WPI Teaching Practicum

An Interdisciplinary Qualifying Project
Submitted to the faculty of
Worcester Polytechnic Institute
in partial fulfillment of the requirements for the
Degree of Bachelor of Science

Submitted By:
Warren Anderson

Submitted To:
Professor John Goulet

Date: April 19, 2011
warrenand@wpi.edu
The simplest schoolboy is now familiar with truths for which
Archimedes would have sacrificed his life.

- Ernest Renan
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Chapter 1: Demographics

Introduction:

In this chapter, several pieces of statistical data pertaining to the specific population Doherty Memorial High School serves is presented and discussed. In education, it is absolutely critical to understand the type of students that teachers must instruct. For example, if the large majority of the student population expresses an interest in graduating college, it is essential that the school provides an abundant foundation to prepare students toward that goal. Similarly, if a large portion of students want to find a job immediately after school, which is especially common among the several technical high schools around the state, the school ought to prepare the students for the workforce. It is also critical that faculty do not forget about the minority, because everyone is a minority in one sense or another. A particular student may have a special need or a handicap, or may be a member of an unusual race or religion. That is, a school should provide a comprehensive education, and without knowing the needs of the students and how diverse they are, that is nearly an impossible goal. Without this knowledge, it is like shooting an arrow in a random direction, and hoping that it meets the desired target.

Worcester Demographics:

Throughout this chapter, numerous comparisons will be made between the demographics between the school and the city, and between the city and the state. Performing comparisons such as these enable an educator to identify how the city is unique compared to the state, and how the school is unique compared to the city. Similarities can also be identified between the three. By making these comparisons, it is possible to identify characteristics that the school may have, but the city lacks, and vice-versa.

From this point forward, when the word school is mentioned, it is understood as Doherty Memorial High School, located in Worcester, MA. The words state is understood as Massachusetts, and the words city and district will be both understood as Worcester, MA. Also, all the data in this chapter are the most recent data, primarily provided by the United States Census Bureau and the Massachusetts Department of Education.

Table 1: State and District Demographics

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Worcester</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>181,045</td>
<td>6,547,629</td>
</tr>
<tr>
<td>People under 5 years</td>
<td>6.60%</td>
<td>6.50%</td>
</tr>
<tr>
<td>People under 18 years</td>
<td>22.10%</td>
<td>24.00%</td>
</tr>
<tr>
<td>People 65 years and over</td>
<td>11.70%</td>
<td>13.00%</td>
</tr>
<tr>
<td>Median Household Income</td>
<td>$45,944*</td>
<td>$64,057*</td>
</tr>
<tr>
<td>Male: Female Ratio</td>
<td>974:1000</td>
<td>968:1000</td>
</tr>
<tr>
<td>Residence Living in Poverty</td>
<td>17.9%*</td>
<td>10.3%*</td>
</tr>
</tbody>
</table>

What is the second largest city in New England? If you ask most people in the region this question, they will say that it is Providence, RI. In actuality, however, Worcester, MA has become the second largest city in New England. (Providence has a declining population of 171,909.2) This common misconception presents an excellent reason why statistical data is necessary. What we believe is not always what is true, and choosing actions based on belief and not on factual data can have great consequences.

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1 Source: http://quickfacts.census.gov/qfd/states/25/2582000.html
2 Source: http://www.city-data.com/city/Providence-Rhode-Island.html
From the data above, it is evident that the age distribution of Worcester is not considerably different from the state as a whole. In particular, the percentage of school age children can be calculated by subtracting the percentage of the population that is under 18 by the percentage of the population that is under 5. The results are 15.5% for the city and 17.5% for the state. Combining this information with the populations of the city and state, the number of school age children can be calculated. There are approximately 28,061 school aged children in Worcester and 1,571,430 school aged children in Massachusetts.

Another important statistic is how the median household income of the city compares to the state. The median household income in Worcester is $45,944, compared to the state average of $64,057. In other words, the household income of an average family in Worcester is approximately 71.7% of that of an average Massachusetts family. Additionally, 17.9% of the people living in Worcester live in poverty (as defined by the national poverty level) compared to only 10.3% of the people living in Massachusetts. The monetary wealth of a city can have great consequences on the education sector. The less income people earn the less money a city can spend on public schools. This is of course alleviated somewhat by federal and state aid, but not entirely. Even if external aid could completely satisfy the needs of such schools, individual families can still struggle paying for school supplies and even basic needs such as food and clothing. A family in poverty is far less likely to have educationally enriching devices, such as the computers, graphing calculators and the internet than wealthier families. It is all too obvious that educational resources cost money, and the less money that is available, the greater the challenge it is to educate without reducing the quality of education. In short, Worcester is a community that requires wise financial distribution in order to deliver a comparable education to that of wealthier communities. This is indeed possible, but achieving this may require students to share more resources, such as calculators, and to use resources more economically, such as using both sides of a piece of paper.

<table>
<thead>
<tr>
<th>Table 2: Ethnic Composition of Worcester³</th>
<th>Table 3: Ethnic Composition of MA⁴</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>White</td>
</tr>
<tr>
<td>59.6%</td>
<td>76.1%</td>
</tr>
<tr>
<td>Hispanic</td>
<td>Hispanic</td>
</tr>
<tr>
<td>20.9%</td>
<td>9.6%</td>
</tr>
<tr>
<td>Black</td>
<td>Black</td>
</tr>
<tr>
<td>11.6%</td>
<td>6.6%</td>
</tr>
<tr>
<td>Asian</td>
<td>Asian</td>
</tr>
<tr>
<td>6.1%</td>
<td>5.3%</td>
</tr>
<tr>
<td>Multi-Race</td>
<td>Multi-Race</td>
</tr>
<tr>
<td>4.0%</td>
<td>2.6%</td>
</tr>
<tr>
<td>Native Hawaiian/Pacific Islander</td>
<td>Native Hawaiian/Pacific Islander</td>
</tr>
<tr>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Native American</td>
<td>Native American</td>
</tr>
<tr>
<td>0.4%</td>
<td>0.3%</td>
</tr>
</tbody>
</table>

Ethnicity is another major demographic that must be considered. When evaluating a community based on its ethnicity, it is vital to avoid stereotypes, (such as Asians are good at math, or African Americans love watermelon) because stereotypes are very rarely accurate, and are plainly offensive. Rather, evaluate ethnicity in terms of cultural diversity. A population with a strong ethnic diversity is far more likely to be culturally diverse than a community that is not ethnically diverse. For example, the cuisine and religions of Caucasians are significantly different than the cuisine and religions of Asians. A culturally diverse community can be more enriching than one that is not, because individuals encounter an abundance of ideas, beliefs, and traditions and will have a greater understanding of humanity as a whole. Although culturally diverse communities can be more simulating, they are arguably more difficult to educate because their ideas and traditions differ. For example, in a Catholic school, there is little question on what religion will be encouraged and taught. However, at a public school in which people have significantly different religions, it would be outrageous to favor one religion over the other. In public schools, religious matters are much trickier to teach than at patriarchic schools.

³ Source: http://quickfacts.census.gov/qfd/states/25/2582000.html
⁴ Source: http://quickfacts.census.gov/qfd/states/25000.html
Segregation and prejudice are also issues regarding ethnicity. Two different things can happen in an ethnically diverse community. Individuals may either favor their own “kind” and develop hatred for those that are unlike them, or they may learn to embrace the richness of various cultures and develop love for one another. It is almost too obvious that the latter is more desirable. Therefore, schools that educate students from several ethnicities (and even schools that are not ethnically diverse) should strive for an appreciative and accepting environment for every race, culture, religion, etc. **A school should be a community in which every culture is celebrated.**

From Tables 2 and 3, it is apparent that Worcester is more ethnically diverse than Massachusetts. In particular, the White population in Worcester is significantly less represented (although still a majority) while every other race on the list is more represented. This fact should be interpreted optimistically, because a greater ethnic diversity means a greater opportunity to learn about each other.

It should be noted that the data from the two tables above are directly from the Census Bureau. The Census Bureau allows certain people to be included in more than one category, so the sum of all the different percentiles is above 100%.

**Enrollment:**

The tables in this section are data pertaining to the enrollment of Doherty Memorial High School.

**Table 4: Student Enrollment by Grade (2010-11)**

<table>
<thead>
<tr>
<th>Grade</th>
<th># of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>330</td>
</tr>
<tr>
<td>10</td>
<td>348</td>
</tr>
<tr>
<td>11</td>
<td>323</td>
</tr>
<tr>
<td>12</td>
<td>326</td>
</tr>
</tbody>
</table>

**Table 5: Enrollment by Gender (2010 - 2011)**

<table>
<thead>
<tr>
<th>Gender</th>
<th>School</th>
<th>District</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>661</td>
<td>12,471</td>
<td>490,363</td>
</tr>
<tr>
<td>Female</td>
<td>666</td>
<td>11,721</td>
<td>465,200</td>
</tr>
</tbody>
</table>

**Table 6: Enrollment by Race (2010-2011)**

<table>
<thead>
<tr>
<th>Ethnicity</th>
<th>School (%)</th>
<th>District (%)</th>
<th>State (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>African American</td>
<td>13.6</td>
<td>13.6</td>
<td>8.2</td>
</tr>
<tr>
<td>Asian</td>
<td>10.9</td>
<td>8.1</td>
<td>5.5</td>
</tr>
<tr>
<td>Hispanic</td>
<td>25.9</td>
<td>38.3</td>
<td>15.4</td>
</tr>
<tr>
<td>Native American</td>
<td>0.7</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>White</td>
<td>47.2</td>
<td>36.5</td>
<td>68.0</td>
</tr>
<tr>
<td>Multi-Race (Non-Hispanic)</td>
<td>1.7</td>
<td>3.1</td>
<td>2.4</td>
</tr>
<tr>
<td>Native Hawaiian, Pacific Islander</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
</tr>
</tbody>
</table>

---

School Ethnic Composition (2010 - 2011)

- African American: 13.6%
- Asian: 10.9%
- Hispanic: 25.9%
- Native American: 0.7%
- Multi-Race (Non-Hispanic): 1.7%
- White: 47.2%

District Ethnic Composition (2010 - 2011)

- African American: 13.6%
- Asian: 8.1%
- Hispanic: 38.3%
- Native American: 0.3%
- Multi-Race (Non-Hispanic): 3.1%
- White: 36.5%
Doherty is a high school that serves grades 9-12. The number of students in each grade level are approximately equal, however there are slightly more students in grades 9 and 10 than in grades 11 and 12. This is partially due to the number of students that drop out or transfer to another school (graduation rates will be discussed in greater detail later). The school serves approximately 1,327 students, or about 4.73% of the city’s school aged children. Of these students, there are slightly more females than males, but the difference is not substantial. Interestingly enough, the school, district, and the state all have slightly more females than males (see table 1). Gender ratios are important, and they can alter the educational environment. When the ratio is close to 50:50 (which is the case for Doherty), then students learn to work and learn with students of the opposite gender as well as their own. They are also exposed to the world’s natural distribution of gender, so these social skills may help them in the workforce or in other everyday task. Of course for adolescence, the presence of the opposite gender may unfortunately increase inappropriate behavior within the school such as “making out” in the corridors. In addition, the opposite gender may be a distraction for some students, especially if a couple happens to be in the same class. On the other hand, all-boy and all-girl schools may have less sexual distractions, but such a dramatic deprivation of the opposite gender is rarely found outside of those institutions. Therefore, it is easy to see how gender ratios play a role in school environment.

From Table 6 and the pie charts that follow, it is clear that the school more ethnically diverse than the state, but not as diverse as the district. Note that this data only includes students in the public school system, and do not include the entire population of the district and state. One reason why the school may not be as ethnically diverse as the district is how people segregate themselves into different neighborhoods in the city. Just like there is a Chinatown and an Italian neighborhood in Boston, there are also different neighborhoods that cater to different ethnicities. Although public busing transports students from all over the city, it is not always logistically possible to equally serve all neighborhoods at each school due to monetary and time constraints. Thus the ethnic composition will be different at one school then it is at another.
Selected Populations:

This section provides information regarding important subgroups of students in the school, compared with the district and the state.

Table 7: Selected Populations (2010-2011)

<table>
<thead>
<tr>
<th>Title</th>
<th>% of School</th>
<th>% of District</th>
<th>% of State</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Language not English</td>
<td>42.7</td>
<td>43.2</td>
<td>16.3</td>
</tr>
<tr>
<td>Limited English Proficient</td>
<td>17.6</td>
<td>31.8</td>
<td>7.1</td>
</tr>
<tr>
<td>Low-Income</td>
<td>48.3</td>
<td>70.1</td>
<td>34.2</td>
</tr>
<tr>
<td>Special Education</td>
<td>15.7</td>
<td>20.9</td>
<td>17.0</td>
</tr>
<tr>
<td>Free Lunch</td>
<td>42.0</td>
<td>63.6</td>
<td>29.1</td>
</tr>
<tr>
<td>Reduced Lunch</td>
<td>6.3</td>
<td>6.5</td>
<td>5.1</td>
</tr>
</tbody>
</table>

The second category, “Limited English Proficient”, is the percentage of students whose first language is not English and who are “unable to perform ordinary classroom work in English.” Therefore, the second category is a subset of the first category. A low-income student is one that is eligible for free or reduced lunch, or that receives Transitional Aid to Families benefits, or that is eligible for food stamps. The number of students in this category is equal or greater than the number of students in the last two categories combined. The “Special Education” category is the percentage of students enrolled in an Individualized Education Program (IEP). It is interesting to note that Doherty has a lower percentage in all of these categories than the district as a whole, but has a higher percentage in all categories than the state, except for special education. This should not be very surprising, because as we have already seen, the school is not as ethnically diverse as the district, but is more ethnically diverse than the state. Since the White population is higher at Doherty than it is at a typical Worcester school, it makes sense that the percentage of students lacking English proficiency is lower. However, there is a larger percentage of White students in the state than there are at the school, so English proficiency is lower at the school than in the state. The number of students that are not proficient in English can partially explain the number of low-income students. Students from families that do not speak fluent English are less likely to have jobs that require English proficiency. Also, jobs that require fluent English usually pay more than jobs that do not.

Technology

The following table shows computer and internet accessibility within the school, district, and state.

Table 8: Technology (2009-2010)

<table>
<thead>
<tr>
<th></th>
<th>School</th>
<th>District</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students per Computer</td>
<td>5.2</td>
<td>3.0</td>
<td>3.3</td>
</tr>
<tr>
<td>Classrooms on the Internet (%)</td>
<td>100.0</td>
<td>100.0</td>
<td>99.9</td>
</tr>
</tbody>
</table>

Almost half of the students at Doherty are low-income students. In such a situation, it is beneficial for a school to have more computers because the likelihood that students do not have computers at home is greater. Since Doherty has a far less percentage of low-income students than the district (see table 7), it makes since that the school has less computers per person then the district. However, the district as a whole ought to have more computers per student.

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8 http://profiles.doe.mass.edu/profiles/student.aspx?orgcode=03480512&orgtypecode=6&leftNavId=303&
9 http://profiles.doe.mass.edu/help/data.aspx
10 http://profiles.doe.mass.edu/profiles/student.aspx?orgcode=03480512&orgtypecode=6&leftNavId=306&
than the state, since the district has a higher percentage of low-income students than the state. More computers in the school and the district will of course be better, but monetary constraints and the amount of space that they take are major challenges.

When I was at Doherty, I noticed that some classrooms did not have a computer. Naturally, the statistic above that indicates that 100% of the classrooms have internet access had be bewildered. How can all the classrooms at the school have internet access if some classrooms do not even have a computer? I believe that at one time, all classrooms did have computers, but some of them broke down and were not replaced. So, when that statistic was made in 2010, all classrooms did have internet access. However, this statistic no longer holds because broken computers were not replaced. Since computers are excellent tools for recording grades and attendance, as well as presenting material to students, it is unfortunate that not all the classrooms have a computer.

**Plans of High School Graduates**

In order for schools to cater to the needs and wishes of its students, it is necessary to ask them what their plans are after high school. The following table summarizes that information.

<table>
<thead>
<tr>
<th>Plan</th>
<th>% of School</th>
<th>% of District</th>
<th>% of School</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-Year Private College</td>
<td>20</td>
<td>19</td>
<td>32</td>
</tr>
<tr>
<td>4-Year Public College</td>
<td>30</td>
<td>21</td>
<td>28</td>
</tr>
<tr>
<td>2-Year Private College</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2-Year Public College</td>
<td>36</td>
<td>40</td>
<td>21</td>
</tr>
<tr>
<td>Other Post-Secondary</td>
<td>3</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>Work</td>
<td>6</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>Military</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Other</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Unknown</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

Almost 90% of high school graduates want to continue their education, while a small portion want to enter the workforce or enter the military. As a matter of fact, the percentage of Doherty students that plan to go to college is significantly higher compared to the percentage of Massachusetts students that plan to go to college. This may be due to the fact that the state has “regular” high schools and technical high schools. Technical high schools are schools in which students specialize in a specific field. A large portion of students that graduate technical high schools enter the workforce after graduation. Therefore, a smaller percentage of Massachusetts graduates plan on going to college than Doherty because Doherty is not a technical high school.

Although most students at Doherty want to go to college, it is very important that the school not only caters to those students. Rather, the school ought to help all students fulfill what they want to accomplish in the future. The school needs to prepare students not only for college, but also for the workforce and the military as well.

It is not necessarily bad that not all students plan on going to college. In fact, in order for our nation to thrive, some people must have careers that do not require a college degree, while other people must be in the military. Actually, the fact that students are able to choose their future for themselves is ideal. What is potentially problematic is the portion of students that do not know what to do. If someone plans to do something, he or she is far more likely to do it. So, a person without any plans after graduation is more likely to do nothing. Also, if a student knows in advance

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what he or she wants to do, then that person can prepare properly for it. For example, if a girl knew since eighth grade that she wants to go to Harvard, she knows that she will have to have excellent grades and outstanding extracurricular activities, among other things, in high school. Or, if a boy knows that he wants to enter the military, he knows he will have to become physically fit and mentally strong. A person that is uncertain about his or her future, on the other hand, will have a harder time preparing for the future, and will be more likely to be unprepared. **Schools, including Doherty, should help students uncertain about what they want to do in the future to find out what path is best for them.** Many schools emphasize going to college after graduation. Rather, schools should provide information to students (and parents) about all possibilities after graduation, not just college.

Finally, note that the percentage of students at Doherty that plan on going to a private institution after graduation is significantly lower than the percentage of students statewide that plan on going to a private institution. This is most likely due to the fact that private institutions are usually more expensive, and so high schools with a higher concentration of low-income students are more likely to choose cheaper alternatives such as public institutions of higher learning. In schools like Doherty, it is critical that information about financial aid, scholarships, and student loans is delivered to students so that they know that they can afford the college they plan on attending.

**Graduates Attending Colleges and Universities**

Table 9 showed the percentage of students that planned continuing their education. Table 10 below shows how many of those students actually attended and to what type of institution they went to. Data on subgroups with fewer than 15 students are not recorded to protect the privacy of those individuals.

<table>
<thead>
<tr>
<th>Student Group</th>
<th># High School Graduates</th>
<th># Attending Coll/Univ</th>
<th>% Attending Coll/Univ</th>
<th>% Private Two-Yr</th>
<th>% Public Two-Yr</th>
<th>% Public Four-Yr</th>
<th>% MA Community College</th>
<th>% MA State University</th>
<th>% MA Univ of Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Students</td>
<td>359</td>
<td>273</td>
<td>76.0</td>
<td>0.0</td>
<td>22.7</td>
<td>44.7</td>
<td>32.5</td>
<td>42.9</td>
<td>18.3</td>
</tr>
<tr>
<td>Lim. English Prof.</td>
<td>33</td>
<td>22</td>
<td>66.7</td>
<td>0.0</td>
<td>9.1</td>
<td>72.7</td>
<td>18.2</td>
<td>68.2</td>
<td>13.6</td>
</tr>
<tr>
<td>Low Income</td>
<td>224</td>
<td>156</td>
<td>69.6</td>
<td>0.0</td>
<td>16.6</td>
<td>58.3</td>
<td>23.1</td>
<td>55.1</td>
<td>13.5</td>
</tr>
<tr>
<td>Special Education</td>
<td>58</td>
<td>38</td>
<td>65.5</td>
<td>0.0</td>
<td>13.2</td>
<td>81.6</td>
<td>5.3</td>
<td>75.3</td>
<td>5.3</td>
</tr>
<tr>
<td>Female</td>
<td>184</td>
<td>154</td>
<td>83.7</td>
<td>0.0</td>
<td>24.7</td>
<td>37.7</td>
<td>37.7</td>
<td>39.4</td>
<td>21.4</td>
</tr>
<tr>
<td>Male</td>
<td>175</td>
<td>119</td>
<td>68.0</td>
<td>0.0</td>
<td>20.2</td>
<td>53.8</td>
<td>25.1</td>
<td>51.3</td>
<td>14.3</td>
</tr>
<tr>
<td>Asian</td>
<td>29</td>
<td>26</td>
<td>78.9</td>
<td>0.0</td>
<td>15.0</td>
<td>50.0</td>
<td>35.0</td>
<td>50.0</td>
<td>20.0</td>
</tr>
<tr>
<td>Black or Afr. Amer.</td>
<td>52</td>
<td>39</td>
<td>75.0</td>
<td>0.0</td>
<td>15.4</td>
<td>59.0</td>
<td>25.5</td>
<td>53.8</td>
<td>10.3</td>
</tr>
<tr>
<td>Hispanic</td>
<td>103</td>
<td>57</td>
<td>67.0</td>
<td>0.0</td>
<td>19.4</td>
<td>62.7</td>
<td>17.9</td>
<td>59.7</td>
<td>13.4</td>
</tr>
<tr>
<td>Multi-Race, Non-Hisp.</td>
<td>3</td>
<td>3</td>
<td>100.0</td>
<td>0.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>White</td>
<td>178</td>
<td>144</td>
<td>80.9</td>
<td>0.0</td>
<td>27.1</td>
<td>31.9</td>
<td>41.0</td>
<td>31.3</td>
<td>22.2</td>
</tr>
</tbody>
</table>

Of the 90% of students that planned on attending college, only 76% actually did. A major question is of course whatever happened to the 14% of students that planned on going to college, but did not. Where they rejected to the college they wanted to go to? If that is the case, then why didn’t they apply to one or more safety schools? Did the students believe that they could not financially afford to go to college? If that is the reason, then did they apply to financial aid? Or, did they simply change their minds and decided to enter the workforce or enter the military instead? Unfortunately, without additional data, the answers to these questions are anyone’s guess. Since the reason(s) why students did not go to college even though they planned to is unknown, it is difficult to solve the problem.

---

Observe that the special education students and students with limited English proficiency have the lowest college attendance rates. This is not a surprising statistic, because most colleges require SAT scores, and people in those categories do not do very well on the SATs. What is perhaps more unexpected is the fact that females (of any ethnicity) have the highest college attendance among all the subcategories. Hence the stereotype that more males go to college than females is simply untrue for Doherty students. In fact, a graduating male is 15.7% less likely to go to college than a graduating female.

**Mobility Rates**

Mobility rates determine how many students enter and leave school throughout the year. It can also be used to determine how many students remain at the school.

