coil so as to alter the impedance matrix given in 5.30 and subsequently $Z_{@2}$. Figure 5.27 shows a simplified T-network model of the coil system with capacitors to alter the impedance matrix, and hence make the real part of $Z_{@2}$ greater than zero.

We begin by removing the capacitors $C_{11}$ and $C_{22}$ from Figure 5.27 and investigating
Figure 5.27: A modified T-network model with capacitors to alter the impedance matrix $Z$ presented in 5.30.

the effects of $C_{33}$ only. We chose the capacitor $C_{33}$ such that

$$j\omega L_3 + \frac{1}{j\omega C_{33}} = 0$$  \hspace{1cm} (5.31)

Hence, $Z_{12} = Z_{21} = Z_3 = R_3$. Now, using 5.27, 5.28 and 5.29, we obtained $Z_{@1}$ and $Z_{@2}$ as

$$Z_{@1} = 0.1571 + j34.2796 \Omega$$
$$Z_{@2} = 0.1571 + j34.5026 \Omega$$

We now use the Matlab program developed in Chapter 3 to determine the tuning and matching capacitors needed to separately tune and match $Z_{@1}$ and $Z_{@2}$ to the external network with characteristic impedance $Z_0$ of 50\Ohm. The main graphical user interface of the program is shown in Figure 5.28. The outputs of the program at port 1 and port 2 are shown in Figure 5.29 and Figure 5.30. There are four unique lumped-element configurations for the L-section network. The desired configuration is shown in Figure 5.31 and the results obtained for $C_1$ and $C_2$ are shown in Table 5.3. From Table 5.3, the value of $C_2$ is quite small and hence can be influence by stray capacitances.
Figure 5.28: Graphical user interface of the Matlab program designed to determine lumped-element components of an L-section matching network.

Figure 5.29: Output of the Matlab program showing the four possible lumped-element configurations of an L-section matching network at port 1 of the coil system.

Figure 5.30: Output of the Matlab program showing the four possible lumped-element configurations of an L-section matching network at port 2 of the coil system.
Figure 5.31: Desired topology and configuration of the L-section matching network.

Table 5.3: Results obtained for $C_1$ and $C_2$ at port 1 and port 2.

<table>
<thead>
<tr>
<th>Ports</th>
<th>$C_1$</th>
<th>$C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>68.6296pF</td>
<td>4.0883pF</td>
</tr>
<tr>
<td>2</td>
<td>68.1859pF</td>
<td>4.0617pF</td>
</tr>
</tbody>
</table>

In order to increase the value of $C_2$, we introduce capacitors $C_{11}$ and $C_{22}$ into the structure of the coil system as shown in Figure 5.27. The effect of $C_{11}$ and $C_{22}$ on the system is a reduction in the reactance of the coil. We now investigate the effects of varying reactance on the input impedance at the ports. This is shown in Figure 5.32 and Figure 5.33. Since 5.31 must always hold true, the only reactance in the coil system are given by

$$X_1 = j\omega L_1 + \frac{1}{j\omega C_{11}} \quad (5.32)$$

and

$$X_2 = j\omega L_2 + \frac{1}{j\omega C_{22}} \quad (5.33)$$

where $X_1 = X_2$ since $C_{11}$ and $C_{22}$ must be of the same value in order to maintain the symmetry in the coil system. From Figure 5.32, we observe that if the values of $C_{11}$ and $C_{22}$ are adjusted such that $X_1$ or $X_2$ varying in the range $j6\Omega$ to $j34\Omega$, then the real part of the input impedance at any of the ports stays relatively constant while the imaginary part varies linear by the changing reactance. Thus, the values of the tuning and matching capacitors $C_1$ and $C_2$ can be increased by varying the reactance.
Figure 5.32: The effects of varying reactance on the real part of the input impedance at port1 and port 2.

Figure 5.33: The effects of varying reactance on the imaginary part of the input impedance at port1 and port 2.

