Financial Computations on the GPU

A Major Qualifying Project Report

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Abstract

This Major Qualifying Project investigates the performance benefits of using the Graphics Processing Unit for algorithmic trading. The accomplished work includes the design, development and rigorous testing of a financial application to analyze real-time market data. Comprehensive analysis and an elaborate discussion of the results show that the GPU outperforms the CPU by several factors.
Acknowledgements

We would like to thank Jacob Lindeman and Professor Emmanuel for the guidance throughout the project and Professor Jon Abraham for comments on this Major Qualifying Project report. We would also like to thank Andrew Brzezinski, Henry Eck, Samer Haj-Yehia and Venkatesh Kidambi, those who work as the Financial Engineering, for their help in retrieving data from the stock KDB database and constructing relevant mathematical models.
1. Introduction

1.1 Stock Market and Algorithmic Trading

Definition of stock market is as follows:

*A stock market (or equity market), is a private or public market for the trading of company stock and derivatives of company stock at an agreed price; these are securities listed on a stock exchange as well as those only traded privately. The stock market is a type of listed market, in which security trades on exchanges, such as the New York Stock Exchange or the International Security Exchange for the public, are executed on an agency basis.* (Hagstrom, 2001)

So brokers, who have no financial interest in the trade, execute the public order against other brokers and charges their clients a commission for the service. This is one way in which investment institutions make profits from the stock market. In order to get more benefits for their customers, companies need to achieve extremely low latency so that they could get the desired stock bid price and enough volume. Decades ago, competed with each other by flying over the country to conduct investment business. Upon entering 1990s, the commerce of electronic trading changed the trading world. The globalization of electronic trading raised the competition between companies to a next level: Simple routine trades were automatically handled by computer algorithms so that the human
traders could focus on more complex trades. It was no longer a competition between representatives, but a more intense competition between algorithms. In a sense, algorithmic trading reduced the human labor in the stock market and started an electronic era. Algorithmic trading is also less prone to human errors and can achieve faster executions.

Let us take a step back to have a better view of how electronic trading reached its zenith at the beginning of the 21st century. Electronic trading was one of the business factors that led to the globalization of capital markets. It brought a thorough revolution to trading strategies and transaction latencies. In a sense, electronic trading is a major reason why global markets got globalized. While companies are less likely to want to see their international business suffering from geographical restrictions and trade barriers imposed by local governments, we witnessed the explosive growth of international e-commerce over the Internet. This is no surprise under the specified circumstances. Although electronic trading was originally designed to make convenient global transactions more convenient, companies then realized that this innovative trading system could benefit the domestic market as well for its competitive trading speed. Realizing this business opportunity, technology vendors started a new competition on low latency solutions, which encouraged the enthusiasm for consumer-based electronic trading (Norman, 2002).
With commerce being conducted increasingly over the Internet, we are entering a period of dynamic pricing because the pressure on sell-side businesses to reduce costs associated with e-commerce means that prices will inevitably fall. Dynamic pricing will force businesses to become more agile, efficient and technology-based. Technology-based business has been designated as a future business type with the rise of electronic trading. Wall Street is also holding annual conferences for technology vendors to introduce highly successful technologies, including software and hardware, for financial development. Technology is indispensable today for investment business operation and technology support. The increasing adoption of algorithmic trading -- "black box trading systems" -- is changing the way Wall Street works and is a source of new royalties to the tune of billions of dollars.

About a third of U.S. equities trading is already being done using algorithmic trading. “With that figure expected to soar to more than 50 percent by 2010”, said Brad Bailey, a senior analyst at the Boston-based researcher Aite Group. "I'm even afraid I'm underestimating that number," Bailey said. The London Stock Exchange estimates that around 40 percent of its trading is algorithmic.

"It's becoming much more mainstream," said Guy Cirillo, manager of global sales channels for Credit Suisse's Advanced Execution Services unit, an algorithmic trading platform that serves major hedge funds and other buy-side clients.
"You are seeing the traditional firms that took longer to adopt have come in strong in the last year to two years," he said. "Realistically, if you are not using this type of technology you are at a serious disadvantage." (Ablan, 2007)
1.2 Hardware Acceleration

The two main criteria for algorithmic trading are speed – that is the speed with which the same set of computations can be performed on multiple sets of data – and programmability. For this principle, general-purpose hardware – such as Intel Central Processing Unit (CPU) – is not suitable. The CPU is designed to execute commands in a linear fashion, however, the task at hand will benefit most from parallelization as the same calculations are required to be performed on multiple data; this is where parallelization and hardware acceleration come into play. Several groups have attempted using hardware acceleration to speed up financial calculations. Hardware acceleration is achieved by utilizing specific hardware to gain higher computational results than those provided by general purpose CPU. Most notable devices intended for intense calculations include Field-Programmable Gate Array (FPGA), IBM’s Cell Broadband Engine Architecture (Cell BE or, simply, Cell) and Graphics Processing Units (GPUs).

An FPGA is a custom integrated circuit that typically consists of a large number of identical logic cells connected to each other by a system of programmable switches (Stokes, 2007). Each logical cell is capable of handling a single task from a predefined set of functions. The customization of an FPGA is achieved by permanently burning instructions that implement functions to be accelerated onto an FPGA according to a design specified by a client’s program. The program can also be loaded into an FPGA from an external source. The program is normally implemented with an assembly-type language and then translated by software.
(supplied by manufacturer) into a design that will eventually appear on the FPGA (FPGA Basics, 2008). FPGAs excel at decision logic – branching and flow control – intensive tasks. However, FPGAs are limited to integer arithmetic due to the complexity associated with encoding floating point operations (Stokes, 2007).

The Cell processor is an architecture jointly designed by Sony, Toshiba and IBM (the union abbreviated to STI). Among other applications it is used for vector processing – also known as SIMD technique that is executing single instruction on multiple data.

Until recently GPU remained on fringes of HPC (high performance computing) mostly because of the high learning curve caused by the fact that low-level graphics languages were the only way to program the GPUs. Now, however, NVIDIA has come out with a new line of graphics cards – Tesla – which they claim to be world’s only C-language development environment for the GPU (High Performance Computing (HPC), 2008). Software development for a Tesla GPU is based on a language called CUDA (Compute Unified Device Architecture) which is a set of libraries that extend the C programming language making it simple for developers that are unfamiliar with graphical languages.

The Cell processor is similar in its capabilities to NVIDIA’s Tesla GPUs since both are used for GPGPU (General Purpose computing on Graphics Processing Units). The two devices share the same idea – using the power of graphics
processors in large-scale, parallelizable computations. The Cell processor and GPU are a good alternative to FPGA. While the computational speeds for the Cell processor and NVIDIA GPU are lower than those of FPGA the difference is not major. What Cell and Tesla GPU lack in speed they make up for in programmability. The two devices are far more flexible – in terms of scalability – and have a much steeper learning curve; both were designed for general purpose computing.
2. Technical Background

NVIDIA’s Tesla C870 GPU computing processor is at the heart of this Major Qualifying Project. It is a massively parallel processor architecture which delivers parallelization requirement necessary for efficiently analyzing streaming real-time market data. It is a multithreaded, many-core processor with performance topped off at 430 GFLOPS. Its 128 streaming processor cores operate at frequency of 1.35 Ghz each and support IEEE 754 single floating point precision (NVIDIA CUDA Programming Guide, 2008).