**Table 11: Mobility Rates (2010)**

<table>
<thead>
<tr>
<th>Student Group</th>
<th>Churn/Intake Enroll</th>
<th>% Churn</th>
<th>% Intake</th>
<th>Stability Enroll</th>
<th>% Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate</td>
<td>1,494</td>
<td>14.6</td>
<td>8.2</td>
<td>1,422</td>
<td>89.7</td>
</tr>
<tr>
<td>ELL</td>
<td>206</td>
<td>25.2</td>
<td>18.0</td>
<td>184</td>
<td>83.7</td>
</tr>
<tr>
<td>Special Education</td>
<td>248</td>
<td>23.0</td>
<td>13.7</td>
<td>229</td>
<td>83.4</td>
</tr>
<tr>
<td>Low Income</td>
<td>843</td>
<td>18.2</td>
<td>11.9</td>
<td>783</td>
<td>88.1</td>
</tr>
<tr>
<td>Black or Afr. Amer.</td>
<td>225</td>
<td>17.8</td>
<td>11.6</td>
<td>208</td>
<td>88.9</td>
</tr>
<tr>
<td>Asian</td>
<td>147</td>
<td>17.0</td>
<td>9.5</td>
<td>136</td>
<td>89.7</td>
</tr>
<tr>
<td>Hispanic</td>
<td>398</td>
<td>19.6</td>
<td>10.6</td>
<td>374</td>
<td>85.6</td>
</tr>
<tr>
<td>Amer. Ind. or Alaska Nat.</td>
<td>8</td>
<td>12.5</td>
<td>0.0</td>
<td>8</td>
<td>87.5</td>
</tr>
<tr>
<td>White</td>
<td>694</td>
<td>10.4</td>
<td>5.6</td>
<td>674</td>
<td>92.3</td>
</tr>
<tr>
<td>Nat. Haw. or Pacif. Isl.</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Multi-race, Non-Hisp.</td>
<td>22</td>
<td>9.1</td>
<td>9.1</td>
<td>22</td>
<td>90.9</td>
</tr>
</tbody>
</table>

*NOTE: Mobility rates will not be publicly reported for enrollments of fewer than 6.*

The “Churn/Intake Enroll” category is how many students that were enrolled during the school year at one point or another. Any student that was eventually transferred in or out of the school is included in this category. The second category, “% Churn” measures the number of students that were transferred in or out of the school during the school year. The third category, “% Intake” measures the amount of students that were transferred into the school over the course of the year. “Stability Enroll” and “% Stability” measures the number of students that were enrolled over the course of the entire school year. Therefore, 6.4% of students were transferred out of Doherty, 8.2% of students were transferred into of Doherty, and 89.7% of students stayed in Doherty throughout the school year. The sum of these percentages is over 100% because more people were transferred in than were transferred out. The average stability rate for Massachusetts is 95.3%, so Doherty has a relatively low stability rate.

A low stability rate can be a great disturbance in a school, because teachers have to accommodate new students while conducting normal operations for the rest of the class. The more people entering and leaving, the greater confusion there is, and the more time that is spent straightening out rosters and the less time that is spent teaching and preparing lessons. When a student shows up in the middle of the school year, teachers have to figure out how they will grade such students.

A low stability rate can also be an indicator on how much students actually like going to school. If students are transferring all over the place, then they are probably unhappy about some aspect of the school environment. Parents can also withdraw their children from schools and place them elsewhere if they are not happy with the...
school their children are attending. So, in order for schools to have high stability rates, students and parents must like the school that the students are attending.

**Graduation Rates and other Indicators**

Indicators are calculations that are used to determine how effective the school is at graduating students and keeping students in school.

**Table 12: Indicators(2009-2010)**

<table>
<thead>
<tr>
<th></th>
<th>School</th>
<th>District</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 9-12 Dropout Rate</td>
<td>2.5</td>
<td>3.8</td>
<td>2.9</td>
</tr>
<tr>
<td>Attendance Rate</td>
<td>93</td>
<td>94.5</td>
<td>94.6</td>
</tr>
<tr>
<td>Average # of Days Absent</td>
<td>12</td>
<td>9.3</td>
<td>9.3</td>
</tr>
<tr>
<td>In-School Suspension Rate</td>
<td>16.8</td>
<td>9.1</td>
<td>3.7</td>
</tr>
<tr>
<td>Out-of-School Suspension Rate</td>
<td>15.8</td>
<td>12.8</td>
<td>6</td>
</tr>
<tr>
<td>Retention Rate</td>
<td>6.6</td>
<td>3.6</td>
<td>2.1</td>
</tr>
<tr>
<td>Graduation Rate</td>
<td>81</td>
<td>71.4</td>
<td>82.1</td>
</tr>
<tr>
<td>Truancy Rate</td>
<td>29</td>
<td>28.5</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Notice that the dropout rate for Doherty is lower than both the district and the state. This is an excellent statistic, because it shows that at this school, students are more willing to continue their education despite any adversities such as failing a grade. The attendance statistic is the percentages of days in which students are in school. Attendance rates at this school are actually worse than in the district and the state. It seems counterintuitive that the school has a lower dropout rate and a lower attendance rate. Perhaps students choose to be absent for a few days (i.e. take a “mental health day”) instead of giving up entirely with their education. While this is a much better alternative to dropping out, attendance is nevertheless important. Therefore, faculty at the school should attempt to improve the attendance rate. **Making the school a safer, more fun, and more invigorating environment is a sure way to do this.** From my experience at Doherty, I witnessed teachers that do an excellent job at achieving all three of those things (and their students actually like showing up for class), while other teachers do a poor job (and their students have worse attendance). Furthermore, parents need to make sure that their children go to school as often as possible. Many parents tolerate an occasional absence (perhaps because they do the same thing), which is very unfortunate. Correcting this problem would be very difficult, because parents tend to be extremely defensive.

Another issue at the school is the suspension rate. The in-school suspension rate is almost double that of the district’s and five times that of the state’s. In addition, the out-of-school suspension rate is one and a quarter more than the district’s and two and two thirds more than the state’s. High suspension rates correlate with lower safety, since students are usually suspended for doing unsafe things. Therefore, high suspension rates can make attendance worse because students do not want to go to an unsafe environment. Furthermore, if a student is suspended, then they are missing class time, and so they are learning very little, if anything at all. It is clear, then, that a high suspension rate can be a large problem. To lower suspension rates, students have to learn to behave appropriately within the school environment. This can be achieved with increased strictness and higher security.

---

At my time at Doherty, I realized an incredible variance in teacher strictness. Some teachers forbid the use of cell phones and mp3 players in class, while other teachers allow them as calculators and “work music”. Some teachers have little tolerance for tardiness and absenteeism, while other teachers do not seem to notice when a student is late. In some classrooms, chatting is forbidden, while in other classrooms, it is a given. If students are ever going to learn what is acceptable and what is unacceptable, then there should be minimal variance of what is allowed and not allowed from classroom to classroom. Also, when I was at the school, I never encountered a security guard. On student career day, there was even a police officer that informed students that there were only five police officers patrolling the numerous schools in the city! Police officers are only present at the school when there is a “problem”. Also, since I was very young at the time, I was able to disguise myself as a student, and I was only asked once where my guest pass was! Who knows how many unauthorized people are in the school at any given time? Clearly, the school lacks vigorous security measures. The need of increased strictness and better security at the school can certainly be improved.

Retention rates are also significantly worse at Doherty than they are at the district and state level. The retention rate is simply the percentage of students that repeat a grade. At the school, the retention rate is nearly double the retention rate of Worcester, and more than triple the retention rate of Massachusetts. A student must repeat a grade if he or she failed a core class without any way of making it up (e.g. summer school or the buyback program) or by having too many absences. So, in order to decrease the retention rate, more students need to pass their classes and more students have to attend school. There are numerous reasons why a student may be failing a class, such as poor interest in the subject matter, no motivation from parents or pairs, lacking the prerequisite skills, poor attendance, etc. Whatever the problem is, students, teachers, and parents need to cooperate with each other to resolve it.

The truancy rate at Doherty is also a notable issue. The truancy rate is calculated as the percentage of students that have nine or more unexcused absences within the course of one year. The truancy rate at the school is slightly worse than the district truancy rate, and is thirteen times higher than the state truancy rate. Ways to improve attendance have already been discussed, but it is worth mentioning that the key to a high attendance rate (and low truancy rate) is to provide an environment in which the students enjoy being in.

The following tables show detailed pertaining graduation rates of various subgroups. The two tables for the adjusted cohort graduation rates are adjusted for transfers in and out of the school, while the other two tables are not.

---

16 http://profiles.doe.mass.edu/help/data.aspx
17 http://profiles.doe.mass.edu/help/data.aspx
18 http://www.doe.mass.edu/infoservices/reports/gradrates/calculating_overview.html
Table 14: 4-Year Graduation Rate (2010)\textsuperscript{19}

<table>
<thead>
<tr>
<th>Student Group</th>
<th># in Cohort</th>
<th>% Graduated</th>
<th>% Still in School</th>
<th>% Non-Grad Completers</th>
<th>% GED</th>
<th>% Dropped Out</th>
<th>% Permanently Excluded</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Students</td>
<td>420</td>
<td>81.0</td>
<td>4.5</td>
<td>1.6</td>
<td>4.0</td>
<td>8.0</td>
<td>0.9</td>
</tr>
<tr>
<td>Male</td>
<td>212</td>
<td>77.8</td>
<td>4.2</td>
<td>1.4</td>
<td>5.2</td>
<td>9.4</td>
<td>1.9</td>
</tr>
<tr>
<td>Female</td>
<td>214</td>
<td>84.1</td>
<td>4.7</td>
<td>1.9</td>
<td>2.8</td>
<td>6.5</td>
<td>0.0</td>
</tr>
<tr>
<td>ELL</td>
<td>42</td>
<td>73.8</td>
<td>4.8</td>
<td>4.8</td>
<td>0.0</td>
<td>16.7</td>
<td>0.0</td>
</tr>
<tr>
<td>Special Education</td>
<td>80</td>
<td>71.3</td>
<td>10.0</td>
<td>5.0</td>
<td>2.5</td>
<td>11.3</td>
<td>0.0</td>
</tr>
<tr>
<td>Low Income</td>
<td>271</td>
<td>77.9</td>
<td>5.5</td>
<td>2.2</td>
<td>4.1</td>
<td>8.9</td>
<td>1.5</td>
</tr>
<tr>
<td>Black or Afr. Amer.</td>
<td>62</td>
<td>79.0</td>
<td>4.8</td>
<td>3.2</td>
<td>3.2</td>
<td>6.5</td>
<td>3.2</td>
</tr>
<tr>
<td>Asian</td>
<td>28</td>
<td>85.7</td>
<td>7.1</td>
<td>0.0</td>
<td>7.1</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Hispanic</td>
<td>132</td>
<td>72.7</td>
<td>7.6</td>
<td>3.0</td>
<td>3.0</td>
<td>12.1</td>
<td>1.5</td>
</tr>
<tr>
<td>Amer, Ind. or Alaska Nat.</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>White</td>
<td>198</td>
<td>87.4</td>
<td>2.0</td>
<td>0.5</td>
<td>4.5</td>
<td>5.6</td>
<td>0.0</td>
</tr>
<tr>
<td>Nat. Haw. or Pacif. Isl.</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Multi-race, Non-Hisp.</td>
<td>5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

* NOTE: Graduation rates will not be publicly reported for cohort counts fewer than 6.

Table 15: 4-Year Adjusted Cohort Graduation Rate (2010)\textsuperscript{20}

<table>
<thead>
<tr>
<th>Student Group</th>
<th># in Cohort</th>
<th>% Graduated</th>
<th>% Still in School</th>
<th>% Non-Grad Completers</th>
<th>% GED</th>
<th>% Dropped Out</th>
<th>% Permanently Excluded</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Students</td>
<td>305</td>
<td>84.9</td>
<td>3.0</td>
<td>1.3</td>
<td>3.3</td>
<td>5.9</td>
<td>0.7</td>
</tr>
<tr>
<td>Male</td>
<td>144</td>
<td>81.9</td>
<td>2.8</td>
<td>1.4</td>
<td>4.2</td>
<td>8.3</td>
<td>1.4</td>
</tr>
<tr>
<td>Female</td>
<td>181</td>
<td>87.5</td>
<td>3.1</td>
<td>2.5</td>
<td>3.8</td>
<td>5.6</td>
<td>0.0</td>
</tr>
<tr>
<td>ELL</td>
<td>15</td>
<td>60.0</td>
<td>0.0</td>
<td>0.7</td>
<td>0.0</td>
<td>33.3</td>
<td>0.0</td>
</tr>
<tr>
<td>Special Education</td>
<td>52</td>
<td>75.9</td>
<td>3.8</td>
<td>7.7</td>
<td>1.9</td>
<td>9.6</td>
<td>0.0</td>
</tr>
<tr>
<td>Low Income</td>
<td>173</td>
<td>81.5</td>
<td>4.0</td>
<td>1.7</td>
<td>4.0</td>
<td>7.5</td>
<td>1.2</td>
</tr>
<tr>
<td>Black or Afr. Amer.</td>
<td>37</td>
<td>78.4</td>
<td>2.7</td>
<td>2.7</td>
<td>2.7</td>
<td>10.8</td>
<td>2.7</td>
</tr>
<tr>
<td>Asian</td>
<td>21</td>
<td>85.7</td>
<td>0.5</td>
<td>0.0</td>
<td>4.8</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Hispanic</td>
<td>92</td>
<td>77.2</td>
<td>5.4</td>
<td>3.3</td>
<td>3.3</td>
<td>9.8</td>
<td>1.1</td>
</tr>
<tr>
<td>Amer, Ind. or Alaska Nat.</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>White</td>
<td>152</td>
<td>91.4</td>
<td>0.7</td>
<td>0.0</td>
<td>3.3</td>
<td>4.6</td>
<td>0.0</td>
</tr>
<tr>
<td>Nat. Haw. or Pacif. Isl.</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Multi-race, Non-Hisp.</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

* NOTE: Graduation rates will not be publicly reported for cohort counts fewer than 6.

\textsuperscript{19} http://profiles.doe.mass.edu/grad/grad_report.aspx?orgcode=03480512&orgtypecode=6&

\textsuperscript{20} http://profiles.doe.mass.edu/grad/grad_report.aspx?orgcode=03480512&orgtypecode=6&
Although Doherty has many challenges with regards to attendance, a redeeming quality is its high graduation rate. The school’s graduation rate is 9.6% higher than the graduation rate of the city as a whole, and only slightly lower than the state’s graduation rate. As an urban school, this is a major accomplishment. The single most important reward a school should provide is a broad, yet vigorous, foundation in which students can use to succeed in their next venture, whether it is college, the workforce, the military, or some other calling. A high school diploma is a physical document that symbolizes this reward. However, it is extremely important to note that just because someone graduated, that does not mean that the student has actually achieved sufficient competence. It is possible that the school failed to provide students with a sufficient education, but graduated them anyway. While this is a horrifying idea, I believe that does happen. Therefore, graduation rates alone cannot predict the quality of education the school provides. MCAS, SAT, and AP results can be powerful indicators of whether or not students have actually received a rigorous education. For example, if MCAS scores are low, but graduation rates are high, then that is a strong indication that the school is graduating students with a minimal foundation to succeed. If on the

Table 16: 5-Year Graduation Rate (2009)

<table>
<thead>
<tr>
<th>Student Group</th>
<th># in Cohort</th>
<th>% Graduated</th>
<th>% Still in School</th>
<th>% Non-Grad Completes</th>
<th>% GED</th>
<th>% Dropped Out</th>
<th>% Permanently Excluded</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Students</td>
<td>432</td>
<td>81.3</td>
<td>0.7</td>
<td>1.4</td>
<td>5.6</td>
<td>10.9</td>
<td>0.2</td>
</tr>
<tr>
<td>Male</td>
<td>193</td>
<td>80.8</td>
<td>0.0</td>
<td>1.5</td>
<td>5.2</td>
<td>11.9</td>
<td>0.5</td>
</tr>
<tr>
<td>Female</td>
<td>239</td>
<td>81.5</td>
<td>1.3</td>
<td>1.2</td>
<td>5.9</td>
<td>10.0</td>
<td>0.0</td>
</tr>
<tr>
<td>ELL</td>
<td>41</td>
<td>80.5</td>
<td>0.0</td>
<td>4.9</td>
<td>4.9</td>
<td>9.8</td>
<td>0.0</td>
</tr>
<tr>
<td>Special Education</td>
<td>73</td>
<td>68.5</td>
<td>1.4</td>
<td>2.7</td>
<td>4.1</td>
<td>23.3</td>
<td>0.0</td>
</tr>
<tr>
<td>Low Income</td>
<td>248</td>
<td>82.7</td>
<td>1.2</td>
<td>2.0</td>
<td>3.6</td>
<td>10.5</td>
<td>0.0</td>
</tr>
<tr>
<td>Black or Afr. Amer.</td>
<td>59</td>
<td>89.8</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>10.2</td>
<td>0.0</td>
</tr>
<tr>
<td>Asian</td>
<td>26</td>
<td>100.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Hispanic</td>
<td>107</td>
<td>70.1</td>
<td>0.9</td>
<td>2.8</td>
<td>5.6</td>
<td>20.6</td>
<td>0.0</td>
</tr>
<tr>
<td>Amer. Ind. or Alaska Nat.</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>White</td>
<td>239</td>
<td>82.0</td>
<td>0.8</td>
<td>1.3</td>
<td>7.5</td>
<td>7.9</td>
<td>0.4</td>
</tr>
<tr>
<td>Nat. Haw. or Pacif. Isl.</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Multi-race, Non-Hisp.</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

*NOTE: Graduation rates will not be publicly reported for cohort counts fewer than 5.

Table 17: 5-Year Adjusted Cohort Graduation Rate (2009)

<table>
<thead>
<tr>
<th>Student Group</th>
<th># in Cohort</th>
<th>% Graduated</th>
<th>% Still in School</th>
<th>% Non-Grad Completes</th>
<th>% GED</th>
<th>% Dropped Out</th>
<th>% Permanently Excluded</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Students</td>
<td>324</td>
<td>87.3</td>
<td>0.3</td>
<td>0.3</td>
<td>4.3</td>
<td>7.7</td>
<td>0.0</td>
</tr>
<tr>
<td>Male</td>
<td>145</td>
<td>86.9</td>
<td>0.0</td>
<td>0.7</td>
<td>4.1</td>
<td>8.3</td>
<td>0.0</td>
</tr>
<tr>
<td>Female</td>
<td>179</td>
<td>87.7</td>
<td>0.6</td>
<td>0.0</td>
<td>4.5</td>
<td>7.3</td>
<td>0.0</td>
</tr>
<tr>
<td>ELL</td>
<td>14</td>
<td>92.9</td>
<td>0.0</td>
<td>0.0</td>
<td>7.1</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Special Education</td>
<td>58</td>
<td>74.1</td>
<td>1.7</td>
<td>1.7</td>
<td>5.2</td>
<td>17.2</td>
<td>0.0</td>
</tr>
<tr>
<td>Low Income</td>
<td>174</td>
<td>85.8</td>
<td>0.6</td>
<td>0.6</td>
<td>3.4</td>
<td>8.5</td>
<td>0.0</td>
</tr>
<tr>
<td>Black or Afr. Amer.</td>
<td>34</td>
<td>91.2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>8.8</td>
<td>0.0</td>
</tr>
<tr>
<td>Asian</td>
<td>22</td>
<td>100.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Hispanic</td>
<td>70</td>
<td>77.1</td>
<td>0.0</td>
<td>0.0</td>
<td>4.3</td>
<td>18.6</td>
<td>0.0</td>
</tr>
<tr>
<td>Amer. Ind. or Alaska Nat.</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>White</td>
<td>197</td>
<td>88.8</td>
<td>0.5</td>
<td>0.5</td>
<td>5.6</td>
<td>4.8</td>
<td>0.0</td>
</tr>
<tr>
<td>Nat. Haw. or Pacif. Isl.</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Multi-race, Non-Hisp.</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

*NOTE: Graduation rates will not be publicly reported for cohort counts fewer than 6.

other hand, test scores and graduation rates are both high, then that is a very good indication that the school is actually providing a comprehensive education.

Recent MCAS Results

The MCAS is the Massachusetts Comprehensive Assessment System, a statewide exam in which all public school students must take. The questions on the MCAS corresponding to the variety of standards outlined in the Massachusetts Curriculum Frameworks. Students from most grade levels are required to take test in various subject areas; however, the 10th grade English and mathematics test are the most important because students must pass them in order to graduate. The test scores range from 200-280, and there are four subcategories, Advanced, Proficient, Needs Improvement, and Warning/Failing, corresponding to grades of 200-219, 220-239, 240-259, and 260-280, respectively. A score of 220 is necessary to pass an exam. MCAS test are scored using a raw score to scaled score conversion. For the 2011 grade 10 mathematics test, a student must have received a raw score of 17-19 out of 60 to receive a 220, or a raw score of either a 59/60 or a 60/60 to receive a 280. For the 2011 grade 10 English test, a student must have received a raw score of 28-31 out of 72 to get a 220, or a raw score of 69-72 out of 72 to get a 280.23 No credit is deducted for incorrect or blank answers. If a student does not pass either 10th grade exam, he or she will be given several opportunities to pass it/them. If a student does not pass either of the tests by graduation, the student will not get a high school diploma. Instead, the student will receive a “Certificate of Attendance”, even if the student fulfilled all other graduation requirements. Therefore, the MCAS is important to measure how well schools fulfill the standards contained in the Curriculum Frameworks and to graduate students.

The following table summarizes the spring of 2010 MCAS results of the school and of the state. Note that the “Proficient or Higher” category is simply the sum of the “Proficient” and “Advanced” categories. The CPI, or the Composite Performance Index, is a measure of how much students are progressing toward proficiency in the mathematics and English MCAS exams. Each student is given a score from 0 to 100 based on their performance. The CPI is the average of all the student’s scores.24 The SGP, or Student Growth Percentile, is a percentile of how much a student or a group of students improved compared to their pairs. So a SGP of 50 means that a student is improving as rapidly as other students in his or her grade level.25

Table 18: MCAS Test of Spring 201126

<table>
<thead>
<tr>
<th>Grade and Subject</th>
<th>Proficient or Higher</th>
<th>Advanced</th>
<th>Proficient</th>
<th>Needs Improvement</th>
<th>Warning/Failing</th>
<th>Students Included</th>
<th>CPI</th>
<th>SGP Included in SGP</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRADE 10 - ENGLISH LANGUAGE ARTS</td>
<td>SCHOOL</td>
<td>STATE</td>
<td>SCHOOL</td>
<td>STATE</td>
<td>SCHOOL</td>
<td>STATE</td>
<td>SCHOOL</td>
<td>STATE</td>
</tr>
<tr>
<td>GRADE 10 - MATHEMATICS</td>
<td>78</td>
<td>64</td>
<td>94</td>
<td>80</td>
<td>28</td>
<td>15</td>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>GRADE 10 - SCIENCE AND TECH</td>
<td>60</td>
<td>87</td>
<td>12</td>
<td>20</td>
<td>48</td>
<td>47</td>
<td>31</td>
<td>27</td>
</tr>
</tbody>
</table>

From this data, it is immediately evident that the school does not perform as well as the state does. In particular, the percentages of students failing the English and the mathematics exams are approximately 60% higher than the state average. However, the fact that 17.6% of the students at Doherty are not proficient in English can partially explain why the school slightly underperforms. English proficiency does not only affect the scores on the English exams, because most of the questions in the other tests are also written in English. Although the performance of the school lags behind the state performance, it is great that the SGP of Doherty students are 50% or higher, which means that the MCAS scores among those students are improving faster than average. Of course, Doherty has plenty of room for improvement, but current MACS scores are not awful considering the limited English proficiency many of the students have.

23 http://www.doe.mass.edu/mcas/2011/results/spring_conversion.xls  
25 www.doe.mass.edu/mcas/growth/GrowthPresentation.ppt  
MCAS Annual Comparisons

In order to prove that students are really improving from year to year, annual comparisons must be made. The following bar graphs display the 2008-2011 grade 10 MCAS results for Doherty, divided by the four grading categories (Advanced, Proficient, Needs Improvement, and Warning/Failing)

---

GRADE 10 - MATHEMATICS
Percentage of Students by Performance Level

GRADE 10 - SCIENCE AND TECH/ENG
Percentage of Students by Performance Level

In 2011, there have been significant gains in all three tests. For the English MCAS exam, less people scored in the Proficient, Needs Improvement, and Warning/Failing categories, and there was a huge increase in the number of students that scored in the Advanced category. For the mathematics exam, there was a slight decrease in the percentage of students that scored in the Proficient and Needs Improvement categories, while the percentage of students scoring in the Warning/Failing section remained the same. Furthermore, there was a modest increase in the percentage of students that scored in the Advanced category. Finally, for the science and technology/engineering exam, there was a sharp decrease in the number of students scoring in the Needs Improvement category, while the number of students in the Advanced category nearly tripled compared to the previous year. The number of students in the Proficient category also increased substantially. The number of students that scored in the lowest category increased by 1%; however, the large gains in the Advanced category far outweigh this. It is important to note that in 2011, more students scored Advanced than in the previous three years in all three exams. Furthermore, more students scored in the Proficient categories than in the previous three years in the mathematics and the science and technology/engineering exams. Therefore, as long as the difficulty and the rigor of the test remained constant from year to year, there was a remarkable improvement in test scores, which suggest an increased mastery of the standards outlined in the Curriculum Frameworks.