$X_1$ and $X_2$ with $C_{11}$ and $C_{22}$ using 5.32 and 5.33.

Since $j\omega L_1 = j\omega L_2 = j34.3904\Omega$, we arbitrarily chose

$$\frac{1}{j\omega C_{11}} = \frac{1}{j\omega C_{22}} = -j24\Omega$$

so that $X_1 = X_2 = j10.3904\Omega$. The input impedance at port 1 and port 2 now become

$$Z_{@1} = 0.1571 + j10.2813\Omega$$

$$Z_{@2} = 0.1571 + j10.5043\Omega$$
Table 5.4: Results obtained for $C_1$ and $C_2$ at port 1 and port 2 with $C_{11}$ and $C_{22}$.

<table>
<thead>
<tr>
<th>Ports</th>
<th>$C_1$</th>
<th>$C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>229.2417pF</td>
<td>14.1186pF</td>
</tr>
<tr>
<td>2</td>
<td>224.3558pF</td>
<td>13.7957pF</td>
</tr>
</tbody>
</table>

Figure 5.34: Output of the Matlab program showing the four possible lumped-element configurations of an L-section matching network at port 1 of the coil system with $C_{11}$ and $C_{22}$.

The tuning and matching capacitors $C_1$ and $C_2$ were again determined using the Matlab program and the results were tabulated in Table 5.4. Figure 5.34 and Figure 5.35 show the four unique lumped-element configurations of the L-section network obtained from the program. The values of $C_{11}$, $C_{22}$ and $C_{33}$ were determined using 5.31, 5.32 and 5.33, and are tabulated in Table 5.5. Figure 5.36 shows the complete T-network model schematic of the RF coil driven by two voltages sources that are 90 degrees out of phase. This is necessary in order to satisfy 5.22 and 5.23. Finally, Figure 5.37 and Figure 5.38 show plots of the magnitude of the reflection coefficients at the input ports as a function of frequency.
**Figure 5.35:** Output of the Matlab program showing the four possible lumped-element configurations of an L-section matching network at port 2 of the coil system with $C_{11}$ and $C_{22}$.

**Table 5.5:** Values of capacitors used in altering the reactance of the coil system.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>$C_{11}$</td>
<td>103.8274pF</td>
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</tr>
<tr>
<td>$C_{22}$</td>
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<td></td>
</tr>
<tr>
<td>$C_{33}$</td>
<td>81.5652pF</td>
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</table>

**Figure 5.36:** Complete T-Network model schematic.
Figure 5.37: Pspice plot of the magnitude of the reflection coefficient versus frequency at port 1.

Figure 5.38: Pspice plot of the magnitude of the reflection coefficient versus frequency at port 2.
Chapter 6