One of NVIDIA GPUs’ main features is ease of programmability made possible with CUDA – Compute Unified Device Architecture. CUDA provides the means to compile and run code for NVIDIA’s GPUs. With a low learning curve, CUDA allows developers to tap into enormous computing power of GPUs yielding high performance benefits.

A typical structure of a CUDA program consists of host and device side code. Host code runs on CPU and can be either C or C++ code. Device code is restricted to C programming language and runs on GPU. Each device function is referred to as a kernel. Kernels are launched from the host in a fashion similar to calling a C function but with one distinction: every kernel call from host specifies addition parameters that describe the thread configuration of the call; in other words, the additional parameters specify how many threads will be spawned to
execute the same piece of device code (NVIDIA CUDA Programming Guide, 2008).

Kernel configuration organizes threads into blocks which are in turn organized into a grid. For convenience a thread block can have one-, two-, or three-dimensions so as to facilitate indexing across elements of a vector, matrix or field. There are three different types of device memory: local (per thread) memory, shared (per block) memory and global memory (Figure 1). Threads of the same block have access to shared memory. Shared memory is low-latency due to its location near each processor. Its use greatly speeds up computations. The amount of shared memory available, serves as a limiting factor to the size of a thread block because all threads of a block are executed on the same processor core (NVIDIA CUDA Programming Guide, 2008). Figure 1 demonstrates CUDA’s memory hierarchy.

As was mentioned earlier, blocks are organized into a grid which can be one- or two-dimensional. The purpose of the grid is to allow more threads to execute on the kernel which otherwise would be limited by the block size. The only requirement for having multiple thread blocks work on a kernel is that they are independent of each other. That is because there’s no guarantee as to the order in which blocks will be executed. The size of the grid – number of thread blocks – is generally dictated by the amount of the data being processed as opposed to the number of processor cores on a GPU. In fact, CUDA is designed in such a
way that the number of thread blocks can greatly exceed the number of processors in the system (NVIDIA CUDA Programming Guide, 2008).

Another important component of this Major Qualifying Project is Kdb+. Kdb+ is an in-memory, column-based database whose purpose in this project is to supply the GPU with real-time market data. Kdb+ functions based on a language called
K which is in turn derived from a much older A Programming Language (otherwise known as APL). Arthur Whitney, the developer of K, has put a great deal of emphasis on brevity in his design of the language. While K may seem somewhat obscure and obfuscated at first, it is actually extremely precise and, more importantly, fast. The succinct quality of K, unfortunately, also reflects in the language’s error handling which is insufficient in Kdb+ (The Kdb+ Database, 2006).
3. System

3.1 Components

As mentioned earlier, the Forecaster application is a financial program that analyzes the real-time market data. It includes five components: a Kdb+ database, two KdbAdapters (1 and 2), a HashMap and, the heart of the system, the Forecaster. Figure 2 depicts the data flow among these five components.

Upon launch, the application is split into three separate processes – the parent and two children processes. Each child process is put in charge of one of the KdbAdapters, while the parent is associated with the Forecaster component that controls both the host and the device sides. The three processes communicate with each other by a means of C-style pipes. Two pipes – input and output – are created prior to spawning the three processes. The write end of the input pipe is handled by the KdbAdapter (I) and the read end of the output pipe is given to the KdbAdapter (O). The read end of the input pipe and the write end of the output pipe is handled by the Forecaster.

The symbol data is stored on the GPU in a simple array. In order to facilitate data storage and management on the GPU, each symbol is assigned a unique integer id which serves as its index in the symbol data array on the GPU; this enables access to any given symbol in a constant time. The required mapping is done in the parent process by the HashMap component – its sole purpose is to map each symbol string from the subscription set to a unique integer within range [0, (set
As mapping of a symbol to a unique integer id may become a possible bottleneck due to large volume of market data, fast hashing is imperative to the system’s success.

The purpose of the two KdbAdapters is to receive the real-time market data from and send algo results to the Kdb+ database. The KdbAdapter (I) establishes a connection with the Kdb+ and subscribes to a specified set of market symbols. Once the Kdb+ receives a subscription request, it immediately begins to send market data for the specified symbols to the KdbAdapter (I). The data is parsed and sent down the input pipe to the Forecaster.

For each raw market data record received by the Forecaster from the read end of the input pipe a symbol string is converted to a unique integer by making a function call to the HashMap (in a fashion mentioned earlier). Then, a kernel call is made from the host and data from the record is copied into a correct place in the device’s memory. As data accumulates on the device, the algos begin to launch. The algo results are copied from the device’s memory to the host’s whereupon the results are sent down through the write end of the output pipe to the KdbAdapter (O).

Similarly to the KdbAdapter (I), the KdbAdapter (O) opens a connection to the Kdb+ database. Once the Forecaster begins sending algo results, the KdbAdapter (O) receives them from the read end of the output pipe and records
them to the Kdb+. For a structural illustration of the steps that occur in the Forecaster System see Figure 3.

Data Flow

Figure 2 Data Flow
3.2 KdbAdapter

As stated earlier a KdbAdapter is the component that deals with moving the data between the Kdb+ database and the Forecaster System. Its functions include: constructor, connect, subscribe, kdbRead and kdbWrite. The constructor is called with a Kdb+ connection parameters – a host and a port number. When calling the connect function a KdbAdapter attempts to connect to a Kdb+
database with parameters specified to the constructor. If the connection settings are invalid an error code is returned. The constructor and the connect methods must always be called in order for a KdbAdapter to function properly. Whether a KdbAdapter is used for reading the data from or writing it to a Kdb+ database determines what other functions will be called.

If a KdbAdapter is used for supplying the system with the market data then the subscribe and the kdbRead functions are used. The subscribe function accepts a symbol set and a table name as its parameters. The symbol set is the set of symbols for which the real-time data will be obtained from a Kdb+ database. The table name is the name of the table from which to draw the data. The KdbRead function is called after subscription is complete; its parameter is the file descriptor, which is used for recording data from the Kdb+ database.

If the role of a KdbAdapter is to write data to a Kdb+ database then only the kdbWrite function is used. Being a mirror image of the kdbRead, the kdbWrite function accepts a file descriptor which is used for outputting the data to a Kdb+ database.

