Evaluating the Robustness and Feasibility of Integer Programming and Dynamic Programming in Aircraft Sequencing Optimization

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Flight delays cost many stakeholders both directly and indirectly
• Airlines
• Customers
• U.S. Economy

One estimate puts the cost of delay for the U.S. in 2007 at
$31.2 billion

Some strategies to reduce delay include:
• Adding new infrastructure
• Increasing peak period pricing
• Limiting landings and takeoffs per hour
Air Traffic Control (ATC)

Figure 1: Generic Airport Configuration (NASA Aviation Systems Division, 2011)
Air Traffic Control (ATC)

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Taxi to a "spot"
Air Traffic Control (ATC)

Figure 1: Generic Airport Configuration (NASA Aviation Systems Division, 2011)

Leave the spot and taxi to a runway queue.
Air Traffic Control (ATC)

Figure 1: Generic Airport Configuration (NASA Aviation Systems Division, 2011)
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Figure 1: Generic Airport Configuration (NASA Aviation Systems Division, 2011)
Air Traffic Control (ATC)

Figure 1: Generic Airport Configuration (NASA Aviation Systems Division, 2011)
Separation Requirements Between Takeoffs

<table>
<thead>
<tr>
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<th>Following</th>
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<th>Leads to</th>
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<tbody>
<tr>
<td></td>
<td>Small</td>
<td>Large</td>
<td>Heavy</td>
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<tr>
<td>Small</td>
<td>60</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Large</td>
<td>90</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Heavy</td>
<td>120</td>
<td>120</td>
<td>90</td>
</tr>
</tbody>
</table>

Delay Benefits of Re-sequencing

Separation times in seconds
Methods to Reduce Delay

1) Re-sequencing

   Reduces delay

2) Metering - holding aircraft until they can taxi unimpeded

   Reduces fuel burn and congestion on taxiways
Project Goal

Compare the feasibility of the Mixed Integer Linear Programming (MILP) and Dynamic Programming (DP) methods and the robustness of the solutions when stochastic variables were added into the optimization problem.
Optimization

Original: A-B-C
205s

Key (times in seconds)
Nose  Spot leave time
Taxi Time
Delay
Takeoff Time
Required Separation
Large Aircraft
Small Aircraft

A
150
B
110
C
75
90
80
60
125

ATC
10/12/2011
Objective:

Minimize departure delay

Constraints:

An aircraft cannot take off before it is ready

Separation times are not violated

Constrained Position Shifting (CPS) is obeyed
Methodology

Unoptimized Schedule + Randomly Perturbed Taxi Times = New Schedule #1

Optimized Schedule + Randomly Perturbed Taxi Times = New Schedule #2

Fuzzy Regression Model: Random Sample of 1000 Flights

- \(|\text{err}| < 30 \text{ sec}: 66.9\%\)
- \(|\text{err}| < 60 \text{ sec}: 90.3\%\)
- \(|\text{err}| < 120 \text{ sec}: 97.5\%\)
- mean \(|\text{err}| = 28.6 \text{ sec}\)

Triangular Distribution Model for Taxi Time Uncertainties

Repeat both of these for 500 iterations
Measuring Robustness and Feasibility

Robustness in stochastic situations
- Departure delay comparison
- Sequence change
- Separation time violations

Operational feasibility in real-time applications
Running times measured on a Dell desktop with:
- Linux
- 4 dual-core processors
- 4GB RAM
Linear Programs plan activities by solving for a set of variables to minimize or maximize an *objective function* while also obeying certain constraints.

A MILP is a Linear Program that has at least one integer constraint. This is the case for the traffic optimization in order to determine the sequence of the aircraft.

Cannot solve for a full day’s worth of data (~400 aircraft)
Dynamic Programming

Breaks a problem down recursively until reaching the simplest sub-problem, then iteratively solves the problem step by step until the entire problem is solved.

The shortest path from s to t is A-C-E with a cost of 17
Departure Delay Results

Deterministic Delay per Aircraft

Delay per Aircraft (seconds)

Baseline | CPS=0 | CPS=1 | CPS=2 | No CPS

Simulated First Come First Serve
Mixed Integer Linear Programming
Dynamic Programming

* CPS = Constrained Position Shifting
Departure Delay Results (cont.)

**Stochastic Delay per Aircraft**

- **Baseline**: 30 seconds
- **CPS=0**: ~35 seconds
- **CPS=1**: ~33 seconds
- **CPS=2**: ~32 seconds
- **No CPS**: ~31 seconds

* CPS = Constrained Position Shifting

- **Unoptimized**
- **Mixed Integer Linear Programming**
- **Dynamic Programming**
# Timing Results

<table>
<thead>
<tr>
<th>Method</th>
<th>Avg. Running Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixed Integer Linear Programming</td>
<td>45 seconds</td>
</tr>
<tr>
<td>Dynamic Programming, CPS=0,1</td>
<td>&lt; 1 second</td>
</tr>
<tr>
<td>Dynamic Programming, CPS=2</td>
<td>30 seconds</td>
</tr>
</tbody>
</table>

* CPS = Constrained Position Shifting
Limitations

Data
• Lack of demand
• Missing data
• Homogeneous aircraft mix

Methods
• Both the Dynamic Programming and Mixed Integer Linear Programming are heuristics

Results
• Arrival crossings not considered
Conclusions

DFW could achieve lower departure delay by not holding aircraft longer than necessary at the runway

CPS needs to be high enough for the deterministic optimizations to improve on Simulated FCFS

Our deterministic optimizations complete in a reasonable amount of time, but are not robust enough for real-world situations
Future Work

Add in arrival crossings

Include priority departures

Execute second optimization at the runway

Consider other stochastic variables
  • Adherence to separation times
  • Spot ready time calculations

Explore different runway layouts

Develop stochastic optimization algorithms
Acknowledgements

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Our project site, Lincoln Laboratory

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Mixed Integer Linear Programming Difficulties

Computationally intractable on a full day’s worth of flights (~400 aircraft)

Necessities:
- Split data into smaller time windows, called bins
- Obey separation requirements at runway
- Obey constrained position shifting (CPS) at spot

Problem:
- Differing unimpeded taxi times can cause the optimization to be unaware of both the spot and runway sequence causing the requirements to not be met
Binning

Bin #1

Bin #2

Time in Seconds

Spot Ready Times

Runway Ready Times
Add aircraft which were ready at the spot before any aircraft in Bin 1.
Add aircraft which were ready at the runway before any aircraft in Bin 1.
Dynamic Programming Difficulties

Optimal substructure:

Step 1 → Step 2 → ... → Step $n$

Our problem:

Step $i$

Optimal schedule for $i$ aircraft

Step $i+1$

Optimal schedule for $i+1$ aircraft