PROTOTYPE CONSTRUCTION

6.1 Coil Construction

The RF coil was constructed on a cast acrylic former. Cast acrylic is a relatively cheap and very durable material with good electrical insulating properties at relatively low frequencies. It is also shatter resistant and very easy to fabricate. The former was fabricated from a cylindrical tube with an outer diameter of 120mm and a thickness of 6.35mm. The dimensions of the tube were chosen based on price constraints and the design parameters. Two cylindrical sections 30mm high were cut from the tube in order to fabricate the base ring former and the upper strap former, respectively. One of the cylindrical sections was divided into two halves along its diameter with one half forming the upper strap former, while the other section formed the base ring former. The upper strap former and the base ring former were held together using fiber glass rods that were dipped in glue and inserted into four cylindrical holes in both former structures. Each hole is 3.18mm in diameter and was drilled 6.35mm deep. The holes in the upper strap former were made to align vertically with the holes in the base ring former. Figure 6.1 shows a schematic layout of the RF coil former in four different projection views.
Figure 6.1: Layout drawing of the cast acrylic coil former: (a) Isometric View, (b) Top view, (c) Front view and (d) Side view.
After the former was constructed and assembled, 30mm wide adhesive copper tapes were placed on the external surfaces to form the RF coil conductors. Tiny gaps, 2mm in length, were left on the surface of the former between the copper tapes where the capacitors on the coil were soldered. The copper tape had a thickness of 38µm which is sufficient to allow RF propagation and impede low-frequency eddy currents caused by switching gradient fields. The capacitors and other components used in building the coil were magnetic since the coil is intended for conceptual studies outside the MRI system. Both variable and fixed capacitors were used since the exact values required were not available. Also, the use of variable capacitors help to compensate for the deviation between the theoretical and physical coil models and most importantly, helps to compensate for anticipated changes resulting from biological loading effects. All fixed capacitors were available in surface mount packaging with a capacitive tolerance of ±5%. The tuning and matching capacitors associated with the two input ports were mounted on two separate 45mm by 10mm rectangular PCBs. These were double-layered PCBs manufactured by ExpressPCB [52]. They were manufactured on a low-loss substrate (GML1000) in order to improve the quality factor of any stray capacitance associated with the board. The PCBs were glued firmly to the base ring former using Araldite glue. SMA connectors, one for each port, were mounted on the PCBs in order to couple the RF signal through the ports to the external network. Figure 6.2 shows the electrical schematic of the RF coil while Figure 6.3 shows the fully assembled coil prototype. The SMA connectors and the variable capacitors on the PCBs are clearly seen in Figure 6.3(a), while Figure 6.3(b) shows the coil in perspective view. A detailed list of all the electrical components used can be found in the Appendix.
Figure 6.2: Circuit schematic drawing of the RF coil featuring distributed and lumped elements.

Figure 6.3: Photographs of the RF coil prototype (a) front view, (b) perspective view.
6.2 Bench Testing and Results

After the coil was constructed, it was tuned and tested in order to investigate the agreement with the theoretical coil model. The first step in the bench testing procedure was to ensure that all the connections of the PCBs were intact and that no short circuit connections have been created. Next, the coil was connected to the HP 8714ES network analyzer for analysis and tuning. One port was connected to the network analyzer using a calibrated BNC cable while the other port was terminated into a 50Ω load. All variables capacitors were set to their maximum values at the onset of the analysis. The variable matching capacitors were then adjusted until the coil was appropriately matched. The $S_{11}$ frequency plot obtained by the network analyzer revealed the existence of two resonant modes as predicted by the theoretical model. This can be seen in Figure 6.4(a). As the variable capacitors on the coil were adjusted, the two resonant modes approached each other until finally coinciding at a single resonance frequency where quadrature operation was achieved. This is shown in Figure 6.4(b)-(d). The final step in the bench testing procedure involves tuning the RF coil and measuring its unloaded quality factor $Q$. The tuning process was accomplished by adjusting the tuning capacitors at the ports until resonance was achieved at a frequency of 63.34MHz. This frequency is very close to the desired resonance frequency of 63.87MHz. Next, the frequency at the lower and upper 3dB points of the $S_{11}$ frequency plot were determined in order to calculate the unloaded quality factor using 3.26. The unload quality factor was determined to be 158.35. A summary of the important coil parameters are tabulated in Table 6.1.
Figure 6.4: Fig. 6.4 Network analyzer S11 frequency plot of the RF coil showing (a) the two resonant modes, (b) the two resonant modes approaching each other in frequency, (c) the lower 3dB point of both modes in quadrature, and (d) the upper 3dB point in quadrature.
Table 6.1: RF coil parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
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<tr>
<td>Coil Diameter</td>
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</tr>
<tr>
<td>Coil Width (strip width)</td>
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</tr>
<tr>
<td>Former outer diameter</td>
<td>120.0mm</td>
</tr>
<tr>
<td>Former inner diameter</td>
<td>107.3mm</td>
</tr>
<tr>
<td>Former material</td>
<td>Cast Acrylic</td>
</tr>
<tr>
<td>Height of former</td>
<td>30.0mm</td>
</tr>
<tr>
<td>Thickness of copper strips</td>
<td>38.0μm</td>
</tr>
<tr>
<td>Operating frequency</td>
<td>63.87MHz</td>
</tr>
<tr>
<td>Quality factor (unloaded $Q$)</td>
<td>158.35</td>
</tr>
<tr>
<td>3dB bandwidth</td>
<td>0.40MHz</td>
</tr>
</tbody>
</table>
Chapter 7