It should be noted that in order to minimize amount of code executed by the kdbWrite and the kdbRead methods and to avoid unnecessary system state checks neither function ever returns – except when the connection to a Kdb+ database is forcibly closed by an external source. In the Forecaster System, this issue is solved by manually “killing” the child processes prior to exiting the parent process.
3.3 HashMap

HashMap provides Forecaster with the ability to quickly convert a symbol string to its appropriate unique integer id; in other words HashMap creates a minimal perfect hash. Constructor for HashMap takes in a file that contains a set of symbols to be analyzed by the system and creates a minimal perfect hash based on that set.

HashMap serves as a wrapper to an application called CMPH. CMPH – C Minimal Perfect Hashing Library – is a free API (Advanced Programming Interface) that enables fast and efficient hashing of large sets of keys. CMPH was developed by Davi de Castro Reis, Djamel Belazzougui, and Fabiano Cupertino Botelho.

3.4 Forecaster

At the heart of the system is the Forecaster component. Its code is divided between the host (CPU) and the device (GPU) memory. The host code, executed on the CPU, handles reading of the market records from an input pipe, writing the algo results to an output pipe and transferring the data to and from the GPU. The device code, on the other hand, is responsible for storage and management of the market records (inside the GPU's memory) as well as the computations.

The Forecaster's launch function accepts three parameters: two file descriptors – one for an input and other for an output streams – and an integer value that
specifies the Forecaster’s run time in seconds. The Forecaster’s launch method returns after the number of seconds specified by the run time constant. This and all other constants are supplied in a configuration file that is passed in as a parameter to the applications at launch time.

The data is stored in the device memory in a sliding window fashion, essentially comprising a cyclical data structure – at any point during the execution of the Forecaster System the amount of data on the card is limited by a time constant specified by the user. The time window is stratified into time buckets – the number of buckets is also defined in the configuration file. For example, if the user chooses to keep fifteen minutes worth of data on the card divided into one-minute buckets then, for the first fifteen minutes, data will be written to “empty” memory locations – every minute data will be written to a new bucket: 15(min) / 1(min per bucket) = 15(buckets) – but when the fifteen minutes expire new market records will overwrite the old ones, starting with the first bucket.

This design is implemented with a two-dimensional array (matrix), where each row represents a time bucket and each cell in a row may potentially contain a market record. The buckets are managed with a help of a variable that always points to the “current bucket” – the bucket to which the data is to be written to at that point in time. Using the previous example, in the beginning the variable points to row zero switching to the next row every minute and after the fifteen minutes it is again set to row zero. Refer to Figure 4 for a visual representation of the way data is stored in the device memory.
The algorithmic computations used in these project are VWAP and TWAP. In order to minimize communication between host and device, both algos are launched with one kernel – the fire kernel. The computations are launched according to a user-specified constant. For example, if the constant is set to ten milliseconds, then the fire kernel will be launched every ten milliseconds. The fire kernel is launched with the following thread configuration:

- blockSize.x = the number of buckets per symbol
- blockSize.y = 1
- gridSize.x = the number of symbols in the set
- gridSize.y = the number of algo types

In this project the number of algo types is two – TWAP and VWAP. This configuration allows to perform both algorithmic calculations on each symbol (possibly at the same time). Each block will execute calculations appropriate to its algo type. First each thread in a block performs computations over a bucket corresponding to its index and records the results. Then one thread from each block computes overall results for a symbol based on the calculations done for each bucket. So, if there are 100 symbols and the desired number of buckets for each symbol is five, then the fire kernel will be launched with 200 blocks (2 algo types by 100 symbols) where every block contains five threads. The blocks will be split into two groups of 100 blocks each – one group for each algo. Within each block a thread will calculate results for an appropriate bucket and then one thread will perform final calculations using results for each bucket.
3.5 Suggested Improvements

The improvements discussed in this section are suggestions and are not used in the actual implementation of the system.

Host-Device Communication Reduction

The best way to optimize system performance is to minimize the communication between the host and the device as much as possible. One of the biggest system bottlenecks is copying each market record to the device memory as it is passed in to the Forecaster component. The best way to minimize the amount of copying of data from the host to the device memory is to use the interval between the algorithmic computations to the application's advantage; that is, storing the new records in the host memory for the length of the “quiet” interval and only coping the data to the device memory before the computations must be made. So, for

PARAMETERS
Data amount: 30 seconds
Bucket count: 5 buckets

Records under 30 seconds old
Records over 30 seconds old

Bucket #0
Bucket #1
Bucket #2
Bucket #3
Bucket #4

Figure 4 Data Storage
example, if an algo is set to fire every 100 milliseconds then the data should be accumulated in the host memory for the length of 100 milliseconds and only copied to the device memory just before firing the algo.

**Asynchronous Device Code Execution**

Another change that may optimize system performance is to combine the method suggested above with asynchronous symbol data updates and algo firing. CUDA supports asynchronous kernel calls by using cudaStreams, which are essentially queues of “orders” to the device code. The cudaStream provides a form of synchronization as the commands in a cudaStream are executed in the FIFO (First In First Out) fashion. It would be beneficial to use the same cudaStream for updating the symbol data and firing the algos. There are three steps in the Forecaster component that can be queued into the same cudaStream to get the following flow of events: the new records, that were stored in the host memory for the interval between the algo executions, are asynchronously copied to the device memory; then using the same cudaStream the update kernel is launched to update the “old” symbol data with the new market records; finally, the algo kernel is launched with the same cudaStream, using it for the third time. In this scenario the only synchronization that will have to be done is calling the cudaStreamSynchronize() function on the cudaStream (that was used for the three asynchronous steps) before adding any new records to the host memory.
Host Memory Subdivision

For further speed up, the new records can be added to two different memory locations. Alternating between these two locations will allow the cudaStreamSynchronize() function to be called later, right before the call that will copy the data from the host to the device memory thereby efficiently reducing the delay between handling the incoming records.

![Diagram of improved storage scheme with cudaStream and host memory locations](image)

Figure 5 Improved Storage Scheme

The cudaStream diagramed above contains two copy instructions. The letters “A” and “B” represent the two host memory locations between which the storing of data will be alternated. The “Tmp” label represents the location in the device memory.
memory where the new records will be temporarily copied to before being used to update the data for each symbol. The new records at location marked by “Tmp” will be considered valid until the update kernel is launched. Once the update kernel finishes – adding the records to the appropriate buckets of the corresponding symbols – the records are considered outdated and will be overwritten once the new set of records is copied from the host to the device memory. The summary of the flow of events is as follows:

1. The cudaStream is synchronized – any previously unfinished instructions are waited upon – to make sure the previous commands were executed successfully
2. The new records are copied from the host memory location “A” to the device memory location “Tmp”
3. The update kernel is launched
4. The algo kernel is launched
5. The new incoming records are being recorded to the host memory location “B” until it’s time for the algo calculations to begin
6. Steps 1 through 5 are repeated for a lifetime of the application, alternating between the host memory locations “A” and “B”
4. Performance

The performance of the GPU versus the CPU was determined by two tests. The dependent variable of each test was the average amount of time it takes to perform a computation. The independent variables were as follows: algo complexity and symbol set size. The results of all tests were in favor of GPU.