**Item by Item MCAS Results**

In order to recognize patterns in the way students are answering questions, and in what areas they still need improvement on, a table of item by item MCAS results would be extremely useful. Fortunately, the Massachusetts Department of Education provides such a table on its website. This section will only emphasize the MCAS 2011 grade 10 mathematics exam, because that is the subject that is of most interest. For item by item results for other tests, please go to:


and select the desired test on the left pane. You will even be able to click on any problem to see exactly what the question was. Keep in mind that Doherty only serves grades 9-12, so test scores for the school are only available at the high school level.

There are several abbreviations in the table below, so here is a list explaining what they all represent:

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC</td>
<td>This is a multiple choice question. There are four choices and only one is the correct answer.</td>
</tr>
<tr>
<td>SA</td>
<td>This is a short answer question. There are one or more solutions and students must write down a correct answer.</td>
</tr>
<tr>
<td>OR</td>
<td>This is an open response question. Students are required to find a solution and explain their reasoning. These usually have multiple parts.</td>
</tr>
<tr>
<td>GE</td>
<td>This is a geometry question.</td>
</tr>
<tr>
<td>ME</td>
<td>This is a measurement question.</td>
</tr>
<tr>
<td>SP</td>
<td>This is a question involving data analysis, statistics, or probability.</td>
</tr>
<tr>
<td>PR</td>
<td>This is a question involving patterns, relations, and algebra.</td>
</tr>
<tr>
<td>NS</td>
<td>This is a number sense and operations question.</td>
</tr>
</tbody>
</table>

One problem that the table reveals is the number of students leaving problems blank. There is no reason why anyone should leave a multiple choice question blank because no points are taken off for incorrect answers. However, 1% of people left seven multiple choice questions blank. Teachers have to make it clear that students should *always* answer multiple choice questions on the MCAS! Even if a student has no idea what the answer is, there is a 25% chance that the student will get the question right anyway. Students are also receiving no credit for
short answers and open responses, but that can happen for two reasons. Either they are leaving them blank, or they got the wrong answer. If students are leaving them blank, then they should not be.

Notice that for every single question in the mathematics MCAS exam, Doherty outperforms Worcester. This is a really impressive statistic that can be improved if people did not leave questions blank. However, as it was discussed earlier, Doherty does not perform as well as the state. The category that the students preformed the best in was GE, and the category that the students did the worst in was PR. The worst multiple choice question was #30, and the worst open response question was #20, both shown below:

![Image of triangle JKL](http://www.doe.mass.edu/mcas/search/question.aspx?mcasyear=2011&district=348&school=512&grade=10&subjectcode=MTH&questionnumber=30)

Triangle JKL is shown on the coordinate plane below.

1. On your grid, draw triangle J’K’L’, the image of triangle JKL after it has been reflected over the y-axis. Be sure to label the vertices.

Triangle J’K’L’ is rotated 90° clockwise about the origin.

2. On your grid, draw triangle J”K”L”, the image of J’K’L’ after it has been rotated 90° clockwise about the origin. Be sure to label the vertices.

3. Suppose the vertices of J”K”L” are reflected over the y-axis and then reflected over the x-axis. Do the vertices of the resulting triangle have the same coordinates as the vertices of triangle JKL? Show or explain how you got your answer.

---

Which of the following absolute-value inequalities best represents the graph shown on the number line below?

- A. $|x - 2| \leq 3$
- B. $|x - 2| \geq 3$
- C. $|x - 3| \leq 2$
- D. $|x - 3| \geq 2$

Therefore, students could really brush up their skills on inequalities and reflections!

---

Table 19: 2011 Item by Item Results for Grade 10 Mathematics

<table>
<thead>
<tr>
<th>ITEM</th>
<th>TYPE</th>
<th>REPORTING CATEGORY</th>
<th>STANDARD SCORE</th>
<th>DOHERTY MEMORIAL HIGH</th>
<th>WORCESTER</th>
<th>STATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>MC</td>
<td>GE</td>
<td>10.47</td>
<td>83%</td>
<td>75%</td>
<td>80%</td>
</tr>
<tr>
<td>02</td>
<td>MC</td>
<td>ME</td>
<td>10.11</td>
<td>60%</td>
<td>53%</td>
<td>62%</td>
</tr>
<tr>
<td>03</td>
<td>MC</td>
<td>SP</td>
<td>10.22</td>
<td>65%</td>
<td>60%</td>
<td>72%</td>
</tr>
<tr>
<td>04</td>
<td>MC</td>
<td>SP</td>
<td>10.01</td>
<td>77%</td>
<td>73%</td>
<td>76%</td>
</tr>
<tr>
<td>05</td>
<td>MC</td>
<td>PR</td>
<td>10.12</td>
<td>51%</td>
<td>47%</td>
<td>53%</td>
</tr>
<tr>
<td>06</td>
<td>MC</td>
<td>NS</td>
<td>10.13</td>
<td>58%</td>
<td>56%</td>
<td>57%</td>
</tr>
<tr>
<td>07</td>
<td>MC</td>
<td>PR</td>
<td>10.14</td>
<td>60%</td>
<td>51%</td>
<td>62%</td>
</tr>
<tr>
<td>08</td>
<td>MC</td>
<td>SP</td>
<td>10.15</td>
<td>78%</td>
<td>81%</td>
<td></td>
</tr>
<tr>
<td>09</td>
<td>MC</td>
<td>NS</td>
<td>10.16</td>
<td>55%</td>
<td>44%</td>
<td>52%</td>
</tr>
<tr>
<td>10</td>
<td>MC</td>
<td>PR</td>
<td>10.17</td>
<td>51%</td>
<td>40%</td>
<td>50%</td>
</tr>
<tr>
<td>11</td>
<td>MC</td>
<td>NS</td>
<td>10.18</td>
<td>58%</td>
<td>48%</td>
<td>60%</td>
</tr>
<tr>
<td>12</td>
<td>MC</td>
<td>NS</td>
<td>10.19</td>
<td>77%</td>
<td>89%</td>
<td>80%</td>
</tr>
<tr>
<td>13</td>
<td>MC</td>
<td>SP</td>
<td>10.20</td>
<td>84%</td>
<td>78%</td>
<td>85%</td>
</tr>
<tr>
<td>14</td>
<td>MC</td>
<td>NS</td>
<td>10.21</td>
<td>56%</td>
<td>49%</td>
<td>67%</td>
</tr>
<tr>
<td>15</td>
<td>SA</td>
<td>NS</td>
<td>10.22</td>
<td>59%</td>
<td>49%</td>
<td>57%</td>
</tr>
<tr>
<td>16</td>
<td>SA</td>
<td>ME</td>
<td>10.23</td>
<td>65%</td>
<td>58%</td>
<td>68%</td>
</tr>
<tr>
<td>17</td>
<td>OR</td>
<td>NS</td>
<td>10.24</td>
<td>2.20</td>
<td>1.76</td>
<td>2.25</td>
</tr>
<tr>
<td>18</td>
<td>SA</td>
<td>PR</td>
<td>10.25</td>
<td>84%</td>
<td>78%</td>
<td>86%</td>
</tr>
<tr>
<td>19</td>
<td>SA</td>
<td>NS</td>
<td>10.26</td>
<td>51%</td>
<td>44%</td>
<td>48%</td>
</tr>
<tr>
<td>20</td>
<td>OR</td>
<td>GE</td>
<td>10.27</td>
<td>1.45</td>
<td>1.33</td>
<td>1.70</td>
</tr>
<tr>
<td>21</td>
<td>OR</td>
<td>PR</td>
<td>10.28</td>
<td>2.16</td>
<td>1.77</td>
<td>2.25</td>
</tr>
<tr>
<td>22</td>
<td>MC</td>
<td>GE</td>
<td>10.29</td>
<td>84%</td>
<td>76%</td>
<td>86%</td>
</tr>
<tr>
<td>23</td>
<td>MC</td>
<td>SP</td>
<td>10.30</td>
<td>73%</td>
<td>68%</td>
<td>66%</td>
</tr>
<tr>
<td>24</td>
<td>MC</td>
<td>PR</td>
<td>10.31</td>
<td>48%</td>
<td>38%</td>
<td>51%</td>
</tr>
<tr>
<td>25</td>
<td>MC</td>
<td>PR</td>
<td>10.32</td>
<td>53%</td>
<td>43%</td>
<td>55%</td>
</tr>
<tr>
<td>26</td>
<td>MC</td>
<td>ME</td>
<td>10.33</td>
<td>50%</td>
<td>53%</td>
<td>60%</td>
</tr>
<tr>
<td>27</td>
<td>MC</td>
<td>SP</td>
<td>10.34</td>
<td>50%</td>
<td>53%</td>
<td>62%</td>
</tr>
<tr>
<td>28</td>
<td>MC</td>
<td>GE</td>
<td>10.35</td>
<td>45%</td>
<td>40%</td>
<td>49%</td>
</tr>
<tr>
<td>29</td>
<td>MC</td>
<td>ME</td>
<td>10.36</td>
<td>66%</td>
<td>60%</td>
<td>66%</td>
</tr>
<tr>
<td>30</td>
<td>MC</td>
<td>PR</td>
<td>10.37</td>
<td>43%</td>
<td>37%</td>
<td>40%</td>
</tr>
<tr>
<td>31</td>
<td>OR</td>
<td>PR</td>
<td>10.38</td>
<td>2.80</td>
<td>2.46</td>
<td>2.90</td>
</tr>
<tr>
<td>32</td>
<td>MC</td>
<td>NS</td>
<td>10.39</td>
<td>89%</td>
<td>84%</td>
<td>90%</td>
</tr>
<tr>
<td>33</td>
<td>MC</td>
<td>GE</td>
<td>10.40</td>
<td>87%</td>
<td>76%</td>
<td>89%</td>
</tr>
<tr>
<td>34</td>
<td>MC</td>
<td>SP</td>
<td>10.41</td>
<td>78%</td>
<td>75%</td>
<td>88%</td>
</tr>
<tr>
<td>35</td>
<td>MC</td>
<td>PR</td>
<td>10.42</td>
<td>45%</td>
<td>41%</td>
<td>50%</td>
</tr>
<tr>
<td>36</td>
<td>MC</td>
<td>ME</td>
<td>10.43</td>
<td>82%</td>
<td>78%</td>
<td>89%</td>
</tr>
<tr>
<td>37</td>
<td>MC</td>
<td>PR</td>
<td>10.44</td>
<td>49%</td>
<td>44%</td>
<td>56%</td>
</tr>
<tr>
<td>38</td>
<td>MC</td>
<td>SP</td>
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</tr>
</tbody>
</table>

Grade 10 Mathematics MCAS Results of all Worcester Public High Schools

The following is a comparison chart of the grade 10 math exam that lists the SGR and the percentage of students that scored proficient or above. As already stated, Doherty preformed better than the district.

Table 20: Grade 10 Math MCAS Comparison Chart

<table>
<thead>
<tr>
<th>School</th>
<th>Median SGP</th>
<th>% Proficient or Higher</th>
<th>Included in SGP</th>
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<tbody>
<tr>
<td>Belmont Street Community</td>
<td>51.0</td>
<td>29</td>
<td>146</td>
</tr>
<tr>
<td>Burncoat Middle School</td>
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<td>61</td>
<td>487</td>
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<tr>
<td>Burncoat Senior High</td>
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<td>63</td>
<td>152</td>
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<td>30.0</td>
<td>32</td>
<td>79</td>
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<tr>
<td>Canterbury</td>
<td>39.0</td>
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<td>31</td>
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<td>Chandler Elem Community</td>
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<td>25</td>
<td>114</td>
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<tr>
<td>Chandler Magnet</td>
<td>60.0</td>
<td>22</td>
<td>151</td>
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<tr>
<td>City View</td>
<td>41.5</td>
<td>25</td>
<td>208</td>
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<tr>
<td>Claremont Academy</td>
<td>53.5</td>
<td>41</td>
<td>172</td>
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<tr>
<td>Clark St Community</td>
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<td>127</td>
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<td>Columbus Park</td>
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<td>60</td>
<td>146</td>
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<tr>
<td>Doherty Memorial High</td>
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<td>76</td>
<td>238</td>
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<tr>
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<td>139</td>
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<tr>
<td>Flagg Street</td>
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<td>Francis J McGrath Elem</td>
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<td>76</td>
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<tr>
<td>Gates Lane</td>
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<td>46</td>
<td>194</td>
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<td>Goddard Sch/Science Tech</td>
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<td>162</td>
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<td>Quinsigamond</td>
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<td>Tatnuck</td>
<td>50.0</td>
<td>62</td>
<td>157</td>
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<td>Thomdyke Road</td>
<td>50.5</td>
<td>50</td>
<td>112</td>
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<td>49.5</td>
<td>23</td>
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<td>75</td>
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<td>West Tatnuck</td>
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<td>91</td>
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<tr>
<td>Woodland Academy</td>
<td>35.0</td>
<td>27</td>
<td>167</td>
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<tr>
<td>Worcester Arts Magnet Sch</td>
<td>55.5</td>
<td>76</td>
<td>124</td>
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<tr>
<td>Worcester East Middle</td>
<td>61.0</td>
<td>57</td>
<td>474</td>
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<tr>
<td>Worcester Technical High</td>
<td>54.0</td>
<td>77</td>
<td>323</td>
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</table>

Grade 10 Mathematics MCAS Results by Subgroup

The table below summarizes student performance on the MCAS grade 10 mathematics test for several subgroups for Doherty High School. The “AYP Part” column is the percentage of students in each subgroup that participated in the MCAS test. Students in the “High Needs” subgroup are low income students, students with disabilities, ELL students, and former ELL students. Note that scores for subgroups with 10 or fewer students are not included, and SGP scores are not included for subgroups with 20 or fewer students.

Table 21: Grade 10 Mathematics MCAS Results by Subgroup

<table>
<thead>
<tr>
<th>Student Group</th>
<th>School</th>
<th>District</th>
<th>State</th>
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<tbody>
<tr>
<td></td>
<td>Std. Inc.</td>
<td>AYP Part</td>
<td>% of Each Perf. Level</td>
</tr>
<tr>
<td>AYP Subgroups</td>
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</tr>
<tr>
<td>Stud. w/Disabs &amp; Former ELL</td>
<td>120</td>
<td>94</td>
<td>90</td>
</tr>
<tr>
<td>ELL &amp; Former ELL</td>
<td>118</td>
<td>95</td>
<td>90</td>
</tr>
<tr>
<td>Low-Income</td>
<td>118</td>
<td>95</td>
<td>90</td>
</tr>
<tr>
<td>African American/Black</td>
<td>118</td>
<td>95</td>
<td>90</td>
</tr>
<tr>
<td>Asian</td>
<td>118</td>
<td>95</td>
<td>90</td>
</tr>
<tr>
<td>Hispanic/Latino</td>
<td>118</td>
<td>95</td>
<td>90</td>
</tr>
<tr>
<td>Native American</td>
<td>118</td>
<td>95</td>
<td>90</td>
</tr>
<tr>
<td>White</td>
<td>118</td>
<td>95</td>
<td>90</td>
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<tr>
<td>Other Subgroups</td>
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<td></td>
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<tr>
<td>High Needs</td>
<td>118</td>
<td>95</td>
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<tr>
<td>Male</td>
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<td>Non-Title I</td>
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</tr>
<tr>
<td>Non-Low Income</td>
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<td>95</td>
<td>90</td>
</tr>
<tr>
<td>ELL</td>
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</tr>
<tr>
<td>Former ELL</td>
<td>118</td>
<td>95</td>
<td>90</td>
</tr>
<tr>
<td>1st YR ELL</td>
<td>118</td>
<td>95</td>
<td>90</td>
</tr>
<tr>
<td>Native Hawaiian/ Pacific Islander</td>
<td>118</td>
<td>95</td>
<td>90</td>
</tr>
<tr>
<td>Multi-race Non-Hispanic/Latino</td>
<td>118</td>
<td>95</td>
<td>90</td>
</tr>
</tbody>
</table>

A very popular stereotype is “Asians are good at math.” Although Asians do outperform all other subgroups on the state and district levels, Whites actually outperform Asians at Doherty. This is an example where stereotypes may be confirmed in some instances (which is why some people believe in them), but contradicted in other cases. It is also interesting that females significantly outperform males at Doherty, but on the state and district level the two subgroups perform more or less alike. A natural question is why this is the case. One possibility is that males are absent more frequently and get suspended more often than females, and therefore miss more class instruction. Another possibility is the fact that there are more female teachers than male teachers, and the female students can associate and learn better from teachers of their own gender.

It is a very desirable idea to have a school that gives an equal education for all students. However, from the table above, it is clear that some subgroups are not as proficient in mathematics as other subgroups. Compare the “High Needs” subgroup with the “Non Low-Income” subgroup, for example. If schools did give everyone an equal education, then MCAS results between one subgroup from another should be more or less equivalent across the board, but that is simply not the case. A major question then, is whether or not schools can even do such a thing. If some students are not proficient in English, could they really receive an equal education to other students if all class work is done in English? While I believe that is possible to narrow the gap, equalizing mathematical proficiency among all subgroups seems like an unrealistic dream, as desirable it may be. Students and teachers lacking a solid medium of communication is an enormous obstacle that can only be solved if the students and teachers are able to understand each other.

**MCAS Participation**

Table 21 shows the percentage of students that participated in the MCAS for each subgroup. Since it is required to pass the grade 10 MCAS test in order to graduate, it is critical that as many students take it as possible. It should not be very surprising that Doherty students with disabilities had the worst MCAS participation, because they may have unexpected medical dilemmas arriving to school. Males, low-income students, and White students have the second-worse participation rate at Doherty at 96%. Only African American and Asian Doherty students had 100% participation. Of all the students in the school, only one student had a medical reason to miss the exam. All the other students were absent for some other reason, or no reason at all, and did not make up the exam that year. Needless to say, this is an alarming statistic. To make matters worse, the MCAS exam takes place for two or three days, so a student has to be absent on all exam days to have not taken even it. There is no excuse for being absent for a graduation requirement, unless some substantial catastrophe occurred. It is nearly impossible for 3% of people to have had a catastrophe at the same time. The 2010 participation rate was much better at 99%, but for one reason or another, the percentage of people not participating in the exam tripled. While it is good that Doherty has higher than average MCAS results than the city, the participation rate is deficient.

**Advanced Placement Results**

MCAS results are not the only useful tool to evaluate student competence. During my time at Doherty, I taught 11th and 12th grade honors students that have already passed the MCAS, so what is perhaps just as important are the test that they will take in the future. Many of the pre-calculus students that I taught will move on to Advanced Placement (AP) calculus and will take the AP calculus AB exam. The AP calculus AB exam evaluates whether or not students have mastered the first semester (or for WPI, first term) of college calculus. If I was teaching pre-calculus for an extended period of time, I can see if the changes that I made teaching had any effect on the students’ future success on the AP exam. Of course, the calculus teacher may have also changed his or her teaching methods, and so AP results are the result of several teachers’ efforts. This is why it is important for teachers to communicate their plans to each other, especially if they are in the same department. Since I know that several of the pre-calculus students will take the AP calculus exam, I need to make sure that they are developing the skills that they need to succeed in calculus. AP results are an indication of whether that was accomplished (if I had continued teaching).

---

Table 22: AP Performance

<table>
<thead>
<tr>
<th>Subject</th>
<th>Tests Taken</th>
<th>% Score 1-2</th>
<th>% Score 3-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Subjects</td>
<td>265</td>
<td>32.8</td>
<td>67.2</td>
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<tr>
<td>English Language Arts</td>
<td>89</td>
<td>24.7</td>
<td>75.3</td>
</tr>
<tr>
<td>English Lang/Comp</td>
<td>55</td>
<td>29.1</td>
<td>70.9</td>
</tr>
<tr>
<td>English Lit/Comp</td>
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<td>17.6</td>
<td>82.4</td>
</tr>
<tr>
<td>Foreign Languages</td>
<td>14</td>
<td>57.1</td>
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<tr>
<td>Spanish Lang</td>
<td>14</td>
<td>57.1</td>
<td>42.9</td>
</tr>
<tr>
<td>History and Social Science</td>
<td>64</td>
<td>29.7</td>
<td>70.3</td>
</tr>
<tr>
<td>Art History</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Govt &amp; Pol: U.S.</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>History: U.S.</td>
<td>48</td>
<td>29.2</td>
<td>70.8</td>
</tr>
<tr>
<td>History: World</td>
<td>13</td>
<td>23.1</td>
<td>76.9</td>
</tr>
<tr>
<td>Psychology</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Math and Computer Science</td>
<td>38</td>
<td>36.8</td>
<td>63.2</td>
</tr>
<tr>
<td>Calculus AB</td>
<td>19</td>
<td>15.8</td>
<td>84.2</td>
</tr>
<tr>
<td>Computer Sci A</td>
<td>10</td>
<td>50.0</td>
<td>50.0</td>
</tr>
<tr>
<td>Statistics</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Science and Technology</td>
<td>60</td>
<td>40.0</td>
<td>60.0</td>
</tr>
<tr>
<td>Biology</td>
<td>24</td>
<td>29.2</td>
<td>70.8</td>
</tr>
<tr>
<td>Chemistry</td>
<td>13</td>
<td>84.0</td>
<td>15.4</td>
</tr>
<tr>
<td>Physics B</td>
<td>23</td>
<td>26.1</td>
<td>73.9</td>
</tr>
</tbody>
</table>

In the table above, performance is not displayed for subjects with ten or fewer participants. Generally, colleges accept scores of 3 and higher, but more prestigious institutions may only accept higher scores for college credit. Note that out of all the AP exams, students perform the best on the calculus exam, in which 84.2% received a score of 3 or higher. This is a very good indication that the mathematics department at Doherty prepares students for college level courses.

SAT Results

The Standardized Aptitude Test (SAT) is a college entrance exam required by many institutions of higher learning, particularly four year institutions. Subjects tested are writing, reading comprehension, and mathematics. Possible scores range from 200-800 in each section, while an average score is approximately 500. The score distribution roughly follows a normal distribution, so that the minimum and maximum scores are quite rare, while scores around 500 are quite common. Colleges are very demanding about the SAT scores that they desire; almost all colleges want above average scores. Students may take the test several times, and colleges will only consider the highest score achieved in each section. There are a few colleges that do not require SAT scores (WPI is among them), but even in those cases an alternative admission requirement is required. Therefore, for the vast majority of college bound students, taking the SAT (and doing well on it) is nothing less but mandatory. From the table below, it is apparent that SAT scores are below average. However, since students have numerous opportunities to retake the test, many students may have achieved higher scores than these in the long term. These low SAT scores may partially explain why some students planned to go to college, but did not attend (i.e. they were rejected because of their SAT scores).

Table 23: SAT Performance

<table>
<thead>
<tr>
<th>Student Group</th>
<th>Test Takers</th>
<th>Reading</th>
<th>Writing</th>
<th>Math</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Students</td>
<td>243</td>
<td>448</td>
<td>444</td>
<td>460</td>
</tr>
<tr>
<td>Lim. English Prof.</td>
<td>16</td>
<td>343</td>
<td>351</td>
<td>407</td>
</tr>
<tr>
<td>Low Income</td>
<td>107</td>
<td>393</td>
<td>396</td>
<td>420</td>
</tr>
<tr>
<td>Special Education</td>
<td>23</td>
<td>358</td>
<td>347</td>
<td>350</td>
</tr>
<tr>
<td>Female</td>
<td>147</td>
<td>448</td>
<td>450</td>
<td>443</td>
</tr>
<tr>
<td>Male</td>
<td>96</td>
<td>447</td>
<td>433</td>
<td>487</td>
</tr>
<tr>
<td>Asian</td>
<td>15</td>
<td>424</td>
<td>411</td>
<td>487</td>
</tr>
<tr>
<td>Black or Afr. Amer.</td>
<td>35</td>
<td>380</td>
<td>387</td>
<td>380</td>
</tr>
<tr>
<td>Hispanic</td>
<td>52</td>
<td>399</td>
<td>405</td>
<td>423</td>
</tr>
<tr>
<td>Multi-race, Non-Hisp.</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>White</td>
<td>139</td>
<td>485</td>
<td>476</td>
<td>492</td>
</tr>
</tbody>
</table>

Teacher Data

The following few sections will deal with information regarding teachers. Table 24 shows how many teachers are hired in the school, district, and state, as well as the student to teacher ratio.