CONCLUSION

7.1 Summary

A new design in RF coil development for breast cancer screening in a Nuclear Magnetic Resonance system has been proposed. The novel approach provides two resonant receiving modes that operate in quadrature, and a region of interest that closely conforms to the volume of the female breast. In this thesis, we discuss the required steps to translate the idea into a working coil prototype operating in a single coil configuration. The result of the implementation is a highly scalable design that provides good spatial coverage and relatively uniform magnetic field distribution around the region of interest.

Although the development is still in its early stages and a number of improvements still need to be made, particularly at higher frequencies where the wavelength becomes comparable to the physical dimensions of the coil, we were able to present a comprehensive lumped-element T-network model that accurately describes the electrical characteristics of the RF coil. We applied two different modeling techniques to obtain the important self-inductance and mutual inductance parameters for various physical dimensions of the coil. The first technique uses an analytical integral
formulation based on Biot-Savarts integral equation, while the second technique utilized an electromagnetic structure simulator based on the Method of Moments with a modified surface RWG basis function. The electromagnetic structure simulator was developed using the Microsoft Visual Studio.Net C++ development environment and the OpenGL 3D graphics API. Results from both modeling techniques were compared with the commercially available ANSOFT HFSS v8.5 package and an excellent agreement was established. Also, the magnetic field distribution in the region of interest was determined, and it was shown that there was indeed ample magnetic field coverage around the diagnostically important lymph node regions of the breast. Furthermore, we were able to show that tuning and matching can be realized using L-section networks when the coil is operating with both modes in quadrature. These facts prove feasible and they re-enforce the ideas presented in this thesis.

7.2 Further Research

As mentioned earlier, the RF coil presented in this thesis is an early prototype and as such there are some considerations that need to be addressed. These include:

- Development of a comprehensive model of the RF coil at higher frequencies. The present model does not account for the transmission line effects that are prominent at frequencies where the wavelength becomes comparable to the physical size of the coil. As such, there were some high order resonant modes in the 500MHz to 1GHz frequency range. We expect these electric phenomena to occur in the range 300MHz to 500MHz which corresponds to magnetic field strength in the range 7T to 11.7T when imaging for protons. The development of an accurate high frequency model would facilitate the design of RF coils that would be used for high-field MRI studies.
• Dual coil configuration. The current design can easily be extended to operate in a dual coil configuration where two RF coils are used for screening both breasts at the same time. Since each coil has two resonant modes operating in quadrature, the MRI system must provide support for two or more signal channels.

• Complex tissue modeling of the breast and associated tissues. Development of a complex tissue model based on the Finite Element Method would provide valuable information regarding magnetic and electric field interactions deep within the breast. Results from such an analysis could be used to improve upon existing coil design and allow for much wider field coverage in and around the volume of the breast. The current model uses the Method of Moments to determine the field distribution around the surface of the breast. This method is very well suited for simulating unloaded coils, but it cannot be employed to simulate a complex biological load.

• Development of a detuning circuitry for the RF coil. The detuning circuitry allows the coil to be isolated from the transmit coil when it is operating as a receiver-only coil.
## Appendix

### Components Listing

<table>
<thead>
<tr>
<th>Qty</th>
<th>Manufacturer</th>
<th>Part Number</th>
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<th>Supplier</th>
<th>Supplier Part Number</th>
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</thead>
<tbody>
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<td>Cap trimmer 1.0-40pF</td>
<td>Digikey</td>
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<tr>
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<td>J495-ND</td>
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ExpressPCB printed circuit board pattern: (a) Top Layer, (b) Bottom Layer.
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