4.1 Hypothesis

Given the GPU’s great potential for parallelization, it was theorized that the CPU will be outperformed in both tests by a factor of at least five.

4.2 Procedure

In the first test the algo complexity was varied to analyze calculation time. Algo complexity is defined by the number of calculations performed every time an algo is fired. To increase the complexity of an algo the computations are repeated a certain number of times by surrounding algo code with a simple loop. Therefore, complexity is determined by the number of iterations through an algo. The first test consisted of measuring the average time it takes – first on the GPU and then on the CPU – for an algo to complete calculations as the complexity (number of iterations) is increased.

The second test was used to establish performance of host and device code, in terms of the amount of symbols used in the calculations. Let the symbol set be defined as the collection of symbols for which data will be processed (refer back to section 3.2 KdbAdapter). During each stage of this test the average time it
took to complete the algo calculations as size of the symbol set was increased was measured.

4.3 Results

The results of both tests showed that an algo running on GPU takes substantially less time to perform computations than on CPU.

The results of tests one and two can be found in Tables 1 and 2; they are also graphically represented in Graphs 1 and 2, respectively. The results for each test show the same trend: as the independent variable was increased, the average time it took an algo to complete calculations also increased; however the rate at which the dependent variable was increased was much greater for tests run on the CPU as opposed to those on the GPU.

<table>
<thead>
<tr>
<th>Complexity Test</th>
<th>Average Calculation Time (msec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complexity</td>
<td>GPU</td>
</tr>
<tr>
<td>(# loops)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.759</td>
</tr>
<tr>
<td>3</td>
<td>7.278</td>
</tr>
<tr>
<td>5</td>
<td>7.881</td>
</tr>
<tr>
<td>10</td>
<td>14.393</td>
</tr>
<tr>
<td>15</td>
<td>23.937</td>
</tr>
<tr>
<td>20</td>
<td>25.900</td>
</tr>
<tr>
<td>25</td>
<td>54.235</td>
</tr>
<tr>
<td>50</td>
<td>86.781</td>
</tr>
<tr>
<td>75</td>
<td>149.788</td>
</tr>
<tr>
<td>100</td>
<td>215.020</td>
</tr>
</tbody>
</table>
Table 2 Size Test

<table>
<thead>
<tr>
<th>Set Size (# symbols)</th>
<th>Average Calculation Time (msec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GPU</td>
</tr>
<tr>
<td>100</td>
<td>1.759</td>
</tr>
<tr>
<td>200</td>
<td>2.697</td>
</tr>
<tr>
<td>300</td>
<td>3.686</td>
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<tr>
<td>400</td>
<td>5.462</td>
</tr>
<tr>
<td>500</td>
<td>6.660</td>
</tr>
</tbody>
</table>

Figure 6 Average Calculation Time vs. Complexity
4.4 Conclusion

The results of both test confirmed the hypothesis that the GPU can outperform the CPU by a factor of at least five in every test. It follows then, that it would be to any brokerage firm’s great benefit to use the GPU for financial computations.
5. Algorithmic Trading Strategy

5.1 Algorithmic Trading Strategy Overview

As we know from the introduction chapter, algorithmic trading is a trading system that utilizes very advanced mathematical models for making transaction decisions in the financial markets. And algorithmic trading strategies are rules built into the models attempting to determine the optimal time for an order to be placed that will cause the least amount of impact on a stock's price. The essential concept of algorithmic trading strategy is to divide large blocks of purchasing requests into smaller blocks, allowing complex algorithms to decide when the smaller blocks are to be purchased. This basic strategy is called "iceberging". The success of this strategy may be measured by the average purchase price against the VWAP for the market over that time period.

There are two elements of an algorithmic trading strategy: the decision of when to trade, or pre-trade analytics, and the decisions of how to trade, or the execution phase of the algorithm.

The decision of when to trade is based on continuously re-calculated analytics. This could include, for example, a moving average crossover algorithm that calculates two moving averages, and analyses, in real time, when they cross one another. It then buys or makes the decision to buy or sell, depending on which average is higher. Volume-weighted-average-price strategy (VWAP) is a
methodology to determine when to trade by continuously re-calculating price average weighted on volume and comparing the average price to current price.

The decision of how to trade, or the order execution element of the algorithm, can be just as complex as the decision of when to trade. For example, once an opportunity is identified by the pre-trade analytic to buy, for example, 10,000 shares of IBM, the order execution element of an algorithmic trading strategy may slice the order up into smaller parts (blocks of 1,000 shares). In conjunction, it may place the order in multiple liquidity pools to take advantage of the prices and availability of liquidity across a ‘virtual’ exchange with multiple participants (Jones, 2007). In conclusion, the decision of how to trade takes consideration of various real-time constraints, such as current market size, stock volatility, news feed on this company and so on. Time-weighted-average price is a strategy to minimize the impact on market volatility and assumes that stock shares are equally traded over the same period of time.

VWAP and TWAP are two mostly often used strategies for algorithmic trading. There are still many more other strategies such as arbitrage strategy, implementation shortfall and trade cost analysis.
5.2 VWAP (Volume-weighted average price) Strategy

- The VWAP Strategy reduces deviation to the Volume Weighted Average Price benchmark with customizable constraints. (MMVI TurboTrade Financial, 2006)

The Volume Weighted Average Price (VWAP) strategy, as the mostly often used algorithmic trading strategy, helps to decide when to trade. VWAP is the most commonly used algorithmic trading strategy, as it provides a fair representation of prices throughout the trading period; but it is inherently an 'at market' strategy. VWAP allows you to achieve the best possible average execution price for a security in without adversely impacting the price. The orders generated by this strategy will vary in size and frequency throughout the duration of the trade. It is often used as a trading benchmark by investors who aim to be as passive as possible in their execution. Most pension funds and mutual funds fall into this category. The aim of using a VWAP trading target is to ensure that the trader executing the order trades in-line with volume on the market. VWAP is often used in algorithmic trading for its convenience and effectiveness.
The VWAP is calculated using the following formula:

$$P_{VWAP} = \frac{\sum_{j} P_j \cdot Q_j}{\sum_{j} Q_j}$$

where:

- PVWAP = Volume Weighted Average Price
- \(P_j\) = price of trade \(j\)
- \(Q_j\) = quantity of trade \(j\)

\(j\) = each individual trade that takes place over the defined period of time, excluding cross trades and basket cross trades.