Table 24: Teacher Data (2010-2011)

<table>
<thead>
<tr>
<th></th>
<th>School</th>
<th>District</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total # of Teachers</td>
<td>90.3</td>
<td>1605.7</td>
<td>68,754.20</td>
</tr>
<tr>
<td>% of Teachers Licensed in Teaching Assignment</td>
<td>98</td>
<td>97.2</td>
<td>97.5</td>
</tr>
<tr>
<td>Total # of Classes in Core Academic Areas</td>
<td>553</td>
<td>8629</td>
<td>335925</td>
</tr>
<tr>
<td>% of Core Academic Classes Taught by Teachers who are Highly Qualified</td>
<td>98.4</td>
<td>96.5</td>
<td>97.7</td>
</tr>
<tr>
<td>Student/Teacher Ratio</td>
<td>14.7 to 1</td>
<td>15.1 to 1</td>
<td>13.9 to 1</td>
</tr>
</tbody>
</table>

It may be strange to see a fractional teachers being reported above. A fraction simply means that a long-term substitute or a part-time teacher replaced or substituted for another teacher. So, if a teacher got pregnant and took three months off, that teacher is still considered a “teacher”, as well as her long-term substitute. Any full-time teacher that got fired or that quit mid-year will also show up as a fraction.

Licensed teachers are instructors that have met or exceeded the minimum state requirements to be a long term teacher within the state. At a minimum, teachers must have a Bachelor’s Degree, a passing MTEL score(s). Additional coursework may be required, or completion of the Educator Preparation Program. Highly qualified teachers are teachers that have a teaching license and that proved competence in the subject matter(s) that they teach. One way to do this is to pass the corresponding MTEL exams, which explains why the percentage of teachers that are licensed and the percentage of teachers that are highly qualified are very close. All teachers hired from 2002 onward that teach a core academic area are required to be highly qualified. Core academic subjects are English,

37 http://profiles.doe.mass.edu/sat/sat_perf_dist.aspx?orgcode=03480512&orgtypecode=6&
39 http://www.doe.mass.edu/Educators/e_license.html?section=k12
40 http://www.doe.mass.edu/nclb/hq/hq_memo.html
reading, or language arts, math, science, foreign languages, civics and government, economics, arts, history, and geography.\textsuperscript{41} Classes like physical education and sewing are not core academic subjects.

A greater percentage of teachers at Doherty are highly qualified than in the district and the state, which is very good. However, the student to teacher ratio is slightly higher than the state average. Student teacher ratios are very important because the lower they are, the smaller classes are as well. Smaller classes mean that teachers give each individual student more attention and guidance, and students and teachers can know each other better. Although it is possible to effectively teach with large student to teacher ratio, teachers are less likely to know a student’s individual needs, learning style, and level of competence. It is true that Doherty has a better student to teacher ratio than the district, but Doherty, as well as other schools in the district, may very well benefit from lower student teacher ratios, especially considering that the district has a much larger percentage of ELL students than the state (see table 7).

**Staffing by Ethnicity and Gender**

Worcester is an ethnically diverse city, and so one would expect the teachers in the city to be ethnically diverse as well. Table 25 reveals that this is not the case. The vast majority of teachers are white, in both the school and the district, while other ethnicities are disproportionately lacking. Furthermore, the gender ratio in Worcester is almost 50:50, yet there are twice as many female teachers than male teachers. Therefore, the teacher population in Worcester and at Doherty is not representative of the diversity found among the student population and in the city itself. The ramifications of this is that minorities and male students may have a harder time associating with their instructors and therefore way struggle coping in class and following the teachers instructions. In addition, ELL students cannot communicate to teachers in their native language because most of the teachers only speak English. Keep in mind that table 25 displays the ethnicity and gender of all staff members, not just teachers, so the number of teachers falling under each subgroup can only be approximated.

**Table 25: Staffing Data by Ethnicity and Gender (2009-2010)\textsuperscript{42}**

<table>
<thead>
<tr>
<th></th>
<th>School</th>
<th>District</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>African American</td>
<td>8.5</td>
<td>110.0</td>
<td>4,094.7</td>
</tr>
<tr>
<td>Asian</td>
<td>1.0</td>
<td>35.0</td>
<td>1,327.5</td>
</tr>
<tr>
<td>Hispanic</td>
<td>18.0</td>
<td>244.0</td>
<td>4,095.3</td>
</tr>
<tr>
<td>White</td>
<td>138.8</td>
<td>2,730.1</td>
<td>112,864.6</td>
</tr>
<tr>
<td>Native American</td>
<td>1.0</td>
<td>4.0</td>
<td>118.7</td>
</tr>
<tr>
<td>Native Hawaiian, Pacific Islander</td>
<td>0.0</td>
<td>1.0</td>
<td>49.9</td>
</tr>
<tr>
<td>Multi-Race, Non-Hispanic</td>
<td>3.7</td>
<td>42.0</td>
<td>611.8</td>
</tr>
<tr>
<td>Males</td>
<td>58.2</td>
<td>633.8</td>
<td>24,313.4</td>
</tr>
<tr>
<td>Females</td>
<td>112.9</td>
<td>2,532.3</td>
<td>98,849.2</td>
</tr>
<tr>
<td>FTE Count</td>
<td>171.1</td>
<td>3,166.1</td>
<td>123,162.6</td>
</tr>
</tbody>
</table>

**Staffing Age**

Staffing age correlates with teaching experience. For example, a teacher that is 22 years old could not have very many years of experience because he or she would have just graduated college. Similarly, a 70 year old teacher is very likely to be experienced, because senior citizens do not typically begin new careers. Therefore, a school with a high percentage of young teachers is likely to have a high percentage of inexperienced ones as well, and vice-versa. From the table below, it is clear that the staffing age at Doherty roughly follows a bell curve, with the most number

\textsuperscript{41} http://www.doe.mass.edu/nclb/hq/hq_mem.html
\textsuperscript{42} http://profiles.doe.mass.edu/profiles/teacher.aspx?orgcode=03480512&orgtypecode=6&leftNavld=817&
of teachers between the ages of 49-56. This suggests that there is a substantial number of teachers with good experience, as well as a significant number of teachers with little experience.

Table 26: Staffing Age Report (2009-2010)\(^{43}\)

<table>
<thead>
<tr>
<th>Age Range</th>
<th>School</th>
<th>District</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 26</td>
<td>8.0</td>
<td>95.0</td>
<td>6,211.9</td>
</tr>
<tr>
<td>26-32</td>
<td>14.2</td>
<td>316.0</td>
<td>18,881.3</td>
</tr>
<tr>
<td>33-40</td>
<td>29.0</td>
<td>547.8</td>
<td>21,291.7</td>
</tr>
<tr>
<td>41-48</td>
<td>45.9</td>
<td>732.1</td>
<td>24,295.6</td>
</tr>
<tr>
<td>49-56</td>
<td>39.3</td>
<td>825.0</td>
<td>30,333.7</td>
</tr>
<tr>
<td>57-64</td>
<td>29.7</td>
<td>548.2</td>
<td>19,700.5</td>
</tr>
<tr>
<td>Over 64</td>
<td>5.0</td>
<td>102.0</td>
<td>2,447.9</td>
</tr>
<tr>
<td>Total</td>
<td>171.1</td>
<td>3,166.1</td>
<td>123,162.6</td>
</tr>
</tbody>
</table>

**Teacher Specialties**

The table below displays the different areas teachers specialize in the school, district, and the state.

Teacher Specialties (2010-2011)\(^{44}\)

<table>
<thead>
<tr>
<th>Specialty</th>
<th>School</th>
<th>District</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Education</td>
<td>80.4</td>
<td>1,346.8</td>
<td>55,732.4</td>
</tr>
<tr>
<td>Special Education</td>
<td>6.9</td>
<td>113.5</td>
<td>9,342.6</td>
</tr>
<tr>
<td>Career Vocational Technical</td>
<td>3.0</td>
<td>84.1</td>
<td>2,107.5</td>
</tr>
<tr>
<td>English Language Learner</td>
<td>0.0</td>
<td>61.3</td>
<td>1,571.7</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td>90.3</td>
<td>1,605.7</td>
<td>68,754.2</td>
</tr>
</tbody>
</table>

The large majority of teachers at Doherty teach general education, while a much smaller portion teaches special and technical education. However, it is interesting to note that there are no ELL teachers, even though 17.6% of the students are not English proficient (see table 7). This is because at Doherty, there are ELL classes in which “regular” general education teachers instruct. These teachers are trained to teach ELL students, but they primarily teach non-ELL classes.

**Conclusion**

Doherty Memorial High school is an urban public high school located in Worcester, MA that serves students from grades 9 thru 12. Like most urban high schools, Doherty has an ethnically diverse student body with a wide range of qualities. Doherty should be noted for its high graduation rate, its low dropout rate, and high MCAS scores compared to other Worcester schools. Doherty, however, is not without its challenges. The school has high truancy and suspension rates, and many students at the school that planned to go to college did not. In order to be an effective teacher, one must embrace and celebrate the accomplishments of the school and its students, while being aware of its weaknesses and attempt to solve them. This challenge never ends, but it is a rewarding one.

\(^{43}\) http://profiles.doe.mass.edu/profiles/teacher.aspx?orgcode=03480512&orgtypecode=6&leftNavId=828&

\(^{44}\) http://profiles.doe.mass.edu/profiles/teacher.aspx?orgcode=03480512&orgtypecode=6&leftNavId=830&
Chapter 2: The Curriculum

Introduction

A curriculum is a sequence of courses that students take in order to gain competency in a particular field, such as mathematics. The contents of a curriculum can be influenced by several factors. The most important factor is the Massachusetts Curriculum Frameworks, which outline the standards that students must master. The MCAS exams are entirely based on the Frameworks and, as discussed in Chapter 1, passing the grade 10 mathematics and language arts exams is a graduation requirement for students. Therefore, teachers that educate grades 10 or lower are under the extra pressure to prepare students for those important exams, which is why the curriculum is heavily modeled after the Frameworks. The curriculum is influenced by many other factors as well, such as the NCTM Curriculum, and the Reauthorization of the Elementary and Secondary Initiatives.

During my experience at Doherty Memorial High school, I had the wonderful opportunity to teach two subjects, pre-calculus and algebra, both on the honors level. In order to teach those subjects, however, it was very important to fully understand the curriculum first. I needed to know what skills the students were expected to have before entering the courses, so that I can extend on topics that they are familiar with. I also need to know what classes they might take in the future, so that I can prepare students for them. Finally, I needed to understand the goals and standards that I had to satisfy for the courses that I taught. Only then could I realize how the smaller piece fits the whole picture.

Course Levels

At Doherty, there are three main levels of rigor a core academic class can have. A level 1 class is the most fundamental level, which provides students with the skills needed to master the standards outlined in the Frameworks, but that covers topics in a simple and less accelerated manner. Although this level lacks the challenge presented in the higher levels, students that take level 1 course are still given a quality education in order to succeed, whatever they may choose to do in the future. Colleges accept level 1 classes at a minimum, but colleges are particularly interested in how many accelerated classes students have taken, and how well they did in them. The honors level is considerably more rigorous and challenging than level 1 classes, and it is these classes the colleges prefer students to take. This is one of the reasons why students are encouraged to take honors level classes as opposed to level 1 classes, as long as they have the ability to keep up with their accelerated pace. Finally, AP level classes are the most accelerated and demanding courses available at the high school. AP level classes are usually year long high school courses that contain the rigor and the content in a typical one semester class in college. Unfortunately, the number of AP classes is limited, partially because the number of subjects in which AP exams are offered are limited. However, if a student receives a high grade on an AP exam, they may be able to gain college credit for the class that he or she has completed in high school. This is a substantial advantage, and so students are especially encouraged to take AP courses if they are capable of the increased demand.

Course levels are important for teachers because they are an indication of the pace they will be teaching and the expectations that they can enforce. As a side note, when I was a student at Lowell High School, I had known teachers that taught “level 1” (actually called “college level” at my high school) and honors level classes in exactly the same way, except that they gave honors students more homework. These teachers are not treating the distinction between level 1 and honors level properly. The two main differences between the levels should be pace and content. The honors level classes should be accelerated and cover more advanced material that goes beyond the minimum requirements of the Frameworks. How these teachers copied and pasted their lessons was a major headache for college level students because they did not fully grasp the topics, but a “blessing” (not really though) for the honors level students because they gain an extra boost to their GPA. What amused me most about these teachers, however, is when they seem bewildered when one class consistently does better than the other!
Course Sequence

At Doherty, I have taught honors pre-calculus and honors advanced algebra. The following diagram shows the mathematics courses a pre-calculus student may take. Most pre-calculus students do not go on to AP calculus because the majority of pre-calculus students are seniors. Currently, there are 7 pre-calculus and 1 AP calculus classes available at Doherty. The chart also shows the typical course flow of an advanced algebra student, except he or she is less likely to take pre-calculus and AP calculus. Although it is recommended that students take honors level courses prior to taking an honors level course, they may take honors level course if they did especially well in a level 1 course. Algebra, Geometry, and advanced algebra are offered at level 1 and at the honors level, pre-calculus is only offered at the honors level, and calculus is only offered at the AP level.

In all Worcester Public Schools, students must take at least three years of mathematics courses in order to graduate. Additionally, most colleges and universities require at least three years of mathematics course. Therefore, most students take at a minimum of algebra, geometry, and advanced algebra.

Typical Course Sequence

Algebra (H)
Grade: 9
Prerequisite: None

Geometry (H)
Grade: 9 or 10
Prerequisite: Algebra (H)

Advanced Algebra (H)
Grade: 10 or 11
Prerequisite: Geometry (H)

Pre-Calculus (H)
Grade: 11 or 12
Prerequisite: Adv. Alg (H)

AP Statistics
And/or
Grade: 12
Prerequisite: Adv. Alg (H)

AP Calculus AB
Grade: 12
Prerequisite: Pre-Calculus (H) or department approval

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45 http://doherty.worcesterschools.org/overview/classroom.phtml?sessionid=361195ddee33375fe65a8202c7a3aad
47 Worcester Public Schools Mathematics Curriculum
Note that if a pre-calculus student wants to take AP calculus, he or she must take four math classes in the first three years of high school. In order to do that, a student must either take geometry and advanced algebra in his or her sophomore year. (It is not recommended for students to take advanced algebra and pre-calculus concurrently, since skills from advanced algebra are required in pre-calculus.) Alternatively, students can test out of algebra 1 and go straight into geometry in their freshman year.

Finally, observe that there is no trigonometric class in the above chart. Unfortunately, Doherty offers only two classes that contain an overview of trigonometry, which are pre-calculus and advanced algebra. In particular, advanced algebra only offers a rather crude introduction of trigonometry. Therefore, many students entering pre-calculus have a minimal understanding of trigonometry, but they do learn about trigonometric functions and their identities in the second semester of pre-calculus. Students taking AP calculus must know trigonometry, so it is imperative that pre-calculus students taking the calculus course understand the trigonometry material well.

It should also be noted that students have the opportunity to take either honors level or AP statistics in the 12th grade. Honors advanced algebra students may choose to go this route, so it is important that students in that class understand how to manipulate equations, interpret data and graphs, and how to convert word problems into algebraic notation, because these skills are essential for statistics. On the other hand, very few pre-calculus students will take statistics, because they are either seniors or because they plan on taking AP calculus as seniors. The few pre-calculus students that do end up taking AP statistics their senior year often take AP calculus concurrently, so they need to be especially prepared for college level mathematics. Preparing these students can be a challenge, considering that there are many students that not interested in AP classes at all. Therefore, two ends of the table need to be satisfied, sort of speak.

**Worcester Public Schools Mission Statement**

The Worcester Public Schools Mathematics Curriculum contains a mission statement which explains the motivation and the influences of how the curriculum was formed.

*Mission Statement*48

“The mission of the Worcester Public Schools mathematics, science and technology curricula is to provide equal opportunities for all students to discover the power of mathematics, science and technology. In conjunction with The Reauthorization of the Elementary and Secondary Initiatives, National Council of Teachers of Mathematics (NCTM) Curriculum and Evaluation Standards, the State Frameworks, this curriculum will be a teachers’ reference for instructional strategies, content and materials.”

The mission statement says that the goal of Worcester Public Schools is to give equal opportunities of education to all students, not to provide an equal education to all students. As discussed in Chapter 1, providing an equal education to all students is not possible, since students have a wide variety of interest and abilities. Rather, the goal of the Worcester Public School system is to give students the same opportunities to receive an education. This is why the mathematics curriculum is united across all the public schools in the district.

The mission statement also clearly indicates how the curriculum was formed. The Reauthorization of the Elementary and Secondary Initiatives, NCTM Curriculum and Evaluation Standards and the State Frameworks all have an influence on the current curriculum.

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48 Worcester Public Schools Mathematics Curriculum
The Massachusetts Curriculum Frameworks and the Common Core Standards

By far the most important influence over the mathematics curriculum for Worcester Schools is the Massachusetts Curriculum Frameworks, or more simply, the State Frameworks. The State Frameworks are a set of standards in which all students must be competent in. The State Frameworks are in several sections, including arts, English language arts, foreign languages, comprehensive health, mathematics, history and social sciences, and science and technology/engineering. Of course, the applicable section for the advanced algebra and pre-calculus is mathematics. The Frameworks are important because the statewide exam, the MCAS, evaluates students’ proficiency meeting the standards in the Frameworks. If a student cannot prove his or her proficiency, then that student cannot graduate high school. So at a minimum, public schools across the state have to ensure that the standards are being addressed. Although there is some opposition to the Frameworks because it dictates what is to be taught and in which grade levels, the Frameworks ensures that students are receiving the skills that they need for college and the workforce.

The most recent version of the mathematics Frameworks was completed in March of 2011. The primary reason why the Frameworks had to be updated was due to the introduction of the Common Core Standards, which came into effect on July 21, 2010. The Common Core Standards are a set of state-driven efforts to produce a uniform set of standards across the nation. That is to say, the Federal government was not involved in its development. These standards were specifically devised by teachers, parents, school administrators, and educational experts in order to prepare the nation’s youth for college and for the workforce, regardless of geographical location. Currently, the Common Core Standards are established for English language arts and mathematics for grades K-12. States could voluntarily adopt the Common Core Standards, and Massachusetts was one of the many that did. The Federal government encouraged states to adopt the Common Core Standards by giving states financial incentives if they did adopt them. How each state chose to adopt the Common Core Standards differs from state to state, the Massachusetts Department of Education decided to incorporate them into the already existing Massachusetts Curriculum Frameworks.

New 2011 Mathematics Standards in the Massachusetts Curriculum Frameworks

Here is a list of all the 2011 state frameworks that cannot be mapped to older standards. Appendix A and B have describe what these standards are in detail. I have used the “Crosswalk of 2011 MA Model Mathematics Course Standards and 2000 MA High School Course Standards” to compile this list.

<table>
<thead>
<tr>
<th>New Algebra 2 Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>N.CN.9, N.VM.1, A.APR.2, A.CED.4, F.IF.6, F.IF.9, S.IC.1, S.IC.2, S.IC.3, S.IC.4, S.IC.5, S.MD.6, S.MD.7</td>
</tr>
<tr>
<td>New Pre-Calculus Standards</td>
</tr>
<tr>
<td>---------------------------</td>
</tr>
<tr>
<td>N.CN.9, A.APR.2, G.SRT.11</td>
</tr>
</tbody>
</table>

The Reauthorization of the Elementary and Secondary Initiatives

As the successor of John F. Kennedy, Lyndon B. Johnson had the mission to continue the Federal government’s role in the civil rights movement. Although many people questioned whether Johnson would be as proactive as Kennedy because he was a southerner, it became clear over time that Johnson would be a faithful leader in the

49 http://www.doe.mass.edu/frameworks/current.html
50 http://www.doe.mass.edu/frameworks/math/0311.pdf, page 9
51 http://www.corestandards.org/frequently-asked-questions
52 www.doe.mass.edu/candi/commoncore/0111mathcrosswalk.xls
movement. One piece of legislation that proved Johnson’s support was the Elementary and Secondary Education Act (ESEA), enacted on April 11, 1965. As part of Johnson’s “War on Poverty” mission, the ESEA is a regulation that funds public and private schools, particularly those with a high proportion (at least 40%) of low-income students. A Title 1 school, as defined in the ESEA, is a school that receives such funds in exchange of the school’s accountability to provide equal educational opportunities for all students and to narrow achievement gaps.53 (Doherty is a Title 1 school.) The ESEA also forbids a national curriculum, hence the state-driven effort of the Common Core Standards, and its voluntary condition.54 ESEA also allows schools to give information about students in the 11th and 12th grades to the military.

Every few years the ESEA is reauthorized. The current form of the ESEA is the No Child Left Behind Act (NCLB) of 2001, put into effect by George W. Bush. NCLB is a standards-based education reform, which allocates funds to states only if they assess the proficiency of basic skills of all students. In fact, schools may not receive federal funding if they do not have a statewide assessment system. States are required to test students in grades three through eight in reading and mathematics on an annual basis. In Massachusetts, the MCAS fulfills this requirement. Like the 1965 version, NCLB does not assert any type of national achieve standards. Rather, standards are to be set by each state individually. As part of NCLB, all teachers instructing core academic subjects must be “highly qualified”. As discussed in Chapter 1, a “highly qualified” teacher is one that is sufficiently proficient in the subject matter he or she teaches and that is licensed to teach. 55 Also important is the fact that the amount of money spent funding education has increased substantially over time. Currently, the federal government spends more than $54 billion to fund education56, compared to just $42.2 billion in 2001.

NCLB increases the importance of the MCAS because schools are required to bring all students to the “Proficient” and “Advanced” level by the end of the 2013-2014 school year. If a Title 1 school does not meet this goal, then a series of actions will pursue. For example, students will be offered to attend other public schools, private tutoring may be offered, or staff members may be replaced or trained.57 Because of NCLB, the curriculum of all Worcester Public schools have to address the various standards outlined in the state Frameworks.

**National Council of Teachers of Mathematics Curriculum and Evaluation Standards**

With over 90,000 members and 230 affiliates, the National Council of Teachers of Mathematics (NCTM) is the world’s largest non-profit organization that supports teachers that strive to give their students the best possible mathematics education possible.58 Although teachers have to pay to become a member of NCTM, the advantages of being a member are numerous. In addition to free periodicals and journals about mathematics in the classroom, members also receive the privilege of accessing lesson plans, activities, and multimedia for classroom use. 59 Members also have access to the NCTM’s curriculum standards and can receive grants for conducting research in the classroom.60

The NCTM Curriculum Standards for Grades 9-12 is in Appendix E. There are 14 NCTM Curriculum Standards in total. The first three standards demand that students recognize and use mathematics as a system of problem solving, as a system of communication, and as a system of reasoning. Students must be able to identify and interpret real world problems into a mathematical model. Students should also refine their ability to express mathematical ideas,

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58 [http://www.nctm.org/about/content.aspx?id=174](http://www.nctm.org/about/content.aspx?id=174)
in both written and oral forms, and to be able to inquire about topics relating mathematics. The Curriculum Standards also encourages students to appreciate the elegance and power of mathematical notation. Finally, as a system of reasoning, students should become competent in formulating, testing, and judging conjectures, counterexamples, and logical arguments.

Standard 4 is about the ability to make mathematical connections, such as the understanding that two mathematical models that may be very different in structure may have similarities. (e.g. in order to identify the zeros of a quadratic function, one can complete the square, use the quadratic formula, factor, or graph the function. These methods are quite different from each other, yet they can all be used to find the zeros of a quadratic.) Standard 4 also contains the expectation that students will be able to identify connections between mathematics and other fields, such as science or engineering.

Standards 5-13 divides a student’s accumulating knowledge of high school mathematics into its core topic areas, including algebra, functions, geometry (from both a visual and algebraic perspective), trigonometry, statistics, probability, discrete mathematics, and pre-calculus. Therefore, according to the NCTM curriculum, students must be competent in all of these fields. This can be a challenge for the students at Doherty because only three courses of mathematics are required, and the fact that algebra, advanced algebra, and geometry alone satisfy the graduation requirements. Therefore, if students are to be exposed to and be proficient in all the topics mentioned above, those three classes must be broad enough to extend into the other fields of mathematics. (e.g. integrating mean, median, and mode in an algebra class, or integrating sine, cosine, and tangent into a geometry class).

The last standard, standard 14, is centered on mathematical structure. Students should be able to understand the real number system and its subsystems, and to be able to compare and contrast them. (e.g. the quotient of two real numbers is a real number, but the quotient of two integers is not necessarily an integer.) Students should also be capable to understand algebraic procedures.

