Chapter 6. Tolerance Analysis

When locators have displacements (caused by manufacturing or positioning error), the workpiece will be displaced, and errors will occur on machining surfaces (Figure 6.1). With the given locator tolerances, can we predict the amount of error it causes for machining surfaces? Can we determine the locator tolerances based on the machining surface tolerance specifications? These questions are to be answered in this chapter.

![Figure 6.1 Tolerance Analysis](image)

Tolerance analysis in CAFDV studies the relationship between locator tolerances and machining surface tolerances within a single setup. The scope does not include studies on fixture assembly and multi-setup tolerance stack up.

In CAFDV, tolerance analysis has two tasks – machining surface accuracy check and locator tolerance assignment. The former calculates the machining surface accuracy with given locator tolerances, and the latter finds the optimal locator tolerances based on
machining surface tolerance. The figure below best illustrates the relationship between accuracy check and tolerance assignment.

**Figure 6.2 Accuracy Check and Tolerance Assignment**

In order for computer implementation, machining surfaces are represented by sample points (Section 6.1). Tolerances are then defined based on surface sample points (section 6.2). Then accuracy check and tolerance assignment are discussed (Section 6.1 and 6.2).

### 6.1. Machining Surface Sample Points

For computer implementation, machining surfaces must be represented with finite points. These points are sampled from the surface contour, since the largest surface deviation always occurs on the contour. And the surface accuracy is defined by finding the largest deviation among its contour points.

**Figure 6.3 Surface Sample Points**
For surface deviation calculation, sample points are taken at each vertex on surface, and more points are taken from a curve to increase precision (Figure 6.3).

The deviations of contour points are calculated based on the workpiece location deviation, and the machining surface error is then calculated by its tolerance type.

### 6.2. Definition of Surface Deviation and Accuracy

For a given tolerance type, the machining surface deviation can be calculated based on its sample point deviations. The calculation follows the standards set in ANSI Y-14.5 (ANSI, 1995). Figure 6.4 shows the target surface and the deviated surface, along with their sample points.

![Figure 6.4 Surface Deviation](image)

**Figure 6.4 Surface Deviation**

**Machining Surface Accuracy**

For a given tolerance type, the surface accuracy is the envelop for all possible deviations, which is equivalent to the maximal deviation (the worst case). For a qualified surface, its accuracy must fall within the specified tolerance.

The calculation for each type of machining accuracy is listed in the following sections.
6.2.1. Surface Profile and Line Profile Deviation

For surface and line profile, they are defined as double the maximum sample point deviation. They can be calculated as (Figure 6.4):

\[ \text{dev} = 2 \times \max\{\Delta p_1^n, \Delta p_2^n, \ldots, \Delta p_n^n\} \]  

(6.1)

where,

\[ \cdot \Delta p_i^n = \Delta p_i \cdot n_i \] is the sample point deviation along surface normal direction

6.2.2. Parallelism, Perpendicularity and Angularity Deviation

For parallelism, perpendicularity and angularity, their surface deviations are calculated as the difference between maximum and minimum sample point deviations (Figure 6.4):

\[ \text{dev} = \max\{\Delta p_1^n, \Delta p_2^n, \ldots, \Delta p_n^n\} - \min\{\Delta p_1^n, \Delta p_2^n, \ldots, \Delta p_n^n\} \]  

(6.2)

6.2.3. Position Deviation

The deviation calculation for position type is a little different from other types. The sample points are derived from the cylinder axis instead of from the surface contour. It is defined to be double the maximum deviation from the target axis (Figure 6.5):

\[ \text{dev} = 2 \times \max\{\Delta d_1^n, \Delta d_2^n, \ldots, \Delta d_n^n\} \]  

(6.3)

Figure 6.5 Position Deviation
6.2.4. Other Types of Deviations

Other types of deviation, such as plane surface flatness, cylindrical surface run-out, symmetry, are not considered in this work. The reason is that they are not affected by locator displacements.