(VWAP, 2008)

To determine whether a transaction is good or not using VWAP strategy is simple. If the current price is below the VWAP benchmark up to the end of a chosen time horizon, the current bid price is considered good for buying in but bad for selling out. Vice versa, if the current price is above VWAP benchmark up to the point, current bid price is considered good for selling out but bad for buying in. How the rule is determined is also straightforward. As VWAP calculates average price weighted on volume, we buy in the stock shares if current price is lower than intra-day average so far and sell it out when the current price is higher than volume-weighted average price. If we could keep trading this way, we could keep ourselves above the market intra-day average, which means we will not lose in short term, as we keep a profit range by buying under the average and selling above the average.
Table 3 shows the way we apply VWAP strategy to real-world data, if we have stock AAA has bid price and traded volume at the following times:

<table>
<thead>
<tr>
<th>Time</th>
<th>Bid Price</th>
<th>Volume</th>
<th>VWAP</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:00</td>
<td>35</td>
<td>100</td>
<td>=35*100/100 = 35</td>
</tr>
<tr>
<td>9:05</td>
<td>40</td>
<td>50</td>
<td>=(35<em>100+40</em>50)/(100+50) = 36.67</td>
</tr>
<tr>
<td>9:10</td>
<td>45</td>
<td>100</td>
<td>=(35<em>100+40</em>50+45*100)/(100+50+100) = 40</td>
</tr>
<tr>
<td>9:15</td>
<td>30</td>
<td>100</td>
<td>=(35<em>100+40</em>50+45<em>100+30</em>100)/(100+50+100+100) = 37.14</td>
</tr>
<tr>
<td>9:20</td>
<td>30</td>
<td>100</td>
<td>=(35<em>100+40</em>50+45<em>100+30</em>100+30*100)/(100+50+100+100+100) = 35.56</td>
</tr>
</tbody>
</table>

And if we graph the sample data, Figure 8 shows that we could see that the VWAP values are smoother than the raw market data values after weighted average.

Figure 8 VWAP Strategy on Sample Data
5.3 TWAP Strategy (Time-weighted average price)

- The TWAP Strategy distributes orders in a linear manner, balancing adverse selection and slippage in real-time. (MMVI TurboTrade Financial, 2006)

The TWAP strategy is also an often used intra-day benchmark. It assumes that a stock volume follows a uniform distribution with respect to time, which means that transaction volumes are equally distributed within a given time horizon. TWAP is effective when we want to minimize the impact by the market volatility in a specified time horizon. TWAP is best for those who want to adhere to a regular trading schedule and execute in equal-size increments regardless of other trades in the market.

TWAP (time-weighted average price) allows traders to time-slice a trade over a certain period of time. Unlike VWAP, which typically trades less stock when market volume dips, TWAP will trade the same number of shares at even intervals throughout the time-period you specify. TWAP is optimal for orders that must be completed by a specific time or for trades in illiquid stocks where you do not want your execution schedule to depend on volumes. This strategy is best utilized in situations where there are little or no liquidity concerns and the trade’s executions can be evenly spread throughout the given timeframe. The cumulative volume profile for a TWAP trade is linear with a positive constant
slope of one, as we could see in the graph below. In addition, orders generated by this strategy tend to be small in size and occur with relatively frequency.

Here is an example indicating how TWAP differs from VWAP. To achieve this Time Weighted Average Price, the BXS engine divides the Order Quantity equally over a number of equally-spaced slices. TWAP differs from the VWAP strategy in that a VWAP trade may buy or sell 30% of a trade in the first half of the day and then the other 70% in the second half of the day. With the TWAP strategy, the trade would most likely execute 50% in the first half and 50% in the second half of the day. (Stanley, 2007) From what is described above, we see that TWAP does not take market traded volume into consideration. If trader is selling under a TWAP strategy, the orders will be evenly time-sliced regardless of the market impact.

A TWAP strategy example using the same data with VWAP is as follows:

<table>
<thead>
<tr>
<th>Time</th>
<th>Bid Price</th>
<th>Volume</th>
<th>Volume Weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:00</td>
<td>35</td>
<td>(100+50+100+100+100)/5 = 90</td>
<td>20%</td>
</tr>
<tr>
<td>9:05</td>
<td>40</td>
<td>90</td>
<td>40%</td>
</tr>
<tr>
<td>9:10</td>
<td>45</td>
<td>90</td>
<td>60%</td>
</tr>
<tr>
<td>9:15</td>
<td>30</td>
<td>90</td>
<td>80%</td>
</tr>
<tr>
<td>9:20</td>
<td>30</td>
<td>90</td>
<td>100%</td>
</tr>
</tbody>
</table>
From the graph below, we could see that TWAP strategy is not based on the volume traded per period of time, but based on time slices. That is why TWAP is also called time-sliced trading strategy.

![Figure 9 TWAP strategy on Sample Data](image)

**5.4 Profit and Loss Analysis**

**5.4.1 Percentage Difference between Executed Quantity and Scheduled Quantity**

The percentage difference between the executed quantity and scheduled quantity provides us with a clearer view of how executed quantity differs from the
volume of shares that we planned to be. Ideally, all the brokers wish for what they exactly need. However, with market prices and volumes fluctuating continuously, it is impossible for brokers to get the desired volume with limited shares of stock in the market. That is why brokers need to run the formula to evaluate their deficiency in the actual executed quantity along the transactions, and adjust the algorithm to face the new situation if necessary. No matter what trading strategy we are using, we are short on purchase amount all the time due to market limitations. That is the reason why we need to know the difference, need to know how much we short and make up the deficiency in future trading.

Table 5 Percentage Difference between Executed Quantity and Scheduled Quantity

<table>
<thead>
<tr>
<th>Sym</th>
<th>time</th>
<th>TotExec</th>
<th>SchdQty</th>
<th>(Execqty-Schdqty)/SchdQty</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEH</td>
<td>9:30:00</td>
<td>2600</td>
<td>4300</td>
<td>-39.53%</td>
</tr>
<tr>
<td>LEH</td>
<td>9:30:10</td>
<td>1800</td>
<td>1800</td>
<td>0.00%</td>
</tr>
<tr>
<td>LEH</td>
<td>9:30:20</td>
<td>1500</td>
<td>1500</td>
<td>0.00%</td>
</tr>
<tr>
<td>LEH</td>
<td>9:30:30</td>
<td>3200</td>
<td>5300</td>
<td>-39.62%</td>
</tr>
<tr>
<td>LEH</td>
<td>9:30:40</td>
<td>2000</td>
<td>3300</td>
<td>-39.39%</td>
</tr>
<tr>
<td>LEH</td>
<td>9:30:50</td>
<td>1900</td>
<td>3200</td>
<td>-40.63%</td>
</tr>
<tr>
<td>LEH</td>
<td>9:31:00</td>
<td>1000</td>
<td>1700</td>
<td>-41.18%</td>
</tr>
<tr>
<td>LEH</td>
<td>9:31:10</td>
<td>700</td>
<td>1200</td>
<td>-41.67%</td>
</tr>
<tr>
<td>LEH</td>
<td>9:31:20</td>
<td>700</td>
<td>1200</td>
<td>-41.67%</td>
</tr>
<tr>
<td>LEH</td>
<td>9:31:30</td>
<td>1400</td>
<td>2300</td>
<td>-39.13%</td>
</tr>
<tr>
<td>LEH</td>
<td>9:31:40</td>
<td>2800</td>
<td>4600</td>
<td>-39.13%</td>
</tr>
<tr>
<td>LEH</td>
<td>9:31:50</td>
<td>2300</td>
<td>3900</td>
<td>-41.03%</td>
</tr>
<tr>
<td>LEH</td>
<td>9:32:00</td>
<td>1500</td>
<td>2500</td>
<td>-40.00%</td>
</tr>
<tr>
<td>LEH</td>
<td>9:32:10</td>
<td>1100</td>
<td>1800</td>
<td>-38.89%</td>
</tr>
<tr>
<td>LEH</td>
<td>9:32:20</td>
<td>900</td>
<td>1500</td>
<td>-40.00%</td>
</tr>
<tr>
<td>LEH</td>
<td>9:32:30</td>
<td>32100</td>
<td>53500</td>
<td>-40.00%</td>
</tr>
<tr>
<td>LEH</td>
<td>9:32:40</td>
<td>9700</td>
<td>9700</td>
<td>0.00%</td>
</tr>
<tr>
<td>LEH</td>
<td>9:32:50</td>
<td>6900</td>
<td>6900</td>
<td>0.00%</td>
</tr>
<tr>
<td>LEH</td>
<td>9:33:00</td>
<td>7000</td>
<td>11700</td>
<td>-40.17%</td>
</tr>
</tbody>
</table>