The above standards are the minimum standards that the NCTM has established. However, the NCTM has expanded the curriculum to include skills that college bound students should master. These additional standards are listed at the end of standards 3-14. Thus, the NCTM can be used as a guide to create different curricula depending if the students want to go to college or not. For example, a traditional high school may have a curriculum that is centered on the extended NCTM curriculum, but a vocational high school may have a combination of both the extended and basic curricula (since a lower percentage plan on going to college."61"

The Effect the MCAS has on Honors Advanced Algebra and Pre-Calculus

The MCAS is extremely important for students in grades 10 and lower. However, all of the students in pre-calculus are juniors and seniors, and all of them have already passed the MCAS exams. In my opinion, a student does not even belong in pre-calculus unless that student is capable of passing the mathematics MCAS exam, because only a partial mastery of the standards are necessary to pass the test. A student entering pre-calculus should have mastered the standards for algebra, geometry, and advanced algebra. Therefore, the MCAS does not have an impact on the pre-calculus students. They are beyond the level of competency that is required to pass the test.

The impact the MCAS has on advanced algebra students is more substantial, because advanced algebra classes do have sophomores that will be taking the tests in the spring. However, in order to enter honors advanced algebra, student had to have done well in honors algebra 1 and geometry (or exceptionally well in the level 1 classes). So, in

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61 Compare the plans of high school graduates of Lowell High School and Lowell Regional Vocational High School, for example. 85% of the graduates from Lowell High School plan on going to college, while 63% of graduates from the vocational school plan on going to college. This is a typical outcome. Sources: http://profiles.doe.mass.edu/profiles/student.aspx?orgcode=08280000&orgtypecode=5&leftNavId=307& http://profiles.doe.mass.edu/profiles/student.aspx?orgcode=01600505&orgtypecode=6&leftNavId=307&
theory, students should have mastered concepts from those two subject areas. In addition, sophomores taking honors advanced algebra are more advanced than their peers, because most students take the class as juniors. Thus, even though many of the standards in the state Frameworks are under the “algebra 2” category (which is comparable to advanced algebra), students entering the class have already mastered the majority of the standards. Passing the MCAS should be no problem for honors advanced algebra students then.

Of course, the state Frameworks are still very important, and just because the students should not have to worry about passing the MCAS, they still have plenty of standards to master. There are several standards in the state Frameworks that are marked with a (+), which are additional standards that college-bound and other advanced students are recommended to fulfill. These standards are not tested in the MCAS, but they are nevertheless important for college preparation. The state Frameworks has standards up to the pre-calculus level, so even those students have plenty of challenges ahead of them.

**Syllabus for Advanced Algebra (H)**

Now that the influences of the Worcester mathematics curriculum has been discussed, the course structure of the advanced algebra (H) class can now be evaluated. The course syllabus is generic and is used for all Worcester public schools for advanced algebra (regardless of level). The course description clearly indicates that the curriculum is centered on the Massachusetts Curriculum Frameworks and the Worcester Public School Mathematics Curriculum, which should be no surprise. The majority of the class will be spent on linear equations, matrices, polynomial functions, exponential and logarithmic functions, and trigonometric functions. If one took a look at the algebra 2 state Frameworks, one would realize that these topics compromise the majority of the standards.

One essential question that students leaving this course should be able to answer is “How can non-linear functions (including trigonometric functions) be used to solve problems in our world?” Answering this question demands students not only to understand the algebra and algorithms behind function operations, but to be able to apply mathematics in the real world. This skill is extremely important, because if mathematics could not be used to solve problems, it would be little more than an artistic manifestation. Standard F-BF 1 from the state Frameworks says that students should be competent in building functions that models relationships between two quantities. In addition, standard 1 from the NCTM curriculum is entirely about mathematics from a problem solving perspective. Therefore, this essential question is not only challenging students to solve real life problems mathematically, but to fulfill state and NCTM standards as well!

The advanced algebra syllabus also lists a set of district-wide reading skills, which is on the syllabus for every class. These reading skills are important because they clearly make the point that language arts are important in *every* subject, including mathematics. These skills are meant be utilized and honed as a study method when the students read the textbook (which will be discussed momentarily). Although this is a great idea, I wonder how many teachers actually take the time to teach students how to study using these methods.
Worcester Public Schools
High School Curriculum

Course Syllabus – Advanced Algebra Honors

Instructor: Mrs. O'Leary  Email: O'LearyK@worc.k12.ma.us

Course Description:
The course will focus on the Algebra II Massachusetts Mathematics Curriculum Framework and the Worcester Public Schools 11th Grade Mathematics Curriculum. This course is a bridge from Algebra I into advanced topics in mathematics and successful completion in Algebra 1 and Geometry are necessary to take this course. Please note that Advanced Algebra is the prerequisite to Pre-calculus and Probability and Statistics.

Course Objectives:
Students will:
• 1 Solve problems using systems of linear equations by graphing, substitution, elimination, and matrices.
• 2 Explore the relationship between tables, graphs, and equations for linear, quadratic, and exponential functions.
• 3 Study the concept of relations/functions and their inverses using function notation, tables, graphs, equations, and composition.
• 4 Continue to practice algebraic skills including addition/subtraction/multiplication/division of polynomials, factoring quadratics, simplifying rational expressions, mathematical conversions
• 5 Explore and solve problems using trigonometry and the unit circle.
• 6 Utilize the TI-83 graphing calculator and scientific calculator for exploration/analysis of mathematical concepts.
• 7 Review test taking strategies & problem solving for SAT/ACT exams.

Essential Questions:
1. How can linear and non-linear functions be used to model real-life situations?
2. What is the difference between a relation and a function?
3. How can we derive the inverse to a relation or function?
4. What are the characteristics of a linear function? Quadratic function? Exponential function? Use tables, graphs and equations in your description.
5. What is the trigonometry of a right triangle?
6. Where do trigonometric functions and solutions occur in our world?

Texts:

District-Wide Reading Skills Across the Curriculum:
• 1 Preview (survey) – note major elements such as organization, vocabulary, summary and graphics.
• 2 Ask Questions - question the text, the author and self.
• 3 Activate Prior Knowledge (schema) – use what is already known to enhance understanding of what is new in the text.
• 4 Make Connections - link text to self, text to world and text to text.
• 5 Visualize - use sensory images to create a mental picture of the scene, story, situation, or process and involve oneself in it.
• 6 Draw Inferences - go beyond the literal information in the text including predicting, figurative meaning and thematic understanding.
• 7 Distinguish Key Ideas - recognize main idea and key concepts.
• 8 Use Fix-Up Strategies - monitor own understanding by pausing to think, re-read, consider what makes sense, restate in own words.

Contextual Vocabulary:
Function families including polynomial, exponential, & trigonometric.
Table/graph/Equation relationships for above functions.
Y-intercepts, zeros, maximum and minimum values.
Relations, Functions, Composition
Law of Sines & Law of Cosines
Radian Measure
Rational expressions

### Grading Policy:

Your quarter grade will be determined as follows:
- **1.** Homework average = 15 %
- **2.** Quiz Average = 15 %
- **3.** Each Individual skill assessment = 60 %
- **4.** Participation/Do Now = 10 %

**Extra credit will not be given!**

***The Worcester School Committee requires that the final grade is determined as 90% year average plus 10% final exam score.***

### Additional Reminders:

<table>
<thead>
<tr>
<th>Student Materials:</th>
<th>Please come prepared to class daily with agenda and 3-ring binder, textbook, scientific or TI-83 graphing calculator, pen/pencil and homework.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Be ready to make positive contributions in class by asking questions, sharing solutions and showing respect for yourself, your peers, and your instructor.</strong></td>
</tr>
<tr>
<td>Extra Help Days:</td>
<td>Room 430 from 1:45 – 2:30 Tuesdays and Wednesdays (subject to change due to unexpected meetings) or by appointment. It is the responsibility of the student to attend extra help sessions as needed.</td>
</tr>
<tr>
<td>Test Day:</td>
<td>Test day is Wednesday. An extra help session will be held one or two days before any assessment.</td>
</tr>
<tr>
<td>Due Dates:</td>
<td>All homework and projects/labs will be given due dates – students must meet the assigned date in order to be eligible for full credit.</td>
</tr>
<tr>
<td></td>
<td>- <strong>1.</strong> Late homework is accepted for one school day with a maximum of half credit. Late homework is not accepted once the unit assessment has been given.</td>
</tr>
<tr>
<td>Make Up Policy for Absent Students:</td>
<td>Students who are absent must abide by the make up policy set in the student handbook. Assessments are to be made up as soon as possible at a mutually convenient time. Students are responsible for getting all notes, handouts, and assignments for days missed.</td>
</tr>
<tr>
<td></td>
<td><strong>Please note that students on project are responsible for meeting all due dates. You are expected to give me advance notice of any projects.</strong></td>
</tr>
<tr>
<td>Absences &amp; Tardiness:</td>
<td>14 absences = loss of credit for the class.</td>
</tr>
<tr>
<td></td>
<td>Tardy to class results in detention.</td>
</tr>
</tbody>
</table>
Vocabulary is important because without it, people would not be able to speak and write intelligently about anything. It is great that the syllabus includes vocabulary words that the students should master, but it is a disappointingly short list. Words like “linear”, “degree”, “inverse” and “determinant” are missing. In reality, a student would probably learn at least fifty vocabulary words in one mathematics class. (The glossary in the back of the textbook has hundreds of vocabulary words, in two different languages!) A vocabulary list would be much more beneficial to students if it included all important terminology that will be used in the course, not just the six most imperative ones. Although this contextual vocabulary section may have been influenced by standard 2 of the NCTM curriculum, it is severely incomplete and mediocre, and any benefit that the list has for any students would be very minimal.

The syllabus also includes a grading policy. A requirement from the Worcester School Committee is that a final assessment is worth 10% of a student’s final grade. (I thought this was a rather weak requirement because in the Lowell school system, the final assessment is worth 25% of a student’s final grade.) My mentor, Kathleen O’Leary does not actually give a formal final assessment, because in her classes, all assessments are worth 60% of the students’ grades and she would never give more than six assessments in one term. All assessments are equally weighted. Homework, which is regularly checked by O’Leary, is worth 15% of a student’s grade, and quizzes are worth another 15%. The remainder 10% of the a student’s grade is his or her participation. At the beginning of nearly every class, a “do-now” is posted on the board for students to work on immediately. A student would typically receive all ten points in this section, unless there is some substantial behavioral or attendance issue. (O’Leary said herself that she is a “soft grader”.) I believe that the grading policy is fair and practical. 25% is given automatically to students that do what they are suppose to be doing (participating and doing homework). The remainder of the grade is test and quizzes, which may seem like a substantial amount, but college bound students have to get use to it, because that is how it will be like in college. It says on the syllabus that extra credit will not be given, however, O’Leary is pretty lax on this rule, and if students in general do poorly on a test, she will give students an additional assignment.

It should be noted that at Doherty, math test are suppose to be given only on Wednesdays. Every subject has its own reserved day for testing. This way, students are never bombarded with several exams on one day. While this can be very beneficial for students, it is an absolute nuisance for teachers. Lesson plans have to be created very carefully so that students are ready to test on Wednesday, otherwise, a test will have to be pushed back a week.

The last section of the syllabus is classroom policies. Cell phones are banned in all of Mrs. O’Leary’s classes, (unless they are used to take pictures of the board). Repeated tardiness, no matter what the excuse is, will result in detention. O’Leary always has extra help days on Tuesdays, which happen to be the day before exams. Late homework will receive half credit, unless the student was absent. There is nothing really surprising in this section, but rules have to be laid out or else students will try to cut corners.

Textbook for Advanced Algebra (H)

The text used for the course is Algebra 2 by Ron Larsen, Laurie Boswell, Timothy D. Kanold, and Lee Stiff. The textbook is absolutely massive (about the size of Calculus Early Transcendentals that all Freshmen at WPI use). Therefore, it is not expected that students lug the book back and forth between home and school. Rather, students are able to access an electronic copy of the book via the internet. Students are also told that they may keep the books at home, because there are enough textbooks in the classroom to share. Textbooks are used on occasion in a classroom setting, but class instruction is usually without the book. Students are expected to read the textbook on a regular basis, especially if the students have difficulty on homework assignments. The textbook is brilliantly colorful and there is a picture on every page (in an attempt to remind students of the real-world applications of
mathematics, I suppose). However, when the textbook is stripped of all of these obnoxious additions, the remainder is quite disappointing. The majority of each section is composed of examples, and the text shows students how to do everything. How are students supposed to come up with their own ideas if there is an example corresponding to every single problem in the book? Also, the examples are not very good. Sometimes, there are far better approaches to solve the problems that they solve. Here is an example of a textbook problem:

To factor \( ax^2 + bx + c \) when \( a \neq 1 \), find integers \( k, l, m, \) and \( n \) such that:

\[
ax^2 + bx + c = (kx + m)(lx + n) = klx^2 + (kn + lm)x + mn
\]

So, \( k \) and \( l \) must be factors of \( a \), and \( m \) and \( n \) must be factors of \( c \).

\[
\textbf{Example 1} \quad \text{Factor } ax^2 + bx + c \text{ where } c > 0
\]

\[
\text{Factor } 5x^2 - 17x + 6.
\]

\[
\textbf{Solution}
\]

You want \( 5x^2 - 17x + 6 = (kx - m)(lx + n) \) where \( k \) and \( l \) are factors of \( 5 \) and \( m \) and \( n \) are factors of \( 6 \). You can assume that \( k \) and \( l \) are positive and \( k > l \). Because \( mn > 0 \), \( m \) and \( n \) have the same sign. So, \( m \) and \( n \) must both be negative because the coefficient of \( x \), \( -17 \), is negative.

<table>
<thead>
<tr>
<th>( k, l )</th>
<th>( 5, 1 )</th>
<th>( 5, 1 )</th>
<th>( 5, 1 )</th>
<th>( 5, 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m, n )</td>
<td>( -6, -1 )</td>
<td>( -1, -6 )</td>
<td>( -3, -2 )</td>
<td>( -2, -3 )</td>
</tr>
<tr>
<td>( (kx + m)(lx + n) )</td>
<td>( (5x - 6)(x - 1) )</td>
<td>( (5x - 1)(x - 6) )</td>
<td>( (5x - 3)(x - 2) )</td>
<td>( (5x - 2)(x - 3) )</td>
</tr>
<tr>
<td>( ax^2 + bx + c )</td>
<td>( 5x^2 - 11x + 6 )</td>
<td>( 5x^2 - 31x + 6 )</td>
<td>( 5x^2 - 13x + 6 )</td>
<td>( 5x^2 - 17x + 6 )</td>
</tr>
</tbody>
</table>

The correct factorization is \( 5x^2 - 17x + 6 = (5x - 2)(x - 3) \).

In my opinion, this is terrible. I had to reread this several times in order to understand all the information presented here. A student would probably have an even harder time understanding what the textbook is trying to explain. Also, this particular section in the book list a bunch of cases, and students are going to be tempted to memorize all the different cases. This is unnecessary, because it is possible to factor the polynomial expression using the so called “grouping method”:

\[
5x^2 - 17x + 6
\]

Want two numbers that multiply to \( 5 \times 6 = 30 \) and add to \( -17 \). Choose \(-15\) and \(-2\).

\[
5x^2 - (15x + 2x) + 6
\]

\[
(5x^2 - 15x) + (-2x + 6)
\]

\[
5x(x - 3) + (-2)(x - 3)
\]

\[
(5x - 2)(x - 3)
\]

This method does not require students to memorize different cases, and there is less guessing involved. I am not aware of any problem where the grouping method fails. This is one of many examples in the book that make things
harder than they have to be. This book has four authors, all of which are either teachers or professors, and I believe it is a shame that they present some material so badly.

There is some merit in the book, however. The book has countless word problems and real life applications are numerous. Converting words into algebraic expressions and symbols is where language arts meets mathematics, and it is an absolutely essential skill. In fact, mathematics would be merely an artistic manifestation if it did not solve anything. Standard 1 of the NCTM curriculum clearly states the need for students to translate word problems into a mathematical model, and the state Frameworks has many standards that require students to obtain the skill (e.g. A-CED-3 and F-BF-1). In fact, all the 2011 state standards that are marked with a ★ are modeling standards. Therefore, the book does offer plenty of opportunities to fulfill these important standards.

**Course Outline for Advanced Algebra (H) (Short Form)**

Below is a brief outline of the honors advanced algebra course. It shows which sections of the textbook are covered in which term. Sections marked with an asterisk (*) are topics that may be skipped if the class is short on time, however, they are especially recommended for honors advanced algebra.
Curriculum Outline for Algebra 2
Doherty Memorial High School

Quarter 1
2.1 Relations and Functions
2.2 Slope and Rate of Change
2.3 Graph Equations of lines
2.4 Write Equations of lines
2.5 Model Direct Variation
1.6 Solve Linear Inequalities
3.1 Solve System Linear Equation by Graphing
3.2 Solve Systems by Substitution
3.3 Solve Systems by Elimination
2.6 Scatterplots

Quarter 2
1.7 Solve Absolute Value Equations and Inequalities
2.7 Absolute Value Functions
4.2 Graph Quadratic Functions in Vertex format
4.1 Graph Quadratic functions in standard form
4.3 Solve $x^2 + bx + c = 0$ by Factoring
4.3 Solve $ax^2 + bx + c = 0$ by Factoring
4.5 Solving Quadratics using the square root
4.7 Completing the Square
4.8 Using the Quadratic formula and the Discriminant

Quarter 3
8.4 Multiply and Divide Rational Expressions
8.5 Add/Subtract Rational Expressions
5.1 Property of Exponents
Revisit Function Notation by evaluating functions
Finding the inverse of the function
Composition of Function
Review of Pythagorean Theorem
Simple Radical Form
Special Right Triangles

Quarter 4
13.1 Trigonometric Ratios
13.2 Define General Angles in Radian Measure
13.3 Define Trig Functions at any angle
13.4 Evaluate Inverse Trig Functions
13.5 Law of Sine
13.6 Law of Cosine
14.1 Graph Sine, Cosine, Tan Functions
14.2 Translate and Reflect Trig Graphs

The ordering above is not set in stone. For example, Mrs. O’Leary decided to teach inverse functions and composite functions in quarter 1 instead of in quarter 3. Also, she switched the ordering of sections 4.1 and 4.2, since she prefers to teach vertex form first.
Course Outline for Advanced algebra (H) (Long Form)

A more detailed outline of honors advanced algebra is below. It lists all of the skills that students should learn in the course, as well as the 2000 state mathematics Frameworks that correspond to each skill. This outline has been used since 2006, and has not been updated to correlate with the current 2011 state Frameworks. I have however, mapped to 2002 standards with the 2011 standards using the crosswalk tool provided by the Department of Education. The 2011 standards are in red. Note that in general, the 2000 standards are broader than the more specific 2011 standards, which is why one 2000 standard is mapped with several 2011 standards. For example, the 2000 standard AII.P.5, which states that students should be able to “perform operations on functions, including composition” and to “find inverses of functions, can be mapped to:

- F.BF.1b - Combine standard function types using arithmetic operations.
- F.BF.4 - Find inverse functions.
- F.BF.4a – Solve an equation of the form \( f(x) = c \) for a simple function \( f \) that has an inverse and write an expression for the inverse.

The 2000 state frameworks for algebra 2 (AKA advanced algebra) are in Appendix C, and the 2011 state frameworks are in Appendix A.

An important question is why advanced algebra is modeled after the 2000 state frameworks and not the current 2011 state frameworks. First of all, it would not be very beneficial to immediately alter all mathematics courses at Doherty just because the state updated the standards. A student that has developed a mathematical foundation based on the 2000 standards may be very confused if a dramatic change in expectations occurs. For example, standard A.APR.2 is a new algebra 2 standard that requires students to know the Remainder Theorem. So current pre-calculus students may not know the Remainder Theorem, and it would be incorrect to assume that students are competent with the most current standards. Of course, the 2011 standards should eventually be implemented into the curriculum, however, it should be done gradually so that the students have time to adjust to the increased expectations. Secondly, as shown above, many of the 2000 standards can be mapped to the current standards. The major difference is that the 2011 standards are more specific than the older ones. The current standards are not all that new then. In fact, only 14 of the 50+ algebra 2 standards cannot be mapped to the 2000 standards. Thirdly, the MCAS will not immediately test high school students on these standards, because the Department of Education realizes that schools and students must adjust to the current standards gradually. Indeed, the DoE still has the old standards readily available on their website, and will remain on their website “until districts have made the transition to the 2011 framework at the beginning of the 2013-2014 school year.” This brings up the fourth point, being that districts are not required to completely transition to the new frameworks until the 2013-2014 school year. Finally, a fifth point why older standards are still being used is because the faculty of Doherty has been primarily occupied with the NEASC evaluation prior to November 2011, and so updating the curriculum to the current standards was not a primary goal in the past, (although it ought to be one now). In my lesson plans, I will map my objectives to the current standards so that they may be useful to me in the future.

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63 http://quadrant-office.worcesterschools.org/modules/locker/files/get_group_file.phtml?gid=1022375&fid=7378352&sessionid=2f6942a99411fbc41ac1a27211d2805
64 www.doe.mass.edu/candi/commoncore/0111mathcrosswalk.xls
65 http://www.doe.mass.edu/frameworks/current.html
# Course Syllabus – Part II, Academic Content for the First Semester

## Advanced Algebra II

<table>
<thead>
<tr>
<th>Content/Topics –</th>
<th>Skills</th>
<th>Required Papers/Projects, Readings, and Final Assessment/Test</th>
<th>Academic Standards (Worcester Benchmarks and State Frameworks)</th>
</tr>
</thead>
</table>
| **Linear Representations** | Find slope and intercepts  
Solve linear equations in two variables | | **AII.P.8** Solve a variety of equations and inequalities using algebraic, graphical, and numerical methods.  
(A.CED.1, A.CED.2, A.REI.2, A.REI.11, F.IF.5, F.IF.7c, F.IF.7e, N.CN.8) |
| **Numbers and Functions** | Use operations with numbers  
Use operations with functions  
Identify properties of exponents | | |
| **Systems of Linear Equations and Inequalities** | Solve systems of equations  
Find solutions to linear inequalities in two variables  
Solve systems of linear inequalities | | **AII.P.9** Use matrices to solve systems of linear equations.  
(N.VM.6, N.VM.8, N.VM.12) |
| **Matrices** | Use matrices to represent data  
Solve using matrix multiplication  
Find the inverse of a matrix | | |
<table>
<thead>
<tr>
<th>Quadratic functions</th>
<th>Solve quadratic equations</th>
<th>AII.P.7 Find solutions to quadratic equations and apply to the solutions of problems. (A.CED.3, F.IF.8c, N.CN.7)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Factor quadratic equations</td>
<td>AII.P.4 Demonstrate an understanding of the exponential and logarithmic functions. (F.IF.7, F.IF.8, F.BF.1, F.LE.4, F.TF.5)</td>
</tr>
<tr>
<td></td>
<td>Use the completing the</td>
<td></td>
</tr>
<tr>
<td></td>
<td>square method</td>
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<tr>
<td></td>
<td>Solve using the quadratic</td>
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</tr>
<tr>
<td></td>
<td>formula</td>
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<tr>
<td></td>
<td>Use complex numbers in the</td>
<td></td>
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<tr>
<td></td>
<td>solution to quadratic</td>
<td></td>
</tr>
<tr>
<td></td>
<td>equations</td>
<td></td>
</tr>
<tr>
<td>Exponential and Logarithmic Functions</td>
<td>Solve problems involving</td>
<td></td>
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<tr>
<td></td>
<td>exponential growth and</td>
<td></td>
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<tr>
<td></td>
<td>decay</td>
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<tr>
<td></td>
<td>Graph and solve exponential</td>
<td></td>
</tr>
<tr>
<td></td>
<td>functions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Graph and solve logarithmic</td>
<td></td>
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<tr>
<td></td>
<td>functions</td>
<td></td>
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<tr>
<td></td>
<td>Use and apply the properties</td>
<td></td>
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<td></td>
<td>of logarithms</td>
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<tr>
<td></td>
<td>Solve problems using base</td>
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</tbody>
</table>
## Worcester Public Schools
### High School Curriculum