6.3. Machining Surface Accuracy Check

The machining surface accuracy is the worst case of all possible surface deviations, so the task is to get a set of locating point deviations, and find the largest machining surface deviation.

As shown by the geometric fixture model, once we know locating point deviations \( \{\Delta d\} \), we can find the workpiece location deviation \( \{\Delta q\} \) as:

\[
\{\Delta q\} = [J]^{-1} \cdot \{\Delta d\}
\]

where:

\[
\{\Delta d\} = \{\Delta d_1 \quad \Delta d_2 \quad \ldots \quad \Delta d_n\}
\]

\[
\{\Delta q\} = \{\Delta x \quad \Delta y \quad \Delta z \quad \Delta \alpha \quad \Delta \beta \quad \Delta \gamma\}^T
\]

![Figure 6.6 Machining Surface Accuracy Check](image_url)
As shown in Figure 6.6, when the locating points have certain deviations, they will cause deviation for the workpiece. Then the sample points will have deviations, and these deviations can then be used to calculate the surface deviation.

Let \( \{q_0\} \) be the ideal workpiece location, \( T_G^W(q) \) be the 4x4 workpiece transformation matrix based on location \( \{q\} \), and \( \{p_i^W\} \) be the surface sample point coordinates in WCS, we can have sample point deviations in GCS \( \{\Delta P_i^G\} \) as:

\[
\Delta P_i^G = P_i^G - P_i^G = [T_G^W(q_0 + \Delta q)] \cdot P_i^W - [T_G^W(q_0)] \cdot P_i^W
\]

\[
\Rightarrow \Delta P_i^G = [T_G^W(q_0 + \Delta q) - T_G^W(q_0)] \cdot P_i^W
\]

\[
\Rightarrow \Delta P_i^G = [T_G^W(q_0 + [J]^{-1} \cdot \{\Delta d\}) - T_G^W(q_0)] \cdot P_i^W
\]

(6.5)

For a given set of locating point deviations \( \{\Delta d\} \), the machining surface deviation can then be calculated following the “definition of machining surface deviation”:

\[
\Delta p_i^n = \Delta p_i \cdot n_i
\]

\[
\text{dev} = \text{dev}\{\Delta p_1^n, \Delta p_2^n, \ldots, \Delta p_n^n\}
\]

(6.6)

By varying the locating point displacements in the locating point tolerance zone, we can get a set of machining surface deviations. The machining surface accuracy is the worst case of all surface deviations.

\[
\text{acc} = \max\{\text{dev}_1, \text{dev}_2, \ldots, \text{dev}_m\}
\]

(6.7)

6.4. Locator Tolerance Assignment

Locator tolerance assignment is to find the tolerance specification for locators, so that all machining surface tolerance requirements can be satisfied. In order to reasonably
distribute tolerances to each locator, first we need to find out how sensitive the machining surface is to each locator. The more sensitive locator should get tighter tolerance specification.

6.4.1. Surface Sensitivity on Locators

Sensitivity analysis is to evaluate how sensitively the surface deviation depends on a certain locating point deviation. It is used for distributing tolerance to locating points according to their sensitivities.

For certain machining surface tolerance $T_j$ ($j = 1 \cdots m$), Let $P_i$ ($i = 1 \cdots n$) be the locating point, $\{d\} = \{0 \cdots 1 \cdots 0\}$ (only the $i$'th element is 1) be the locating point normal deviations, then the surface deviation based on this unit locating point deviation is:

$$\delta_{ij} = \delta(d)$$  \hspace{1cm} (6.8)

And the sensitivity for the tolerance upon the locating point $S_{ij}$ can be found by normalizing the deviations for all locating points:

$$S_{ij} = \frac{\delta_{ij}}{\delta_{ij} + \delta_{2j} + \cdots + \delta_{nj}} \left( \sum_{i=1}^{n} S_{ij} = 1 \right)$$  \hspace{1cm} (6.9)
A sensitivity matrix can then be constructed for all surface tolerances and locating points:

<table>
<thead>
<tr>
<th>Sensitivity $S_{ij}$</th>
<th>Machining Surface Tolerances $T_j$ ($j = 1\cdots m$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Locating Points $P_i$ ($i = 1\cdots n$)</td>
<td>$S_{11}$ $S_{12}$ $\cdots$ $S_{1m}$</td>
</tr>
<tr>
<td></td>
<td>$S_{21}$ $S_{22}$ $\cdots$ $S_{2m}$</td>
</tr>
<tr>
<td></td>
<td>$\vdots$ $\vdots$ $\vdots$</td>
</tr>
<tr>
<td></td>
<td>$S_{n1}$ $S_{n2}$ $\cdots$ $S_{nm}$</td>
</tr>
</tbody>
</table>

Table 6.1 Sensitivity Matrix

6.4.2. Tolerance Distribution

For each machining surface tolerance, the locating point tolerances are assigned based on their sensitivities. In the case of multiple machining surface tolerances, the tightest tolerance is selected as the final tolerance for each locating point. This procedure is detailed below.

For machining surface tolerance $T_j$ ($j = 1\cdots m$), a reference tolerance $t_0$ is picked (the selection of $t_0$ is detailed later) to assign the locating point tolerances $t_{ij}$ ($i = 1\cdots n$), based on their sensitivities. This is done through a weight factor $w_j$:

$$t_{ij} = w_j \cdot t_0$$  \hspace{1cm} (6.10)

Points that has larger sensitivity should have tighter tolerance, so $w_i$ is designed as:

$$w_i = 1 - k \cdot S_{ij}$$  \hspace{1cm} (6.11)
The factor ‘k’ is to prevent zero tolerance when the sensitivity $S_{ij} = 1$. It can be tuned to achieve optimal result. In our implementation of locator tolerance assignment, $k = 0.9$ is assumed. Combining above equations together, locator tolerances are assigned as:

$$t_{ij} = t_0 \cdot (1 - k \cdot S_{ij}) \quad (6.12)$$

In the case of multiple tolerances on a machining surface, first the locating point tolerance is assigned for all surface tolerance, and then the tightest tolerance among them is selected as the final locating point tolerance. This is shown in the table below.

<table>
<thead>
<tr>
<th>Locating Point Tolerance $t_{ij}$</th>
<th>Machining Surface Tolerances $T_j$ $(j = 1 \cdots m)$</th>
<th>Final Locating Point Tolerance $t_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{11}$ $t_{12}$ $\cdots$ $t_{1m}$</td>
<td>$t_{21}$ $t_{22}$ $\cdots$ $t_{2m}$</td>
<td>$t_1 = \min{t_{11}$ $\cdots$ $t_{1m}}$</td>
</tr>
<tr>
<td>$\vdots$ $\vdots$ $\vdots$</td>
<td>$\vdots$ $\vdots$ $\vdots$</td>
<td>$t_2 = \min{t_{21}$ $\cdots$ $t_{2m}}$</td>
</tr>
<tr>
<td>$t_{n1}$ $t_{n2}$ $\cdots$ $t_{nm}$</td>
<td></td>
<td>$t_n = \min{t_{n1}$ $\cdots$ $t_{nm}}$</td>
</tr>
</tbody>
</table>

Table 6.2 Tolerance Assignment for Multiple Surface Tolerances

With the assigned tolerances, the locators can ensure that all machining surface tolerances will be satisfied.

6.5. Summary

Given the locator tolerances, we can predict the machining surface accuracy, based on its tolerance type. On the other hand, given the machining surface tolerance, we are able to determine the locator tolerances.
For computer implementation, machining surfaces are represented by its sample points.
Six fixture-related tolerances are then defined with the surface sample points.

In locator tolerance assignment, surface sensitivity on locating point is defined to best
distribute tolerances among locating points.