1 Sym = Symbol name  
Time = Transaction time for every 10 seconds  
TotExec = Cumulated Executed Quantity within 10 seconds  
SchdQty = Cumulated Scheduled Quantity within 10 seconds  
2 (Execqty-Schdqty)/SchdQty calculates the percentage difference between cumulated executed quantity and cumulated scheduled quantity within 10 seconds
We are the seller’s position in the table 5 above. For buyer’s position, negative percentage rate indicates that demand volume is greater than supplied volume. So when the percentage difference between executed quantity and scheduled quantity is negative for buyers, the cumulated volume of buying requests is greater than that of selling requests. On the other hand, for seller side position, negative percentage rate indicates that supplied volume is greater than demand volume. At that time, cumulated volume of selling requests is greater than that of buying requests. Usually, we have the percentage rate controlled within the range of 10%. However, every entry of percentage rate is either negative or zero, which points to the fact that everyone is trying to sell the stocks, resulting in the great decline of the stock price. Below is a more straightforward diagram using data above. Figure 10 shows that most stock holders that day were trying to sell their shares so that there was no available stock buyer in the market. After analyzed the difference between executed quantity and scheduled quantity, we knew that how bad performances our orders were experiencing from the pure negative percentage rates honestly reflected on the diagram. Knowing how hard it was getting sold reminds us to change a trading strategy. As we could see in Figure 10, the seller’s executed quantity could not reach the scheduled quantity for every single transaction. Data source from Lehman Brothers, within 3 minutes after market opens on Tue, Sept 23.
Price improvement is different in constructing algorithm depending on which position we are at. When we are at buyer’s position (indicated by number 1 when programmed in Excel Macro), price improvement = ask price – executed vwap price. Similarly, when we are at seller’s position (indicated by number 2 when programmed in Excel Macro), price improvement = executed vwap price – bid price.

Figure 11 denotes how excel macro automatically generates price improvement result for a large amount of data. The algorithm in Excel Macro is compiled in
Visual Basic. The algorithm below shows how we judge our position at first and then calculate the price improvement per share.

```
' depending on position of buying or selling calculating loss/profit with the difference of 
EXECVWAP and bid or ask price

Range("AT1").Select
ActiveCell.FormulaR1C1 = "PRICE IMPROVEMENT ANALYSIS"

For j = 3 To Sheets("sheet1").UsedRange.Rows.Count
    If Sheets("Sheet1").Cells(j, 6).Value = 2 Then
        Do While Sheets("Sheet1").Cells(j, 43).Value <> 0
        End Do
    Else
        Do While Sheets("Sheet1").Cells(j, 43).Value <> 0
            Sheets("Sheet1").Cells(j, 46).Value = Sheets("Sheet1").Cells(j, 30).Value - Sheets("Sheet1").Cells(j, 43).Value
        End Do
    End If
Next j
```

Figure 11 Excel Macro for Price Improvement Analysis

What the algorithm basically produces is that it judges the trader’s position, either a seller or a buyer, and then applies the price improvement formula according to the first-step judgment. The output price improvement will be located at column 46. And column 43 indicates the executed price in the market recommended by VWAP strategy, while column 29 indicates the bid price that we desired to be.

After we run the algorithm, the truncated price improvement result table looks as indicated in Table 5.
Table 6 Price Improvement Per Share

<table>
<thead>
<tr>
<th>time</th>
<th>TotExec</th>
<th>VwapImpPerSh</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:30:00</td>
<td>2600</td>
<td>-0.01180457</td>
</tr>
<tr>
<td>9:30:10</td>
<td>1800</td>
<td>-0.01584118</td>
</tr>
<tr>
<td>9:30:20</td>
<td>1500</td>
<td>0.1786206</td>
</tr>
<tr>
<td>9:30:30</td>
<td>3200</td>
<td>0.02531019</td>
</tr>
<tr>
<td>9:30:40</td>
<td>2000</td>
<td>0.0283421</td>
</tr>
<tr>
<td>9:30:50</td>
<td>1900</td>
<td>-0.01259497</td>
</tr>
<tr>
<td>9:31:00</td>
<td>1000</td>
<td>0.005915136</td>
</tr>
<tr>
<td>9:31:10</td>
<td>700</td>
<td>-0.008636364</td>
</tr>
<tr>
<td>9:31:20</td>
<td>700</td>
<td>-0.001304348</td>
</tr>
<tr>
<td>9:31:30</td>
<td>1400</td>
<td>-0.03763335</td>
</tr>
<tr>
<td>9:31:40</td>
<td>2800</td>
<td>-0.04418033</td>
</tr>
<tr>
<td>9:31:50</td>
<td>2300</td>
<td>0.02314607</td>
</tr>
<tr>
<td>9:32:00</td>
<td>1500</td>
<td>-0.01787879</td>
</tr>
<tr>
<td>9:32:10</td>
<td>1100</td>
<td>-0.02409091</td>
</tr>
<tr>
<td>9:32:20</td>
<td>900</td>
<td>0.02138085</td>
</tr>
<tr>
<td>9:32:30</td>
<td>32100</td>
<td>0.02149423</td>
</tr>
<tr>
<td>9:32:40</td>
<td>9700</td>
<td>-0.007844351</td>
</tr>
<tr>
<td>9:32:50</td>
<td>6900</td>
<td>0.04710667</td>
</tr>
<tr>
<td>9:33:00</td>
<td>7000</td>
<td>0.02960919</td>
</tr>
</tbody>
</table>

1 TotExec = Cumulated Executed Quantity within 10 seconds
VwapImpPerSh = Price improvement per share under VWAP method

With the data above, we could diagram the relationship between executed quantity and price improvement per share in Figure 12.
Figure 12 Executed Quantity and Price Improvement Per Share

From Figure 12 above, the blue bar indicating total executed quantity follows the right-hand side axis and the red line follows the left-hand side axis. Data in Figure 12 also comes in on Oct 2, the night before news announced the new hope for the bailout plan. As we could see in Figure 12, data varies greatest from 9:30:10 to 9:30:20 because most people in the market were trying to buy in stock shares since people found back confidence in the stock market. A while later between 9:32:30 and 9:32:40, actual transactions were made.