**Course Syllabus – Part II, Academic Content for the Second Semester**

**Advanced Algebra II**

<table>
<thead>
<tr>
<th>Content/Topics –</th>
<th>Skills</th>
<th>Required Papers/Projects, Readings, and Final Assessment/Test</th>
<th>Academic Standards (Worcester Benchmarks and State Frameworks)</th>
</tr>
</thead>
</table>
| Polynomial Functions | Graph polynomial functions  
Find products and factors of polynomials  
Solve polynomial equations  
Find the zeros of polynomial functions | | AII.P.8 Solve a variety of equations and inequalities including polynomial, exponential, and logarithmic functions.  
(F.IF.5, F.IF.7c, N.CN.8, A.APR.1, A.APR.2, A.APR.4, A.APR.6, A.APR.7, A.CED.1, A.CED.2, A.REI.2, A.REI.11) |
| Rational & Radical Functions | Identify inverse, joint and combined variation  
Graph rational functions  
Multiply and divide rational expressions  
Add and subtract rational expressions  
Solve rational equations and inequalities  
Identify radical expressions and functions  
Simplify radical expressions | | AII.P.5 Perform operations on functions, including composition.  
(F.BF.1b, F.BF.4, F.BF.4a) |
| Conic Sections | Find parabolas, circles, ellipses, and hyperbolas | | AII.G.3 Relate geometric and algebraic representations of lines, simple curves, and conic sections.  
(A.APR.3, F.IF.4, F.IF.7) |
<table>
<thead>
<tr>
<th>Counting Principals</th>
<th>Series and Patterns</th>
<th>Trigonometric Functions</th>
<th>AII.D.2 Use combinatorics to solve problems, in particular, to compute probabilities of compound events. (CC.9-12.S.CP.6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use Permutations and combinations</td>
<td>Identify independent events</td>
<td>Solve situations involving dependant events and conditional probability</td>
<td></td>
</tr>
<tr>
<td>Identify independent events</td>
<td>Solve using arithmetic and geometric sequences</td>
<td>Solve using arithmetic and geometric series</td>
<td></td>
</tr>
<tr>
<td>Solve situations involving dependant events and conditional probability</td>
<td>Use Pascal’s triangle in the solution to problems</td>
<td>Use the binomial theorem</td>
<td></td>
</tr>
<tr>
<td>Solve using arithmetic and geometric sequences</td>
<td>Solve using arithmetic and geometric series</td>
<td>Solve trigonometric functions</td>
<td></td>
</tr>
<tr>
<td>Solve using arithmetic and geometric series</td>
<td>Use Pascal’s triangle in the solution to problems</td>
<td>Find radian measure and arc length</td>
<td></td>
</tr>
<tr>
<td>Use Pascal’s triangle in the solution to problems</td>
<td>Use the binomial theorem</td>
<td>Graph trigonometric functions</td>
<td></td>
</tr>
<tr>
<td>Use the binomial theorem</td>
<td>AII.P.2 Identify arithmetic and geometric sequences and finite arithmetic and geometric series. (A.SSE.4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AII.G.1 Define the sine, cosine, and tangent of an acute angle. (F.TF.2)</td>
<td>AII.G.2 Derive and apply basic trigonometric identities and the laws of sines and cosines. (F.TF.8)</td>
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<td></td>
</tr>
</tbody>
</table>
Prerequisite Skills for Advanced Algebra (H)

Theoretically, students should have mastered everything that they have “learned” in algebra 1 and geometry. In reality, however, students forget much of what they have learned during summer vacation, or may have not learned everything that they were supposed to. That is why the majority of the first quarter of advanced algebra is actually a review. Although students should know how to graph a line, some of them will not remember. Fortunately, graphing lines is part of the review, so graphing lines is not a prerequisite per se, but it is a topic that students should have already seen at least. Other parts of the review include functions, domain and range, equations of lines, direct variation, inverse functions, composite functions, and the Pythagorean Theorem. All of these skills should have been learned in algebra 1, but a surprising portion of the students (but not all) do not remember these things. In addition, although algebra 1 and geometry are prerequisites for the course, some students take advanced algebra and geometry at the same time so that they can take AP calculus in their senior year. Therefore, only very basic algebra and geometry skills are necessary to do well in this course. The minimum skills that students need to succeed are:

- Arithmetic, including adding, subtracting, multiplying, and dividing real numbers (including fractions, decimals, and percentages).
- Understand and apply the order of operations, and use the distributive property.
- Evaluate and simplify algebraic and arithmetic expressions.
- Solving single variable first order polynomial equations.
- Graph points on a Cartesian plane, and identify the coordinates of a point on a Cartesian plane.
- Manipulate equations to isolate variables.
- Know the absolute value of a real number.
- Compute the square of a real number, and approximate the square root of a real number.
- Identify basic properties of shapes such as a triangles, squares, circles, and parallelograms. For example, know that a circle has area \( \pi r^2 \) and diameter \( 2\pi r \).

In addition, students should be able to do the following, although these concepts will be reviewed in the course:

- Model a linear phenomena with a linear equation.
- Convert an equation of a line to point-slope form, standard form, and slope-intercept form.
- Graph an equation of a line.
- Identify the slope, y-intercept, and x-intercept of a line.
- Find the inverse of a function, graphically and algebraically.
- Identify the domain, range, zeros, and y-intercepts of a function.
- Compute composite functions, and use composite functions to solve real world problems.
- Understand and apply the Pythagorean Theorem.
- Model using inequalities with one and two variables.
- Graph and solve inequalities in one and two dimensions.
- Simplifying square roots.
- Rules of exponents, and graphing exponential functions.
- Special right triangles.
- Factoring and graphing quadratics.

Again, all of these concepts listed should have been learned in algebra 1 and geometry, but several of these concepts will be reviewed in advanced algebra. As a reference, Appendixes F and G show the course outlines for algebra 1 and geometry respectively.

Syllabus for Pre-Calculus
Just like the syllabus for advanced algebra, the pre-calculus syllabus is used district-wide. The format of the syllabus is the same as the syllabus for advanced algebra. Since this reading skills and grading sections are the same for both syllabi, this section will only discuss the other sections of the pre-calculus syllabus.

Worcester Public Schools Syllabus
Pre-Calculus

Instructor: Mrs. O'Leary
Email: O'LearyK@worce.k12.ma.us

Course Description:

The goal of this course is to provide a strong background in preliminary calculus concepts for those juniors who might wish to take A.P. Calculus AB as a senior and for those seniors who might take calculus in college. This course is generally intended for students who have successfully completed Honors Algebra 1, Honors Algebra 2, and Honors Geometry. The course will be guided by the Pre-Calculus Massachusetts Mathematics Curriculum Framework and the Worcester Public Schools 11th and 12th Grade Mathematics Curriculum.

First semester topics will include: Linear Relations and Functions, Systems of Equations and Inequalities, the Nature of Graphs, Polynomial and Rational Functions, and Trigonometric Functions.

Second semester topics will include: graphs and inverse trig functions, trig identities and equations, exponential and logarithmic functions, intro to sequences and series, intro to conics, intro to calculus – limits, derivatives and integrals.

Essential Questions:

- 1. Importance of Graph Analysis?
- 2. Characteristics of Linear, Quadratic, Rational, Exponential, Logarithmic and Trigonometric functions?
- 3. How to apply mathematical models to real-world situations?
- 4. Value of graphing calculator as a tool for organizing and analyzing data sets?
- 5. Define slope as a rate of change?
- 6. Concept of a Limit? Definition of Derivative? Definition of Integral?
- 7. How can calculus be used as an alternative approach to solving common problems?

Text:

- 1. Merrill, Advanced Mathematical Concepts;
- 2. College Board, Pre Calculus Through Modeling;

District-Wide Reading Skills Across the Curriculum:

- 1. Preview (survey) – note major elements such as organization, vocabulary, summary, and graphics.
- 2. Ask Questions - question the text, the author and self.
- 3. Activate Prior Knowledge (schema) – use what is already known to enhance understanding of what is new in the text.
- 4. Make Connections - link text to self, text to world and text to text.
- 5. Visualize - use sensory images to create a mental picture of the scene, story, situation, or process and involve oneself in it.
- 6. Draw Inferences - go beyond the literal information in the text including predicting, figurative meaning and thematic understanding.
- 7. Distinguish Key Ideas - recognize main idea and key concepts.
- 8. Use Fix-Up Strategies - monitor own understanding by pausing to think, re-read, consider what makes sense, restate in own words.

Contextual Vocabulary (includes but is not limited to):

Rate of change * Multivariable systems * Limit * Derivative * Integral* Functions
*Composition of Functions*Maximum/Minimum Value*Point of Inflection*Horizontal & Vertical Asymptotes
*Zeros of a Function*Restrictions on the Domain
The course description indicates that students typically take honors level algebra 1, advanced algebra, and geometry prior to taking pre-calculus, but students that did not take all three classes at the honors are not necessarily unprepared to take this course. This course is highly recommended for students that wish to take calculus in the future (which is virtually all college-intending students). The course curriculum is modeled after the state Frameworks and the Worcester Public Schools Mathematics curriculum. Topics included in the course are listed in the syllabus. Just like the advanced algebra class, the pre-calculus begins with a review of critical topics, such as functions, composite functions, inverse functions, domain and range, and graphing.
There are seven essential questions that students should be able to answer by the end of this course, compared to just one essential question. For pre-calculus, the essential question from advanced algebra lives on: “How to apply mathematics models to real-world situations?” The 7th essential question is similar, except the question is specifically about calculus, not mathematics in general. By the end of the course, students should master a wide variety of functions and their graphs, and should be able to compute and approximate simple derivatives, integrals, and limits. Question 4 is an important one because it demands that students understand technology’s proper role in the classroom (as a tool and not as a brain). I really enjoy it when students have a surprised look on their faces that says, “That can’t be right! The calculator is wrong,” because then I know for sure that they are not taking the calculators’ answers for granted.

The vocabulary section is a bit more extensive than the corresponding advanced algebra section, but it is still lacking many important terms. At least there is a warning on this syllabus that says that the vocabulary “includes but is not limited to…”!

Textbook for Pre-Calculus

The course syllabus indicates that there are two textbooks for pre-calculus, and that is correct. All pre-calculus teachers were suppose to be issued new textbooks, but because of some mishap, only about half of the textbooks were actually delivered. So, the teachers decided that some of them will use the older textbook for their classes, and the rest of the teachers will continue using the older textbook. Mrs. O’Leary was one of the teachers that were stuck with the old text, Advanced Mathematical Concepts. I will discuss the merits and demerits of this textbook only, since I have only seen the cover of the newer textbook.

The authors of Advanced Mathematical Concepts are educators Berchie W. Gordon, Lee E. Yunker, F. Joe Crosswhite, and Glen D. Vannatta. None of these people obtained a Ph.D. in mathematics, and the quality of the textbook shows this. There are many errors in the book, and the book has a rather soft presentation of mathematics. The text has are far more words than math, and the word problems are painfully outdated and impractical. (What city dweller needs a linear programming problem on maximizing a farm’s profit?) The textbook has sections that “teach” students how to use their graphing calculators, that is, if they have an archaic TI-82! The textbook also “teaches” students how to use their computers to solve problems using BASIC. In this day and age, students could just look up applets or search on Wolfram Alpha (i.e. no programming required). What makes this textbook worse is the nonsensical ordering of topics. The sections that show students how to use graphing calculators are before the applicable sections, so if a teacher instructs by the book, students know how to abuse their calculators from the get-go. Good thing that no one actually has a TI-82 anymore. In my opinion, students should learn how to use calculators after they have had some manual exposure of the topic. What also “grinds my gears” is the fact that the textbook starts calculus topics in the middle of the third chapter, even before students learn about polynomial functions. The book does not mention calculus again until chapter 17. I do not know why the authors ordered the sections as they did, but I am not impressed.

Despite all of these flaws, the textbook does have some merit. Unlike the advanced algebra textbook, in which the examples show students how to do everything, the examples in this textbook does not. Rather, the examples show students how to do the most critical steps, but leave out simple steps that the students should be able to do on their own. Here is an example from a section on maximizing functions restricted on a convex polygonal set:
This example does not explain how to graph the system of inequalities, because that a skill that students should have already mastered. A graph is provided as a visual aid for students to compare with. Furthermore, students should be able to identify the vertices, so they are simply listed and not explained. There is a computational explanation on how to evaluate the function on each vertex and how to identify the maximum, because this the new skill that the example is presenting. This example is a good balance between explaining and telling.

Notice that in the example above, there is a graphical aid, which is another advantage of this book. Adolescence tend to be visual learners (thanks to video games, television, and computers), and so any visual representation of a mathematical concept is good, and this textbook has an abundance of them.

A third merit that this textbook deserves is its weight and size. Compared to the advanced algebra textbook, this one is as light as a feather. It amazes me that that a hardcover textbook that has more than one thousand pages can be so light. However, since there is an abundance of these textbooks available in the classroom (since some teachers are using the new textbooks) there is no reason for students to bring the book back and forth between home and school. Therefore, students are allowed to keep their books at home, although a few insist on bringing their books to class every day (perhaps to do homework at the last minute).

**Course Outline for Pre-Calculus (Short Form)**

The following is a course outline pre-calculus. Just like the advanced algebra outline, it shows which sections from the textbook are covered in the course. The additional topics are optional, depending on time constraints. The material in the additional topics section will be taught in calculus, and is therefore a preview AP calculus.

---

66 Advanced Mathematical Concepts, p.88
Doherty Memorial High School

Curriculum Outline For Pre-Calculus

Quarter 1

1.1 Linear Relations and Functions
1.2 Composition and Inverse Functions
1.3 Linear Functions and Inequalities
1.4 Distance and Slope
1.5 Forms of Linear Equations
1.6 Parallel and Perpendicular Lines
2.1 Solving Systems of Equations
2.2 Introduction to Matrices
2.3 Determinants & Multiplicative Inverses of Matrices
2.4 Solving Systems of Equations
2.5 Solving Systems of Inequalities
2.6 Linear Programming

Quarter 2

3.1 Symmetry
3.2 Families of Graphs
3.3 Inverse Functions & Asymptotes
3.4 Rational Functions & Asymptotes
3.5 Graphs of Inequalities
3.6 Tangent to a Curve
3.7 Graphs & Critical Points of Polynomial Functions
3.8 Continuity & End Behavior
4.1 Polynomial Functions
4.2 Quadratic Equations and Inequalities
4.3 Remainder and Factor Theorem
4.4 Rational Root Theorem
4.5 Locating Zeros of a Function
4.6 Rational Equations and Inequalities

Quarter 3

5.1 Angles and their Measure
5.2 Central Angles and Arcs
5.3 Circular Functions
5.4 Trigonometric Functions of Special Angles
5.5 Right Triangles
5.6 Law of Sines
5.7 Law of Cosines
5.8 Area of Triangles
6.1 Graphs of Trigonometric Functions
6.2 Amplitude, Period, and Phase Shift
6.3 Graphing Trigonometric Functions
6.4 Inverse Trigonometric Functions
6.5 Principal Values of Inverse Trigonometric Functions

Quarter 4

7.1 Basic Trigonometric Identities
7.2 Verifying Trigonometric Identities
7.3 Sum and Difference Identities
7.4 Double-Angle and Half-Angle Identities
7.5 Solving Trigonometric Equations
11.1 Rational Exponents
11.2 Exponential Functions
11.3 The number e
11.4 Logarithmic Functions
11.5 Common Logs
11.6 Exponential & Logarithmic Equations
11.7 Natural Logarithms

Additional Topics

12.1 Limits
12.2 Derivatives and Differentiation Techniques
12.3 Area Under a Curve
12.4 Integration
12.5 The Fundamental Theorem of Calculus
12.6 The Binomial Theorem
Course Outline for Pre-Calculus (Long Form)

Below is a detailed course outline for pre-calculus that is very similar to the one for advanced algebra. The skills that students should learn were originally mapped to the 2000 state Frameworks, but I have mapped most of them to the current 2011 state Frameworks, marked in red. This curriculum outline has been in use since the 2008-2009 school year. Note that the “Assessment” column is filled out. This column was completed and used by former mathematics department head of Doherty, Victoria DeSimone, but Mrs. O’Leary does not use the same assessments as she did. I did not delete that column because it is an example of what a teacher may put there.

Although most of the 2000 state standards were mapped to 2010 standards, not all of them could be accurately be mapped. For example, a 2000 state standard, PC.P.1 states that students should be able to “use mathematical induction to prove theorems and verify summation formulas”. However, there is no 2011 state standard that states that students need to know mathematical induction. The 2011 standard A-APR.5 is about the Binomial Theorem and its proof. It can be proved by induction or by a combinatorial proof. That is the only 2011 standard that mentions proof by induction, and even that standard does not make a proof by induction mandatory. Additionally, that standard is marked by a (+), indicating that it is not compulsory. So, not all 2000 state standards can be mapped to 2011 state standards. (Standard PC.P.1 can be mapped to the additional standard 3 of the NCTM curriculum, however.)

As a reference, the 2010 state frameworks for pre-calculus are in Appendix B and the 2000 state frameworks for pre-calculus are in Appendix D. The NCTM curriculum is in Appendix E.
## Curriculum Mapping Template

**Name:** Victoria DeSimone  
**Subject Area:** Pre-Calculus Honors  
**Grade:** 11 & 12  
**Year:** 2008-09

## Full Year

<table>
<thead>
<tr>
<th>Time Frame</th>
<th>Content</th>
<th>Skills - Students will:</th>
<th>Assessment</th>
<th>Standards</th>
</tr>
</thead>
</table>
| September  | **Linear Functions**  
- Table, Graph & Equation relationship  
- Function Notation  
- Composition & Inverses  
- Construction of Linear Model  |  
- Read/Write/Apply function notation in real-world setting  
- Apply Graph-Equation-Table to Construction/Interpretation of a Linear model  
- Define and Interpret Domain and Range (and restrictions) for linear models  
- Interpret slope as rate of change for linear model  
- Identify & Interpret meaning of zeros and y-intercepts for linear models (M)  
- Identify & Construct Inverse relations and functions (M)  
- Composition of relations/functions (M)  
- Calculate/Interpret Slope as rate of change (M)  
- Construct/Interpret Line of best fit (M)  
- Interpolate and extrapolate data for given application (M)  |  
- Concept Map – table/graph/equation relationship  
- Projects - Create & Analyze Linear Models:  
  ~ Stack of Cups  
  ~ Mile Run  
- TI-83 GC linear modeling applications  
- Unit Assessment Vocabulary & Function Notation Assessment  |  
- 12.P.10 (9-12.A.REI.11)  
- 12.P.5 (9-12.F.BF.1b, 9-12.F.BF.4)  
- PC.D.2 |
**Name:** Victoria DeSimone  
**Subject Area:** Pre-Calculus Honors  
**Grade:** 11 & 12  
**Year:** 2008-09

## Full Year

<table>
<thead>
<tr>
<th>Time Frame</th>
<th>Content</th>
<th>Skills - Students will:</th>
<th>Assessment</th>
<th>Standards</th>
</tr>
</thead>
</table>
| September & October | **Systems of Equations**  
- Matrix Algebra  
- Linear Programming | - Operations with Matrices including addition, subtraction, scalar multiplication, multiplication  
- Manipulate Matrices – 2x2, 3x3 & higher - determinant, transpose, inverse, identity  
- Set up Matrix Equations for 3x3 or higher  
- Use TI-83 for solving systems 3x3 or higher (matrices/row reduction applications)  
- Demonstrate equivalency of methods to solve system – ~graphing  
~ elimination  
~ substitution  
~ matrix algebra  
- Use Linear Programming for solving systems of 2 or more inequalities  
- Prove/Construct Parallel & Perpendicular Lines (M)  
- Identify type of system – Consistent, Inconsistent, | - Project – Analysis of a System of Equations: ~ CD Music Lab  
- Project: Solving Real world application of system of equations with Linear Programming Application  
- TI-83 for application of system of equation using matrices  
- Project: Equivalency of methods for solving a system of equations  
- Unit Assessment  
- 12.P.10 (9-12.A.REI.11) |
Full Year

<table>
<thead>
<tr>
<th>Time Frame</th>
<th>Content</th>
<th>Skills - Students will:</th>
<th>Assessment</th>
<th>Standards</th>
</tr>
</thead>
</table>
| October & November | **Graph Analysis**  
- Parent Graph Characteristics  
- Translations and scale changes for Parent Graphs  
- Definition of Polynomial Function  
- Complex Number System  
- End Behavior  
- Rational Root Theorem  
- Fundamental Theorem of Algebra  
- Identify/Describe/Compare Parent graphs: ~Linear (M) ~Quadratic (M) ~Cubic ~Absolute Value (M) ~Square root ~Cube root ~Rational  
- Identify and use Symmetry to determine odd/even function  
- Determine & Analyze End behavior  
- Operations with Complex Number System – complex conjugates  
- Calculate Real Roots of a Polynomial using | Project -Analysis of functions: ~ Fences Lab ~ Summer Camp Lab ~ Rational Functions Lab  
- Vocabulary Assessment  
- Assessment - translation of parent graph by equation, graph and verbal description  
- Complete 4-Square organization chart for graph analysis of any polynomial function up to degree 4  
- Assessment: Identify graph family and sketch |  
12.P.13 (9-12.F.BF.4)  
PC.P.2  
PC.P.9 (9-12.F.IF.6, 9-12.S.ID.7)  
12.P.12 (9-12.F.BF.Ib)  
12.N.1 (9-12.N.CN.1, 9-12.N.CN.2) |
Name: Victoria DeSimone  Subject Area: Pre-Calculus Honors  Grade: 11 & 12  Year: 2008-09

Full Year

<table>
<thead>
<tr>
<th>Time Frame</th>
<th>Content</th>
</tr>
</thead>
</table>
|            | • Descartes Rule of Signs  
            | • Remainder Theorem  
            | • Odd/Even Functions  
            | • Critical Points  
            | • Tangent to a curve  
            | • 1st & 2nd Derivatives  
            | • Asymptotes (Horizontal, Vertical)  
            | • Holes  
            | • Continuity and Discontinuity of Functions |

<table>
<thead>
<tr>
<th>Skills - Students will:</th>
</tr>
</thead>
</table>
| factoring and synthetic division --Review of factoring techniques as needed to find roots  
| • Construct/Interpret Tangent to a curve  
| • Interpret meaning of tangent line with slope of 0  
| • Locate critical points using derivatives (1st & 2nd)  
| • Write and describe translations of parent graphs  
| • Locate Holes/asymptotes  
| • Explain Continuity and discontinuity and analyze functions as continuous or discontinuous on an interval |

<table>
<thead>
<tr>
<th>Assessment</th>
</tr>
</thead>
</table>
| accurate graph for given data  
| • Use theorems/skills to algebraically determine roots  
| • Assessment: Apply 1st & 2nd derivative concept to real-world problems in business and science |

<p>| Standards |</p>
<table>
<thead>
<tr>
<th>Time Frame</th>
<th>Content</th>
<th>Skills - Students will:</th>
<th>Assessment</th>
<th>Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>December</td>
<td><strong>Trigonometric Functions</strong></td>
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<td></td>
<td>• 6 Trigonometric Functions and geometric relationship</td>
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<td></td>
<td>• Reference &amp; Co-terminal Angles</td>
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<tr>
<td></td>
<td>• Symmetry and Geometry of the Unit Circle</td>
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<td>• Law of Sines</td>
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<td>• Law of Cosines</td>
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<td></td>
<td>• Arc Length, Sector Area &amp; Angular velocity</td>
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<tr>
<td></td>
<td>• Area formulas for a triangle:</td>
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<tr>
<td></td>
<td>• Convert radians/degrees (M)</td>
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<td></td>
<td>• Construct unit circle using special angles (M)</td>
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</tr>
<tr>
<td></td>
<td>• Use reference and co-terminal angles to solve problems (M)</td>
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<td>• Review and apply concepts of right triangle trig (angle and trig value relationship) (M)</td>
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<td>• Apply Law of Cosines and Law of Sines to real-world situations (vectors) (M)</td>
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<td>• Develop and apply formulas for arc length and sector area using geometry</td>
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<td>• Demonstrate the equivalency of methods for finding area of</td>
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<td>• Construction of unit circle using reference angles, symmetry and trig values</td>
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<td>• Project – Law of Sines and Law of Cosines: Discus Throw Lab</td>
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<td>• Assessment Triangle Area given varying information</td>
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<td>• Unit assessment</td>
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<td>• TI-83 for trig calculations – finding trig or angle value</td>
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<td>• PC.M.1 (9-12.F.TF.1)</td>
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<td>• PC.M.2 (9-12.A.SSE.1)</td>
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<td>• PC.P.3 (9-12.F-IF.10, 9-12.F.TF.8, 9-12.G.SRT.6-8)</td>
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<td>• PC.G.2 (9-12.N.VM.1-2)</td>
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**Full Year**

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</table>
|            | ~Heron’s ~.5absinC | a triangle – Heron’s, .5absinc, .5bh  
- Calculating the angle from a given trig function  
- Unit conversions for angular velocity |           |           |
| January    | **Graphs and Inverses of Trigonometric Functions**  
- Graphs of 6 trig functions:  
  ~Amplitude  
  ~Period  
  ~Phase Shift  
  ~Vertical Shift  
- Inverses of Trig Functions  
- Real-world Applications including Harmonic motion |  
- Create graphs of trig functions from values on unit circle  
- Describe parent graphs for sin x, cos x, tan x  
- Describe effects of parameter changes - amplitude, period, phase shift, vertical shift – to parent graphs  
- Construct tables/graph/equation for translation of above parent graph  
- Read/Write/Interpret inverse trig functions and calculate values |  
- Project – Analysis of Sine Wave:  
- Construct graphs of 6 trig functions  
- Assessment: Describe translations to parent graphs given table/graph or equation  
- Calculate values of inverse trig functions  
- Interpret or create models for real-life applications |  
- PC.P.5 (9-12.F.TF.9)  
- PC.P.6 (9-12.F.BF.3)  
- PC.P.4 (9-12.F.TF.9) |
### Full Year

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<td>January &amp; February</td>
<td><strong>Trigonometric Identities and Equations</strong></td>
<td>• Discover and apply reciprocal id’s, quotient id’s, and Pythagorean Trig ID’s</td>
<td>• Create trig formula sheet</td>
<td>• PC.P.5 (9-12.F.TF.9)</td>
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<td></td>
<td>• Pythagorean Identities</td>
<td>• Analyze/apply sum and difference id’s</td>
<td>• Assessment: Prove Geometrically the Pythagorean/Reciprocal/Quotient Identities</td>
<td>• PC.P.6 (9-12.F.BF.3)</td>
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<td></td>
<td>• Reciprocal Identities</td>
<td>• Analyze/apply half-angle, double-angle id’s</td>
<td>• Assessment: Verify Trig Identities</td>
<td>• PC.P.4 (9-12.F.TF.9)</td>
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<td>• Quotient Identities</td>
<td>• Simplify trig expressions</td>
<td>• Assessment: Solve Trig Equations</td>
<td>• 12.G.2 (9-12.N.VM.1-2)</td>
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<td>• Angle Sum/Difference Identities</td>
<td>• Verify Trig identities</td>
<td>• Assessment: Apply id’s to problem solving</td>
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<td></td>
<td>• Double Angle Identities</td>
<td>• Solve trig equations</td>
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<td>• Half Angle Identities</td>
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*Standards:

- PC.P.5 (9-12.F.TF.9)
- PC.P.6 (9-12.F.BF.3)
- PC.P.4 (9-12.F.TF.9)
- 12.G.2 (9-12.N.VM.1-2)
## Full Year

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<tr>
<td></td>
<td></td>
<td>• Graph exponential functions &amp; inequalities (M)</td>
<td>• Project: Saving to buy a car</td>
<td>All.P.4 (9-12.F.IF.7, 9-12.F.IF.8, 9-12.F.BF.1, 9-12.F.LE.4, 9-12.F.TF.5)</td>
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<td>• Define the number “e” and apply to problem solving</td>
<td>• Application Problems using logs &amp; exponential functions in science, economics, nature, population</td>
<td>12.P.10 (9-12.A.REI.11)</td>
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<td>• Apply exponential functions to real-world situations involving finance, science, economics</td>
<td>• Unit Assessment</td>
<td>12.P.11 (9-12.F.IF.5, 9-12.F.IF.7)</td>
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<td>• Evaluate expressions with logarithms</td>
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<td>12.P.13 (9-12.F.BF.3)</td>
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<td>• Solve equations &amp; inequalities with logarithms and exponential functions</td>
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<td>• Explore and analyze the relationship between logarithmic and exponential functions</td>
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<td>• Compare/Contrast natural and common logs</td>
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<td>Time Frame</td>
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<td>Skills - Students will:</td>
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<tr>
<td>April &amp; May</td>
<td>Limits, Derivatives &amp; Integrals</td>
<td>• Define “limit”&lt;br&gt;• Use limit theorems to evaluate the limit of a polynomial function&lt;br&gt;• Use differentiation techniques to find the derivative of a function&lt;br&gt;• Review definition of derivative&lt;br&gt;• Apply derivative to real-world situations including business, science&lt;br&gt;• Explore Area Under a Curve as method of integration (Riemann Sums)&lt;br&gt;• Use integration techniques to find indefinite integrals&lt;br&gt;• Calculate definite integrals using FTC</td>
<td>• Assessment“ Graph f(x), f'(x) and f''(x) and explain relationship between the 3 functions&lt;br&gt;• Equivalency of methods calculating area under a curve using Riemann Sums and Integration&lt;br&gt;• Explain Fundamental Theorem of Calculus&lt;br&gt;• Derivatives to find marginal cost and revenue functions&lt;br&gt;• Using the graphing calculator for integrating and taking derivatives</td>
<td>• PC.P.9 (9-12.F.IF.6)&lt;br&gt;• PREP FOR AP CALC</td>
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<td>Time Frame</td>
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<td>Skills - Students will:</td>
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<td>May &amp; June</td>
<td><strong>Sequences &amp; Series</strong></td>
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<td>• Arithmetic Sequence</td>
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<td>• Geometric Series</td>
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<td>• Convergent v. Divergent Series</td>
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<td>• Sigma and Factorial Notation</td>
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<td>• Binomial Theorem</td>
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<td>• Calculate the nth term and arithmetic mean of an arithmetic sequence</td>
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<td>• Calculate the sum of n terms in an arithmetic series</td>
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<td>• Construct an arithmetic sequence from given information</td>
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<td>• Calculate the nth term and geometric mean of a geometric sequence</td>
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<td>• Calculate the sum of n terms of a geometric series</td>
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<td>• Find the limit of the terms of an infinite sequence</td>
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<td>• Find the sum of an infinite geometric series</td>
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<td>• Determine whether a series is convergent or divergent</td>
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<td>• Use sigma and factorial notation</td>
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<td>• Use the binomial theorem to expand a binomial</td>
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<td>• Project – Construction &amp; Analysis of a Geometric Sequence: Quilt Lab</td>
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<td>• Compare and contrast arithmetic and geometric sequences and series</td>
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<td>• Lab: Exploring series and determining if convergent or divergent</td>
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<td>• Project Exploration: Pascal’s Triangle, Fibonacci Sequence, and Fractals</td>
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<td>• PC.P.1</td>
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<td>• 12.P.3 (9-12.A.APR.5)</td>
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<td>• 12.P.1 (9-12.A.SSE.4)</td>
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Prerequisite Skills for Pre-Calculus

Just like the advanced algebra course, most of the first quarter of pre-calculus is review. The review is on functions, composite functions, inverse functions, domain and range, graphs of functions, zeros, y-intercepts, and x-intercepts. Therefore, the prerequisite skills to succeed in pre-calculus are relatively minimal. Students should nevertheless have these minimum skills:

- Arithmetic, including adding, subtracting, multiplying, and dividing real numbers (including fractions, decimals, and percentages).
- Understand and apply the order of operations, and use the distributive property.
- Evaluate and simply algebraic and arithmetic expressions.
- Solving single variable first order and second order polynomial equations.
- Graph points on a Cartesian plane, and identify the coordinates of a point on a Cartesian plane.
- Manipulate equations to isolate variables.
- Know the absolute value of a real number, and find solutions of absolute value equations.
- Compute the square of a real number, and approximate the square root of a real number.
- Identify basic properties of shapes such as a triangles, squares, circles, and parallelograms. For example, know that a circle has area πr² and diameter 2π.
- Rules of exponents and square roots.
- Factoring and graphing quadratics.
- Model a linear phenomena with a linear equation.
- Convert an equation of a line to point-slope form, standard form, and slope-intercept form.
- Graph an equation of a line.
- Identify the slope, y-intercept, and x-intercept of a line.
- Simplifying square roots.
- Understand and apply the Pythagorean Theorem.

Additionally, students will benefit having these skills (many of which will be reviewed):

- Find the inverse of a function, graphically and algebraically.
- Identify the domain, range, zeros, and y-intercepts of a function.
- Compute composite functions, and use composite functions to solve real world problems.
- Model using inequalities with one and two variables.
- Graph and solve inequalities in one and two dimensions.
- Graph cubic, exponential, square root, trigonometric, and absolute value functions.
- Understand sine, cosine, tangent, and trigonometric identities.
- Solving a system of equations using elimination and substitution.

Revisiting MCAS Question #20

Before concluding this chapter, I had to point out something that struck my attention. In Chapter 1, the 2011 mathematics open response MCAS question that the students at Doherty got incorrect involved reflecting a triangle over the y-axis and rotating it 90°. If you look at the algebra 1 and geometry course outlines in Appendices F and G, you will not find anything about translations, rotations, or reflections. In fact, only pre-calculus has translations in its curriculum. However, by the time a student takes pre-calculus (if the student even does), he or she would have already taken the MCAS exam. In addition, only advanced students that have passed the MCAS would even take pre-calculus. Therefore, it is my opinion that the geometry curriculum should be modified to fulfill the standards regarding rotations, translations, and reflections in order for students to do better on the MCAS.
According to 2011 data from the National Center for Education Statistics, 8th grade mathematics students perform higher than students from all other forty-nine states and Washington, D.C. In order to assess the performance of students, the NAEP test was used. The maximum score on the exam is a 500, and the lowest score possible is a 0. 8th grade Massachusetts students scored an average of 299 points, while the national average was 283 points. Of all races in the state, Asians preformed the best with an average of 321 points and Hispanics preformed the worst at 273 points. There was a negligible performance gap between males and females. (Males had an average score of 299, compared to 298 for females.) In 2009, the average 8th grade Massachusetts student scored 299 on the mathematics NAEP, so there was no improvement statewide. In 2009, the average 8th grade U.S. student scored a 282, so there was negligible improvement nationwide.

Although Massachusetts students perform better in mathematics than students from all the other states, Massachusetts still lags behind students from numerous Asian and European countries. In 2007, 40% of grade 4 Hong Kong students, for example, were qualified for the advanced TIMSS level, compared to only 22% of Massachusetts students. In other 2011 study, comparing NAEP test results with other international test reveals that Massachusetts was the only state in which the majority of 10th grade students were proficient in mathematics. However, even Massachusetts compares only to the entire nations of Canada, Japan, and Switzerland. Shanghai students were the top performing in the world, with 75% of its students proficient in mathematics.

Conclusion

The curriculum for Worcester Public Schools is primarily modeled by the state Frameworks, but the Reauthorization of Elementary and Secondary Initiatives and the NCTM curriculum also influenced its making. Although the current curriculum is still modeled by the 2000 state Frameworks, the curriculum must be modified in the very near future to accompany the current 2011 state Frameworks, which was updated due to the integration of the Common Core Standards. The state requires the curriculum to be updated no later than the 2013-2014 school year. The curriculum could use a major update, and so this intervention is welcome. Furthermore, President Barack Obama is currently in the process of reauthorizing the No Child Left Behind Act, which ought to strengthen the educational system even further. The following is the opening of the Letter from the President regarding the Reauthorization of the Elementary and Secondary Education Act:

> Every child in America deserves a world-class education.

> Today, more than ever, a world-class education is a prerequisite for success. America was once the best educated nation in the world. A generation ago, we led all nations in college completion, but today, 10 countries have passed us. It is not that their students are smarter than ours. It is that these countries are being smarter about how to educate their students. And the countries that out-educate us today will out-compete us tomorrow.

> We must do better. Together, we must achieve a new goal, that by 2020, the United States will once again lead the world in college completion. We must raise the expectations for our students, for our schools, and for ourselves—this must be a national priority. We must ensure that every student graduates from high school well prepared for college and a career.

> -President Barack Obama

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68 [http://www.air.org/expertise/index/index.cfm?fa=viewContent&content_id=49](http://www.air.org/expertise/index/index.cfm?fa=viewContent&content_id=49)
69 [http://www.doe.mass.edu/frameworks/current.html](http://www.doe.mass.edu/frameworks/current.html)
70 [http://www2.ed.gov/policy/elsec/leg/blueprint/publication_pg2.html#part2](http://www2.ed.gov/policy/elsec/leg/blueprint/publication_pg2.html#part2)
Chapter 3: Course Materials

Introduction

In Chapter 2, the quality of the textbooks for Pre-Calculus and Advanced Algebra was discussed since the curriculum was heavily structured on them. In this chapter, all other course materials will be discussed and the methodology behind them. Fortunately, I only taught students at the honors level, and many of them take advanced placement classes in other subjects. All the students that I taught were in general independent, self-motivated, and well-behaved. Therefore, I was able to teach in a similar manner for all of my classes. The structure of a typical class, the amount of homework given and the level of expectations that I had was relatively consistent in all of my classes. Of course, not all of the students were the same, and individuals needed to be addressed. Chapter 4 will discuss how I chose to account for the diversity of the classroom population. Much of the course materials that I have developed were motivated by what I have observed at Doherty before I started teaching, as well as what I have learned being a student at Lowell High School.

Do Now

At the beginning of each class, I have the students work on one to four problems written on the board. The purpose of this exercise is multifold. First, as the students work on the problems, it allows the teacher to take attendance and to have a few extra minutes to prepare for the next class. At a school like Doherty in which there are only four minutes between classes, a few extra minutes can really make a difference to prepare between classes. Secondly, students will occasionally be late to class, and so it is better to have missed the do now than to miss a lesson. Thirdly, students have something to work on immediately when they enter the classroom, instead of fiddling their thumbs waiting for the teacher to teach.

The do now is usually related to the current subject being taught. On occasion, I will give students a do now that is on a previous topic or I will give them a math SAT question. After about five minutes, I always go over the do now. Depending on the difficulty of the problems, I will either go over them with the students or have students write the solution on the board and explain their answers.

Do nows are something that I learned from observing my IQP mentor. She implemented do nows on a daily basis and her classes are more productive than other classes that I have observed. When I was a student at Lowell High School, do nows were never implemented. I always waited for the teacher to tell my pairs and me what to do. From what I have observed at Doherty really brought to my attention how much time was wasted without the do now. When I started teaching, I had to remind the students constantly that they should be working on the do now, because they would not do anything unless I told them to. Over time however, do nows became routine and I did not have to remind students to do them.

I did not grade do nows in a formal fashion. Instead, I went around the room and informally checked whether or not students were working on them, and if so, how they approached the problem and if they got the correct answer. Occasionally, students will have more than one way approaching a problem. In that situation, I discussed both methods with them. (Is one method quicker than the other? Is one way more intuitive than the other?) 10% of the students’ grades is participation, so if I know that a particular student does not do the do now, his or her grade will suffer. When a student is not working on the do now, I remind them of the consequences it will have on their grade. However, the vast majority of the students attempted do nows on a daily basis.

Since I did not grade do nows for correctness, the purpose of them was not as an assessment of mastery, but as an assessment of the way the students approached problems. For example, do students attempt to draw a geometric interpretation of the problem, or do they use a strictly algebraic approach? Do they write down key information first, and then attempt the problem, or do they use information as they need it? Do they write down sentences or words, or do they use math only. Form my observation, most students do not draw pictures to model problem, they prefer a strictly algebraic approach, and they use information only as they need it. I try to encourage students to try different ways of approaching the same problem, but I do not force them to.
Example Do Now

The following is a typical do now that was for the advanced algebra class. Solving systems of (in)equalities was a skill that they have recently developed.

Solve the following system of equations graphically:

1. \(\begin{align*}
   & y = 3x - 9 \\
   & y = x + 1 \\
   & \text{(5,6)}
\end{align*}\)

2. \(\begin{align*}
   & y = |x - 1| \\
   & y = |x + 1| \\
   & \text{(0,1)}
\end{align*}\)

Of course, I did not show the class the solutions to the problems until they were done on the board. I provided the solutions only for my own reference as I checked students’ work. For this particular do now, I passed out small pieces of graph paper; otherwise, the answers may be varied. Even then, some students did not get the correct solutions because they did not graph very accurately. This was a result that I expected, and I wanted it to happen, because it lead into the need of that day’s lesson on solving a system of equalities algebraically rather than graphically. Using a graphical approach is often just an estimate. The students were lucky the solutions were whole numbers. However, if the solutions where irrational numbers, then graphing the equations would never yield the exact solution. An algebraic method, on the other hand, would.

Homework

When I started teaching, I assigned homework primarily from the textbooks. However, as I continued teaching, I discovered that the problems in the textbooks did not always correlate with what I taught, especially for the pre-calculus class. For example, when I taught the class some of the applications of the derivative, the book did not include very many applications of the derivative. The problems in the book were all about taking the derivative of a polynomial function. In that situation, I had to make my own world problems. On occasion, I would have students make up a word problem and solve it for homework, and have students solve each other’s problems the next day. This has been very fun for some students, but other students seemed to be frustrated because they could not come up with a good problem.

Normally, I reviewed homework problems before I started the day’s lesson. I would either have students do the more difficult problems on the board and explain their answers, or if a problem was particularly challenging, I would simply explain how it is done. However, if no one had any questions on the homework, I may decide not review the assignment. I occasionally collected the homework assignments and graded them. I always collected homework before reviewing it. Otherwise, students would copy what is on the board instead of keeping what they had, and then homework would no longer be a valid assessment of how much students learned. I have always told the students which homework assignments were to be collected. Otherwise, some students would hand in scribbled or incomplete work. The disadvantage of telling students in advanced which homework assignments were to be collected was that some students did not bother doing homework assignments unless they knew they would be collected. My advanced algebra students took particular advantage of this fault. To counter this, I decided to check off random homework assignments while students worked their do now. Therefore, some homework assignments were handed in and graded, while other homework assignments were simply checked for completeness. I could not collect all homework assignments and grade them because that would be way too much work for me. Occasionally, I would even have students grade each other’s homework assignments. However, this was time-consuming so I only did this a few times. It also violated the students’ privacy because students would know each other’s grades.

Homework was a mixture of dealing with algebraic expressions, graphing and solving word problems. The purpose of the homework assignments was to reinforce the concepts that the students have learned in class. The graded homework assignments were also a form of assessment of what exactly the students knew what to do and how they done what they did. I did not require students to explain their answers or to write sentences. In fact, a student could
not have done well on homework assignments unless he or she understood what was going on, so requiring students to explain their work would be redundant. If I taught a class with a high proportion of ESL students, I may have required them to write sentences. However, I felt that it would have been an unnecessary burden to have honors students explain what they have done in words. When I picked problems from the textbooks, I chose mostly even-numbered problems, because the odd-numbered ones had answers in the back of the book. I did not avoid assigning odd-numbered problems, however, because I believe that it is good for students to have some problems to check their answers with. Also, if I did not assign odd-numbered problems, very few students would probably bother looking at them. I typically allow students to use calculators on homework and I allow collaboration on homework. Even if I did forbid these things, some students may ignore such restrictions anyway. (I know that when I was in high school, I used calculators when I was not supposed to.)

Homework assignments were graded with a 0, ✓, ✓, and ✓+. A 0 was given if a student did not do the assignment at all. If any serious attempt was made to complete the homework assignment, and if the majority of homework questions were done incorrectly or the homework was extremely messy, a ✓ was earned. Even if every question was done wrong, but an attempt was made, a ✓ was given. If at least half of the homework questions were done correctly, but if serious flaws were still present, then a ✓ was earned. If the homework was done completely and satisfactorily, a ✓+ was given. In order to get a ✓+, a student must have attempted all the problems. A student does not have to get all homework questions correct to get a ✓+; however the majority of the questions must be done correctly. If a homework assignment was particularly messy, the student would not get a ✓+. Late homework assignments would automatically be given a maximum of a ✓, unless the student was absent. Grades of 0, ✓, ✓, and ✓+, correlate with numeric grades of 0, 33, 67, and 100 respectively. Although this grading scheme may seem a bit harsh, the large majority of students did very well on homework assignments. Also, I have learned that having high expectations for my students resulted in high quality work and slackers quickly learned to increase the quality and completeness of their homework assignments.

When I first started teaching, the quality of the homework assignments was poor. Over time, however, I have seen great improvement in students’ homework. The grading system that I implemented was not the only motivator students had to produce high quality work. When a student does not get a ✓+, I always indicate on their assignments why they did not receive the grade, whether it be a skipped problem, messiness, or serious algebraic mistakes. That way, students always know how to improve. When students do well on homework assignments, I always write a positive remark and I draw a star or a smiley on their paper. Something as simple as a positive comment can make a student proud of their work and continue to produce nice work. Students that continuously do poorly on homework would eventually have to face me or Mrs. O’Leary. We would ask those students why they are not completing homework assignments satisfactorily. Usually, students explain that they have no time or that they have other priorities. However, I also know that these students play video games constantly (that is all they talk about in class) or are on Facebook several hours a day, so that excuse is a very bad one. I always explain to them the importance of doing homework and give these students a warning that they need to spend more time on homework before I give them a detention. Sometimes they actually do improve, other times they do not. When they do not improve, then I give them a detention. During detention (which Mrs. O’Leary covers) students are required to do a homework assignment that they did not complete. Finally, after all of this, the students realize that it is easier just to do a good job on homework in the first place instead of staying after school and doing it, and they improve their work. When I notice improvement, I always complement the students verbally. Very few students have went through this entire disciplinary routine, but it has proved to be very effective. Unfortunately, I do not have student samples of homework, but I have seen wonderful improving in terms of neatness and completeness.

Homework assignments are included at the end of each lesson plan.

**Example Homework Assignments**

Below is a homework example that I have given the pre-calculus students when I was just beginning teaching.

Pages 89-90, #11-14, 22 (Just graph. Do not find the max or min of the objective function)
Before I gave this assignment, students have just learned how to graph a system of equations, what a convex polygon is, and how to identify the vertices of a convex polygon. As you can see, the problems in the textbook did not correlate to exactly what I was teaching. That is why I had to add the extra direction to ignore the function at the end of each problem. As I have found out, this can be very confusing for some students, especially those who rush out of the classroom exactly when the bell rings. Even though I clearly told the students to ignore the last line on problems 11 thru 14, many students were conflicted over what I told them to do and what the book told them what to do. The next day, several students asked me how to maximize and minimize those problems.

The world problem was given because I wanted students to practice applying the new mathematical concept that they leaned. I have gone over a very similar problem in class. However, the problem was rather week because of the absurd numbers involved in the problem. It is apparent that the textbook is dated because the problem states that no one can sell bicycle tires at more than $5 to remain competitive! To make matters worse, this supposed “recent college graduate” does not make more than a $3 profit for each tire.

Here is a more recent assignment, which was given after I introduced the limit definition of the derivative. I have given them examples in class so they had something to reference to as they calculate the limits.

Use the limit definition of the derivative to find the derivatives of the following functions:

1. \( y = x^2 + 5 \)
2. \( y = x^2 - 2x \)
3. \( y = 3x^2 - 8x + 5 \)
4. \( y = 2x^3 - 1 \)

This homework assignment correlated precisely to what I was teaching. The textbook was devoid of any exercise that had students evaluate limits. The textbook also had several functions that were not polynomials that would make calculating the limit more difficult and beyond the scope of the lesson. Every student clearly knew what to do, but not all of them knew how to. Problem 4 was particularly challenging because students had to expand a third degree expression. I always tried to include such challenging problems in homework so that the more advanced students in the class have something to ponder about also.

**Instruction and Lesson Plans**

The topics that I have covered were approximately those that are outlined in the curriculum. Although I do not entirely agree with the ordering of the topics in the course outlines, I decided to teach them in order. I did this because I only taught for a few months, and when I left, the teacher that normally teaches the classes that I taught
had to resume teaching. The teacher instructs three pre-calculus classes and two advanced algebra classes, and I taught two pre-calculus classes and one advanced algebra class. The regular teacher chose to teach courses in the order in the course outlines. If I decided to teach out of order then, the pre-calculus and advanced algebra classes would have been out of sync when I left, making the situation more difficult for the regular teacher. When I left Doherty, the classes that I taught were only a few days ahead of the other pre-calculus and advanced algebra classes, making it much more manageable for the regular teacher to resume teaching all of her classes.

After going over the do now and homework, I usually instruct the class. From ID3100, I learned that there are many ways students learn. Some students are verbal learners; others are symbolic learners or kinesthetic learners. Some students are visual learners, while others learn by reading words. Many students learn by a combination of the above. In order to cater to all students, I had to acknowledge all the different ways students learn. To do this, I often accompanied an algebraic (symbolic) representation of a word problem (linguistic) with a graph (visual). I will also explain (verbal) what is going on and students produce notes (kinesthetic) on what is being taught. Of course, it is not always possible to integrate every possible learning style in every single lesson. For example, computing the determinant of a 3x3 matrix is not easily explained by words or by pictures and a symbolic approach must be given:

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

There is a trick so that no memorization is required, however even this trick requires the use of symbols. The trick is to circle a $a_1$, and to cross out its corresponding row and column. Multiply $a_1$ with the determinant of the remaining 2x2 matrix. Do the same thing for $a_2$ and $a_3$. Add the terms containing $a_1$ and $a_3$, and subtract that result with the term containing $a_2$. This is the determinant of the 3x3 matrix. Although this process can be explained linguistically, it must accompany the use of symbols.