Retrieving the price improvement per share, we multiply it by executed quantity to get the accumulated price improvement within 10 seconds. If we have to get total accumulated price improvement up to this moment, we could add up to this point to retrieve the accumulated price improvement. All the price improvement units are in dollar.
In Figure 12, we applied price improvement method on the sample data which was 3 minute within the market opens. When analyzing the real data in Figure 13, we weighed the price improvement rate so that the data will be less fluctuated. The way we weigh the data is to apply the following formula:

\[
\text{Weighted average price improvement per share} = \frac{\text{Total Aggregated Price Improvement in Dollar to the end of the time ticket}}{\text{Total Aggregated Executed Volume in Share to the end of the time ticket}}
\]

As we could see in Figure 13, weighted-average price improvement looks much smoother along the time comparing to Figure 12, and almost remains constant in the end. Price improvement per share in the above diagram follows right-hand side axis. The algorithm we ran to attain weighted-average price improvement is
very similar to the loop we used for participation rate calculation. Figure 14 below shows the main part of Excel Macro which aggregates data and calculates the weighted-average price improvement.

```
'SumTotExecQty
```

```
Sheets("Sheet2").Cells(2,12).Value = Sheets("Sheet2").Cells(2,4).Value

'TotPriceImprovement
```

```
Sheets("Sheet2").Cells(2,14).Value = Sheets("Sheet2").Cells(2,13).Value

For a1 = 3 To Sheets("Sheet2").UsedRange.Rows.Count

'Weighted-average price improvement per share
```

```
Sheets("Sheet2").Cells(a1,18).Value = Sheets("Sheet2").Cells(a1,14).Value / Sheets("Sheet2").Cells(a1,12).Value

Next a1
```

**Figure 14 Excel Macro for Weighted-average price improvement per share**
5.4.3 Participation Rate Analysis

First of all, there are two kinds of participation rates involved in our future calculation: period participation rate and cumulative participation rate. Period participation rate is the executed volume every 10 seconds weighted by the actual market volume every 10 seconds. Cumulative participation rate is the cumulative executed volume by the end of the time ticket weighted by the cumulative market volume by the end of the time ticket. Both formulas are listed as follows:

\[
\text{Period Participation Rate} = \frac{\text{Executed Volume every 10s}}{\text{Actual Market Volume every 10s}}
\]

\[
\text{Cumulative Participation Rate} = \frac{\text{Cumulative Executed Volume by the end of the time ticket}}{\text{Cumulative Market Volume by the end of the time ticket}}
\]

Here we will raise a simple example to show how the actual calculation works to achieve both participation rates. Table 7 below shows how to calculate the cumulated participation rate, where intvol stands for current market volume, TotExec stands for current executed volume, SumIntVol stands for cumulative market volume by the end of the time ticket and SumTotExecQty stands for cumulative executed volume by the end of the time ticket. Table 8 below shows the calculation for period participation rate using the same data in Table 7, where 10sIntVol stands for cumulative market volume for every 10 seconds and 10sTotExec stands for the cumulative executed volume for every 10 seconds.
Table 7 Participation Rate on Sample Data

<table>
<thead>
<tr>
<th>time</th>
<th>intvol</th>
<th>TotExec</th>
<th>SumIntVol</th>
<th>SumTotExecQty</th>
<th>Cumulative participation rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:30:00</td>
<td>500</td>
<td>200</td>
<td>500</td>
<td>200</td>
<td>200/500 = 0.400</td>
</tr>
<tr>
<td>9:30:08</td>
<td>900</td>
<td>300</td>
<td>1400</td>
<td>500</td>
<td>500/1400 = 0.357</td>
</tr>
<tr>
<td>9:30:09</td>
<td>1000</td>
<td>600</td>
<td>2400</td>
<td>1100</td>
<td>1100/2400 = 0.458</td>
</tr>
<tr>
<td>9:30:10</td>
<td>400</td>
<td>250</td>
<td>2800</td>
<td>1350</td>
<td>1350/2800 = 0.482</td>
</tr>
<tr>
<td>9:30:11</td>
<td>300</td>
<td>50</td>
<td>3100</td>
<td>1400</td>
<td>1400/3100 = 0.452</td>
</tr>
<tr>
<td>9:30:20</td>
<td>600</td>
<td>100</td>
<td>3700</td>
<td>1500</td>
<td>1500/3700 = 0.405</td>
</tr>
</tbody>
</table>

Table 8 Participation Rate on Sample Data ||

<table>
<thead>
<tr>
<th>time</th>
<th>10sIntVol</th>
<th>10sTotExec</th>
<th>Period Participation Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:30:00</td>
<td>500+900+100=2400</td>
<td>200+300+600=1100</td>
<td>1100/2400 = 0.458</td>
</tr>
<tr>
<td>9:30:10</td>
<td>400+300=1700</td>
<td>250+50=300</td>
<td>300/1700 = 0.176</td>
</tr>
<tr>
<td>9:30:20</td>
<td>600</td>
<td>100</td>
<td>100/600 = 0.167</td>
</tr>
</tbody>
</table>

The data we actually use is already aggregated for every 10 seconds. In the table below, intvol and TotExec are both aggregated values within 10 seconds. So for period participation rate, we could directly use intvol and TotExec values without any change. It is still the same way obtaining the cumulated participation rate. Since both entries are aggregated, we only need to apply formulas below to get both participation rates. And results are listed in Table 9.

\[
\text{Period Participation Rate} = \frac{\text{TotExec}}{\text{intvol}}
\]

\[
\text{Cumulative Participation Rate} = \frac{\text{SumTotExecQty}}{\text{SumIntVol}}
\]
## Table 9 Period and Cumulative Participation Rate

<table>
<thead>
<tr>
<th>Time</th>
<th>intvol</th>
<th>TotExec</th>
<th>SumIntVol</th>
<th>SumTotExecQty</th>
<th>Cumulative participation rate</th>
<th>Period Participation Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:30:00</td>
<td>3807</td>
<td>2600</td>
<td>3807</td>
<td>2600</td>
<td>68.30%</td>
<td>68.30%</td>
</tr>
<tr>
<td>9:30:10</td>
<td>6372</td>
<td>1800</td>
<td>10179</td>
<td>4400</td>
<td>43.23%</td>
<td>28.25%</td>
</tr>
<tr>
<td>9:30:20</td>
<td>34000</td>
<td>1500</td>
<td>44179</td>
<td>5900</td>
<td>13.35%</td>
<td>4.41%</td>
</tr>
<tr>
<td>9:30:30</td>
<td>11235</td>
<td>3200</td>
<td>55414</td>
<td>9100</td>
<td>16.42%</td>
<td>28.48%</td>
</tr>
<tr>
<td>9:30:40</td>
<td>14265</td>
<td>2000</td>
<td>69679</td>
<td>11100</td>
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<tr>
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<td>113842</td>
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<tr>
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</table>
The algorithm which we applied to calculate both participation rates is simple. Basically, we just need to construct a loop that aggregates the cumulative sum for market volume and executed volume. The Excel Macro for this algorithm is shown in Figure 15.

Figure 15 Excel Macro for Participation Rate
What is indicated in Figure 15 above is how we aggregates executed volume and total market traded volume. After retrieving the aggregated value, we need to judge whether the market traded volume is zero before division, for it is possible that no share was traded for a particular stock during a specified period of time. If the market traded volume is non-zero, we divide the executed volume by the total market traded volume to get cumulative participation rate. Period participation rate calculation uses the same way except that instead of total executed volume and total market volume, we use aggregated executed volume and market volume for every 10 seconds. Both cumulative participation rate and period participation rate will be compared to the configured rate, which is 10% as a constant, in the graphs below.

The configured rate remains 10% as a constant, for it is an optimized number based on previous experiences. Given the above algorithm, we could graph data listed in Table 9 as shown in Figure 16. Notice that the data we used in Table 9 is truncated, which only contains 3-minute data after the market opens. In Figure 17, we graphed complete data received on the particular morning. In the Figure 17, it is more obvious that the cumulative participation rate looks smoother and closer to the configured rate, which is set as 10%, even though the period rate still maintains high volatility.
Figure 16 Participation Rate on Truncated Data

Figure 17 Participation Rate on Complete Data
6. Future Analysis

As we know, news-driven algorithms have recently become popular. News algorithms attempt to analyze news stories and make trades based on their predicted impact on the underlying stock. Traditional algorithmic trading system decides what to do after analyzing the market data. In other words, algorithmic trading system generates no result without history market data. However, market prices nowadays could change greatly in a millisecond, and company could even go bankrupt before people get a chance to sell the stock shares they hold.

Another issue that brought to our concern is that news today has a greater impact on stock market than ever. Rumors about bankruptcy of United Air Lines on Sept. 26th caused UAL stock prices dipped 15% in 30 minutes. On Oct 8th, U.S. stocks plunged and the major indexes fell to five-year lows as traders acted on rumors and hopes about recapitalization for some of the biggest financial institutions in Britain and the U.S., including Morgan Stanley, Bank of America and Royal Bank of Scotland Group. The Chicago Board Options Exchange volatility index, which measures premiums paid for protection against stock-market swings, closed at its highest level of the crisis, up 3.1% at 53.68 on that day (Curran, 2008).

The Dow Jones company saw the necessity of upgrading the current algorithmic system due to the current market environment. That is how Dow Jones decided to cooperate with Ravenpack on the development of Dow Jones News Analytics project. DJ Analytics reads news posted on Dow Jones News Feed with an ultra-
low latency. While Dow Jones News Feed covers world-wide financial news from Dow Jones Newswires, Wall Street Journal, Barron’s and all other trust-worthy major news sites, DJ Analytics covers all headlines and full text of news stories completely and generates analysis for every piece of news indicating the impact on the oil price, bank interest rate, currency rate and other 21 major indexes and rates. While automated, computer-based trading is widely used to capture and leverage predictable events, the ability to correlate breaking news with streaming financial data in real time is the next frontier for algorithmic trading. We will be expecting a built-in news analytics as part of algorithmic trading system on GPU in the future.
Bibliography

   http://www.reuters.com/article/sphereNews/idUSN2931330320070531?sp =true&view=sphere


   http://content.members.fidelity.com/Inside_Fidelity/fullStory/1,,1212,00.htm


of IBM, the order execution element of an algorithmic trading strategy may slice the order up into smaller parts (blocks of 1,000 shares).

   http://www.turbotrade.com/content/view/116/94/


   http://en.wikipedia.org/wiki/VWAP

Appendix A

Forecaster Component

\[ \textbf{Forecaster}(\text{HashMap } *\text{hmap}, \text{ int } \text{algoFireInterval}) \]

The constructor; initializes the Forecaster component

- \text{hmap} \quad \text{An initialized Hashmap of symbols}
- \text{algoFireInterval} \quad \text{The frequency of algorithmic computations}

\[ \textit{void launch}(\text{int } \text{inFd}, \text{ int } \text{outFd}, \text{ int } \text{runTimeSec}, \text{ const char } *\text{memUsageFilepath}) \]

Function to launch the Forecaster component

- \text{inFd} \quad \text{The file descriptor to the input channel from which the market records are to be streamed real-time}
- \text{outFd} \quad \text{The file descriptor to the output channel into which the results of algorithmic computations will be recorded to}
- \text{runTimeSec} \quad \text{The amount of time in seconds the Forecaster component is to be launched for}
- \text{memUsageFilepath} \quad \text{The file path to where the symbol usage statistics will be recorded to}

HashMap Component

\[ \textbf{HashMap}(\text{const char } *\text{keysFilepath}, \text{ int } \text{hashAlgoType}=\text{CMPH\_BDZ}) \]

The constructor; initializes the HashMap component

- \text{keysFilepath} \quad \text{The file path to a set of keys for which a minimal perfect hash map is to be generated}
- \text{hashAlgoType} \quad \text{The type of algorithm to be used in generating the hash}

\[ \textit{const char **getKeys}() \]

Returns the full set of keys in the HashMap

\[ \textit{int getVal}(\text{const char } *\text{key}) \]
Returns a value of the specified key

    key   The key for which a value is requested

*const char *getKey(int val)*

Returns a key to which the specified value maps to

    val   The value for which a key is requested

*int getSize()*

Returns the size of the HashMap

*int getMaxKeyLen()*

Returns the maximum number of characters in a key

KdbAdapter Component

*KdbAdapter(const char *host, const int port)*

The constructor; initializes the KdbAdapter component

    host   The hostname of the Kdb+ database
    port   The port to connect to

*void connect()*

Connect to a Kdb+ database using the parameters specified in the constructor

*void subscribe(const char* tableName, const char **symSet, int symSetSize)*

Subscribe to a certain set of symbols from the specified table

    tableName   The name of the table from which to get the data
    symSet      The symbol set to subscribe to
    symSetSize  The number of symbols in the subscription set

*void kdbRead(int inFd)*

Continuously read the data for the subscribed symbols

    inFd   The file descriptor to a channel into which the subscription data will be recorded to
void kdbWrite(int outFd)

Continuously write the data to the Kdb+ database

  outFd   The file descriptor to a channel into which the output
data will be recorded to