As I teach, I always ask students if they have any questions, and if they do, I try to answer them. If it is a question that students should know the answer to, I ask the class if they can answer the student’s question. There have been occasions when I did not know what a student was asking and there were other occasions when I did not know the answer. However, I tried my best to answer questions. If I did not know the answer to a question, someone else in the classroom almost always knew the answer. I encourage students to help each other learn, so I appreciate it when a student answers another student’s question.

An occasion when I did not know that answer to a student’s question came when I was teaching how to compute determinants. For many high school students, including myself when I was one, did not really understand the use of such a thing except when it came to solving $Ax=b$, and determining if a solution of $Ax=b$ exists. Sure enough, a student asked if determinants had any other use. It the time, I did not know the answer and neither than Mrs. O’Leary. I told the student that I was sorry I did not have an answer for her. I should have found out other uses for the determinant, but I did not because I forgot to do so, and the student did not ask again. Although I did not know the answer to the student’s question then, I would be able to answer it now. In vector calculus I learned that a determinant can be used to find the area of a parallelogram and the volume of a parallelepiped. Determinants are also involved in cross products, scalar triple products, vector triple products, and the curl of a vector field. From this experience, I learned that in order to be a teacher, a person must have an education that goes beyond the high school level. Advanced subject matter in college often relies on elementary concepts learned in high school. When a teacher knows how the concepts students learn in high school connect with college material, he or she is able to show students that what they are learning is practical and useful. Showing students the usefulness of what they are learning can be a great motivator because you have given meaning and purposes to what they are learning about.

After I have taught the day’s lesson, I would have students work on problems that I have either got from the textbook or the internet or problems that I have made myself. I usually work out the problems before I have the students work on them; however there have been occasions when I did not. If I did not work out the problems...
myself, students are more likely to encounter problems that are too difficult or too easy, so I quickly learned that it is best to try problems myself to ensure they are appropriate. I allow students to work together on problems. In fact, if the problems are from the textbook, students have to work together because there are not enough textbooks for everyone. I encourage students to ask questions and to help one another. When students work in pairs, students are more likely to procrastinate. In this case, I may assign any unfinished problems as homework. Doing this has been an effective way to keep students on task.

The appendix has all the lesson plans that I made for each class that I taught. Each lesson plan contains the day’s objectives, curriculum frameworks addressed, do now, lesson, and homework assignment. Some lesson plans are for more than one day. When I started teaching, I overestimated how much time I had, so lesson plans were long enough for multiple days.

Example Lesson Plan

In this section, a lesson plan is shown as an example. All lesson plans are in the appendix. The first section of each lesson plan is the overview, which contains information about the state the students are currently in and the objectives or outcomes of that day’s lesson. The next section is a list of applicable Massachusetts Curriculum Frameworks that are addressed in the lesson. The third section contains the day’s do now problems. There is usually a section after the do-now when I review the previous homework assignment, but since the previous class was on a Friday I chose not to assign homework. (There was a massive snow storm that weekend and school was cancelled Monday and Tuesday, hence this Wednesday class was the first one in the week.) After reviewing homework, I go over the day’s lesson. Then, I usually give students some class work so that they have a chance to practice what they just learned and to ask questions. Finally, at the end of each lesson plan is the day’s homework assignment.

Pre-Calculus

Wednesday, November 2\textsuperscript{nd}

Overview:
Previously, students have learned how to find the inverse of a function algebraically and graphically. Students also know how to graph simple functions that are translations of their parent functions. Within the next few weeks, students will be introduced to the concepts of limits and derivatives. Students will learn the concept of a limit by graphing rational functions. By the end of today, students will be able to identify rational functions and to find vertical and horizontal asymptotes algebraically.

Massachusetts Curriculum Frameworks Addressed in this Lesson

F-BF 4  Find inverse functions. (Do-Now)

F-IF 7d  \textbf{Graph rational functions}, identifying zeros and \textbf{asymptotes} when suitable factorizations are available, and showing end behavior.

Do Now:

Without doing any algebra, graph the inverse of the following functions:

\[ y = x^2 \]
\[ y = 3x - 2 \]
\[ y = |x - 2| \]
Green graphs are the original functions, and red graphs are the inverses.

**Vocabulary:**

A **rational function** is a function in the form \( y = \frac{a(x)}{b(x)} \) where \( b \neq 0 \) and \( a(x) \) and \( b(x) \) are polynomials in terms of \( x \). \( y = 1/x \) is the parent graph of rational functions.

**Example:**

\[ y = \frac{3x^2}{x(x-2)} \] is a rational function because it can be written in the form \( y = \frac{a(x)}{b(x)} \).

\[ y = \frac{\sqrt{x}}{x+1} \] is not a rational function because \( \sqrt{x} \) is not a polynomial.

An **asymptote** is a line in which a graph of a rational function approaches as \( x \) or \( y \) gets very large or very small.

**Example:**

The graph of \( y = \frac{1}{x} \) has two asymptotes. One is the line \( y = 0 \), and the other is the line \( x = 0 \).
A **vertical asymptote** is a vertical line in which a graph of a rational function approaches, but does not intersect with. The line $x = a$ is a vertical asymptote of $f(x)$ if $f(x)$ approaches $\infty$ or $-\infty$ as $x$ approaches $a$ from either the left or from the right. A vertical asymptote occurs when the denominator of a rational function in terms of $x$ “tries” to be zero.

Example:

$y = \frac{2x}{x-3}$ has a vertical asymptote at $x=3$ because that is when the denominator is zero. Notice that the rational function is in terms of $x$.

![Graph showing vertical asymptote at x=3](image)

A **horizontal asymptote** is a horizontal line in which a graph of a rational function approaches. It is possible for a graph to intersect the horizontal asymptote. The line $y = b$ is a vertical asymptote of $f(x)$ if $f(x)$ approaches $b$ as $x$ approaches $\infty$ or $-\infty$. A horizontal asymptote occurs when the denominator of a rational function in terms of $y$ “tries” to be zero.

Example:

$x = \frac{1}{y+2}$ has a horizontal asymptote at $y = -2$ because that is when the denominator is zero. Notice that the rational function is in terms of $y$. 

![Graph showing horizontal asymptote at y=-2](image)
A **slant asymptote** is an asymptote that is neither horizontal nor vertical. The line $l$ is a slant asymptote of $f(x)$ if $f(x)$ approaches $l$ as $x$ approaches $\infty$ or $-\infty$. A slant asymptote occurs when the degree of the numerator is exactly one greater than the degree of the denominator.

**Example:**

$$y = \frac{x^3}{x(x-1)}$$ has a slant asymptote because the degree of the numerator, which is three, is exactly one greater than the degree of the denominator, which is two. The slant asymptote has the equation $y = x + 1$. The vertical asymptote $x = 1$ is not a slant asymptote.

You should be able to recognize when slant asymptotes occur, but I will not test you on your ability to derive the equation of such asymptotes (it involves long division of polynomials).

**Rational Functions without Asymptotes**

Not all rational functions have asymptotes. If the denominator is one, then the rational function is simply a polynomial function that is completely smooth. If you can factor the numerator and denominator and cancel out terms so that it becomes a polynomial function, then it has no asymptotes. However, the function will still have holes where the denominator could not be zero.
Example:

\[ y = \frac{x^2 + x - 6}{x + 3} = \frac{(x - 2)(x + 3)}{x + 3} = x - 2, \ x \neq -3 \]

Notice that when the \( x + 3 \) terms cancel, the restriction that \( x \neq -3 \) is added, since the denominator cannot be zero. Therefore, the graph will look like the line \( y = x - 2 \), except it has a **hole** when \( x = -3 \).

When you have a rational function, **always** try to factor the numerator and denominator and cancel out terms. If you can and you are left with a polynomial function, then the function has no asymptotes, but it may have holes. If you cannot cancel out terms, and the denominator is not one, then it does have asymptotes.

**Class work:**

**Find any asymptotes and holes.**

\[ y = \frac{x}{x - 1} \]

\[ y = \frac{x^2}{x^2 - 1} \]

\[ y = \frac{x^2}{x^2 + 1} \]

\[ y = \frac{x^2 + x - 12}{x + 4} \]

\[ y = \frac{1}{x - 2} \]
Homework:

Page 140, Guided Practice 11-14, Exercises 15-18. If a slant asymptote exists, explain why. It is not necessary to find the equation of slant asymptotes. To be collected.

The lesson plan is very colorful and orderly because I decided to project this lesson directly on the whiteboard using the ELMO (discussed momentarily).

Using Color with the White Board

The advantage of a white board is that I was able to highlight information in color. Although the school did not always deliver a fresh supply of markers in different colors, I was able buy my own to use. I usually write sentences in black and important vocabulary words in red. I used different colors for the axes of a graph and for each function. So for example, if I wanted to graph \( f(x) = x^2 \) and \( f(x) = x \) I would write the following on the board:

\[
\begin{align*}
  f(x) &= x^2 \\
  f(x) &= x
\end{align*}
\]

Drawing functions in different colors became extremely helpful when I taught transformations in pre-calculus, because I wanted to draw several functions on the same graph. Having a color coordinated scheme reduced the confusion among students because they knew which function was which. They could then understand what was being taught instead of wondering with graph is which.

The ELMO

The ELMO is not a Sesame Street character. Instead it is a modern projection device that enables me to hook up my laptop to it and project my computer screen on the board. The ELMO also has a camera so that worksheets and even a graphing calculator can be projected on the board. (Luckily, the board is white.) This device has been tremendously useful because I can show students what buttons to press on the graphing calculators, and what their graphing calculator screens should look like. Hooking up my computer to the ELMO has allowed me to show the class information that I could never have dreamed of when I was in high school. I could project 3D graphs on the board to show them that the intersection of three plans is the solution to a system of equations with three variables. I have shown the class an applet of a graphical demonstration the derivative as a limit. (Move one point closer to another and the line going through the two points becomes the tangent.) Instead of wasting valuable class time writing everything on the board, I can just project the information on it. When a student solves a problem in an interesting manner, I can ask the student to project his or her solution on the board instead of having to rewrite it on
the board. I was even tempted to show the class videos from www.khanacademy.org, but I did not. In retrospect, I think that I really should have, because that website has even helped me understand concepts from my calculus days.

The ELMO has been an incredibly helpful tool, and the amount of time saved using it was astonishing. The ELMO is not perfect however. Sometimes, the projector gets overheated and will not turn back on for several minutes. Other times it does not recognize the signal from my computer. Sometimes, the camera cannot focus correctly. White lined paper is notably blurry. By and large though, the benefits using this device far outweigh its imperfection.

Graphing Calculators

Since a great portion of the pre-calculus and advanced algebra classes involve functions and their graphs, graphing calculators are very useful tools. Some students have their own graphing calculators, but most of them do not. Fortunately, there is a classroom set enough for everyone. I usually teach students how to use graphing calculators after they learned how to do a particular task manually. For example, after I believed that the algebra students had enough practice graphing parabolas and finding their vertexes and intercepts by hand, I taught them how to graph parabolas with the calculator and how to locate critical points with the calculator. Sometimes, I give students a set of problems to do, and then I ask them to use the graphing calculators to check their answers. I encourage students to compare their own solutions with the calculator results. For example, many students have asked me what 1.2E-47 means and how that is approximately 0. I told them how the calculator expresses extremely small and extremely large numbers using scientific notation, and that the seemingly outrageous result the calculator produced is simply a small round off error. Questioning and interpreting the results given by technology is a very critical skill to have, because computers compute everything by adding binary numbers, and the end result may be much different than if the problem is solved using “human methods”. Sometimes the solutions computers generate are represented differently than but equivalently to a manual solution, like in the example above. Other times a computer result is simply wrong. An example where a graphing calculator produces a wrong result is when it graphs vertical line on a graph when in actuality there is only a vertical asymptote (another concept I have explained to the pre-calculus classes).

One occasion in which graphing calculators really helped students understand the concepts being presented was when I was teaching the pre-calculus students translations of graphs. I could teach them this concept in one of three ways. I could just tell them how changing the function changes the graphs of the functions. I could have also had students graph a bunch of graphs on graph paper and see the relationship themselves. Alternatively, I could have had the students graph a series of functions with the graphing calculator and discover the relationship that way. If I just told them the relationships, then the students are not learning how to recognize patterns and to think critically. If I have students graph on graph paper, then that would take up a huge amount of time. Also, there is a limited supply of graph paper. However, by using the graphing calculators, students were able to recognize the patterns themselves without spending a huge amount of time that it would take to graph functions by hand. Students were able to see exactly the effects of adding a constant to a function, for example, and they were able to conclude that adding a constant of a function resulted in a vertical translation up or down, depending on the sign of the constant.

Paper

One would think that a discussion on paper would be silly. However, it is easy to take for granted such trivial course materials until you are deprived of it. Unfortunately at Doherty (and at other Worcester public schools), paper is a rare resource. Teachers have told me various reasons why the school has such a low paper supply. Some say that the greedy politicians at city hall rather have a six figure salaries than to give schools enough money to afford paper. Others say that it is for environmental reasons. Whatever the reason may be, the reality is that paper is scarce and must be rationed carefully. In fact, when a teacher “wants to” (as opposed to “has to”) make copies of something, they must write down the number of copies they are making on a log sheet. If a teacher makes too many

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71 The KhanAcademy is a website that has thousands of instructional videos made by an MIT professor. There is a tremendous mathematics section covering almost every conceivable topic, and it is growing.
copies then he or she will get yelled at. To me this is absurd, but there is little I can do about the situation. In order to reduce paper consumption in my classroom I made as much use of the ELMO as possible, and printed out my lesson plans at home. One in a while, I would even project quizzes on the board. However, I have never done that for an exam because I do not like all of the head movement. Exams are the only things that have always been on paper. Graphing paper is even rarer, so I had to cut up the sheets into small pieces for the students to use and I told students to use both sides. Graphing by hand is an essential skill to have (what if no computer is in reach?), so I could not just allow students to use the calculators all the time for graphing. Fortunately, the students have an abundant supply of their own paper, so I never have to supply them paper for note taking or homework. (Many level 1 classes, however, are not so well prepared.)

Writing Utensils

I do not prefer my students to write in pen, because when students make a mistake in pen, their work becomes a mess. On exams, a pencil is required or else five points will automatically be deducted from their grade. I have not been a strict with other assignments because it is extremely rare that a student would submit work in pen in the first place. When a student does submit a homework assignment in pen, there is no penalty since I never had to make one. I simply request that they write in pencil next time and they obey. Also, students are almost always prepared for class, and I almost never have to lend a student a pencil. However, I understand if a student lost his or her pencil and needs to borrow one. (When I was in high school, I seemed to lose a pencil every day. I also found a pencil every day.)

Email

During one pre-calculus class, I had students complete a survey instead of a do-now. One of the questions was “Do you have internet access at home?” Every student answered yes, which surprised me. I decided to email students homework assignments and even my lesson plans so that students had an electronic copy of important information. For many students, this saved time and reduced frustration because they did not have to copy the homework assignment on the board. In addition, if they did not write something important in their notes, then the lesson plan that I sent them helped them identify what they were missing. Email was also helpful when students had questions at home, in which case I emailed them back before next class session. Of course, the disadvantage of emailing is time, and I had to email homework assignments in the early afternoon or else students would not have the assignment on time. Although it had been slightly challenging to communicate via email in a punctual manner, I believe that the benefits outweighed the disadvantages.

Tests

Tests are the most important tools for assessment because they should reflect students’ ultimate mastery of the subject matter. If students do well on the test, and the test correlated with the material students should have mastered, then the teacher and the students have succeeded in their educational tasks. If on the other hand, students have done poorly on a test, then that is an indication that either the test was not an accurate form of assessing what was being taught or that the teacher was not effective teaching the material. Of course, students are liable to some degree in their own education, but teachers have a tremendous amount of power to inspire and challenge students to succeed. Hence, it is easy to blame students when they do poorly, but often times the teacher is also at fault. Tests will be discussed in greater detail in Chapter 5.

Example of a Test

The following test was given after pre-calculus students have learned how to solve a system of equations by substitution and elimination and how to use linear programming to solve problems. Students have also learned the vertex theorem, which states that a maximum and minimum of a function restricted on a polygonal convex set is on a vertex of the polygon. In terms of the syllabus, the test covered sections 2.4 thru 2.6.
The first two problems on the test were review problems. I have warned students in advanced that previous material would end up on the exam. The reason why I did this was so that students would not forget what was taught previously in the class, since much of that information is important for calculus and other fields. For example, an understanding of the domain of a function is very important for limits, and an understanding of composite functions is needed to comprehend the chain rule.

The next three questions were multiple choice questions. I chose to put such questions on the test for several reasons. One reason is because the SATs and AP exams have a large portion of them, so students have to learn how to identify correct answers among a list of items. Secondly, many students are not so stressed out about tests if they know that the answers lie in front of their faces. Thirdly, multiple choice questions allow me to ask more challenging questions than I could ask in a fill-in format. This is because the fill-in format requires students to recall information, while the multiple choice format requires students to only recognize the correct answer. Also, I do not have to determine how much credit to give to vague responses, which brings up the final point - multiple choice questions are easiest to grade.

Questions 6 and 7 asked students to solve a system of equations, using whatever algebraic method that they chose. I gave partial credit for these problems, depending on the severity of the mistake(s). For example, if a student got the x and y coordinates right for number 6, but got the sign of z incorrect, only one point was deducted. If however, a student could only get one coordinate right, but attempted a complete algebraic solution, only half credit was given.

Question 8 required students to apply the vertex theorem, and the last question was a word problem that required students to solve a linear programming problem. Partial credit could also be earned for these problems.

Below is a table of test results for three pre-calculus classes. Between A and B term I continued to teach at Doherty, so for a short time I was able to instruct all three pre-calculus classes, not just the two I normally taught. The table shows how many students got each question wrong on the exam (including those students that only earned partial credit).

Please note that only period 1 received the test below. The other two periods got very similar tests. Questions 3 thru 9 were the same, except the numbers were changed. Students got two of three possible questions for questions 1 and 2. The question not included in the test for period 1 was to compute a 3x3 determinant. The other tests are included in Chapter 5.

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<tbody>
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<td># of students that got #1 incorrect</td>
</tr>
<tr>
<td>Period 1</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>21</td>
</tr>
<tr>
<td># of students that got #2 incorrect</td>
</tr>
<tr>
<td>Period 1</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td># of students that got #3 incorrect</td>
</tr>
<tr>
<td>Period 1</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td># of students that got #4 incorrect</td>
</tr>
<tr>
<td>Period 1</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td># of students that got #5 incorrect</td>
</tr>
<tr>
<td>Period 1</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td># of students that got #6 incorrect</td>
</tr>
<tr>
<td>Period 1</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td># of students that got #7 incorrect</td>
</tr>
<tr>
<td>Period 1</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td># of students that got #8a incorrect</td>
</tr>
<tr>
<td>Period 1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td># of students that got #8b incorrect</td>
</tr>
<tr>
<td>Period 1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td># of students that got #8c incorrect</td>
</tr>
<tr>
<td>Period 1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td># of students that got #8d incorrect</td>
</tr>
<tr>
<td>Period 1</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td># of students that got #9 incorrect</td>
</tr>
<tr>
<td>Period 1</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>Average Grade</td>
</tr>
<tr>
<td>Period 1</td>
</tr>
<tr>
<td>84.1</td>
</tr>
<tr>
<td>Highest Grade</td>
</tr>
<tr>
<td>Period 1</td>
</tr>
<tr>
<td>97</td>
</tr>
<tr>
<td>Lowest Grade</td>
</tr>
<tr>
<td>Period 1</td>
</tr>
<tr>
<td>55</td>
</tr>
</tbody>
</table>

Questions 1 and 2 were review questions, so I expected students to do well on them. However, that was not the case. Almost all the period 1 students got question 1 wrong! I took the time to review the exam solutions with the
students so that they understood what they did incorrect, but I was very surprised that finding the domain of a function was so challenging for the students. I also told them that a question such as this one would appear on the test. Many students have done a simple algebraic mistake on the last problem, hence the high number of students that did not get the final answer correct. Otherwise, I believe that students did well on the exam, and the students could proficiently solve systems of equations and linear programming problems. Note that the average grade for all the periods is between a 76 and 81. This means that the exam was not too difficult or too easy. Competent students obtained an ‘A’, while students that goofed off failed.
1. (5 points) What is the domain of \( f(x) = \frac{3x}{\sqrt{x^2 - 9}} \)?

2. (5 points) If \( f(x) = 3x - 2 \) and \( g(x) = 2x^2 - 3 \), then what is \( f(g(x)) \)?

3. (5 points) Which of the following are not all convex polygons?
   a. All regular polygons
   b. All quadrilaterals
   c. All triangles
   d. All polygons in which all interior angles are less than or equal to 180°

4. (5 points) A system of equations with four variables and four equations can never be solved with which method:
   a. Elimination
   b. Substitution
   c. Graphing
   d. Guess and check

5. (5 points) A feasible region is:
   a. The set of all the vertices of a convex polygon.
   b. The function that is to be maximized or minimized in a linear programming problem.
   c. The solution to a system of inequalities.
   d. The maximum or minimum value for the objective function.

6. (20 points) Solve the system of equations algebraically:
   \[
   \begin{align*}
   x + y + z &= 6 \\
   -x + 2y + 3z &= 12 \\
   x - 2y - z &= -6 \\
   \end{align*}
   \]

7. (10 points) Solve the system of equations algebraically:
   \[
   \begin{align*}
   2x - y &= 9 \\
   x + 3y &= -6 \\
   \end{align*}
   \]
8. (25 points)
   a. (10 points) Graph the system of inequalities below.
   b. (5 points) Determine if the feasible region is a convex polygon.
   c. (5 points) Determine the vertices of the polygon.
   d. (5 points) Find the maximum of \( f(x, y) = x - y \) restricted on the polygonal set.

\[
\begin{align*}
0 &\leq x \leq 5 \\
0 &\leq y \leq 6 \\
2 &\leq x + y \leq 8
\end{align*}
\]

9. (20 points)
   A company produces remote controls and televisions. Long-term projections indicate an expected demand of at least 100 remote controls and 200 televisions each day. Because of limitations on production capacity, no more than 200 remote controls and 300 televisions can be made each day. To satisfy a shipping contract, a total of at least 400 items must be shipped each day.

If each remote control sold results in a $5 loss, while each television sold results in a $50 profit, how many of each item should be made daily to maximize net profits?
Conclusion

The lesson plans that I have created correlate strongly with the Worcester Public Schools Curriculum and the Massachusetts Curriculum Frameworks. By linking my class lesson plans on these two things, I was not able to instruct the students satisfactorily to state standards and district guidelines, but I was able to teach classes that had purpose and structure. One purpose is to prepare the students for their future math classes. (The advanced algebra students would take either AP statistics or pre-calculus, and the pre-calculus students would take AP calculus). Another purpose is to prepare the students for college, since the large majority of honors students plan on attending college. A third purpose, not less important that the other two, is to embrace an appreciation for mathematics and to inspire students. By integrating the use of technology and by making the best out of the limited resources such as paper at Doherty, I was partially able to fulfill these three purposes. I write the word partially, because Mrs. O’Leary would certainly continue what I did not finish.
Chapter 4: The Students

Introduction

I have mentioned in chapter 3 that since all of the students that I taught were honors upperclassmen, the same strategies that worked in one class tended to work in another class. However, individual differences were still present and I had to acknowledge them. One size does not fit all, and so as a teacher I had to adapt to the various needs of the students. Some students had poor attendance, behavioral problems, physical handicaps, medical issues, or overbearing parents. Students also have a variety of learning styles and students preferred different classroom activities. Some students are less competent than others and require things at a slower pace. However, going at a slower pace can bore advanced students, so delicate equilibrium had to be established. Students also had a variety of interest. They had very diverse answers to what their favorite class was and what their hobbies were. Some played sports, and others played video games. One student liked to knit, while another student liked to row. Furthermore, Worcester is a culturally diverse city. Some students were Catholics; others were Jews, Protestant, Orthodox, Muslim, or nonbelievers. Accommodating this varied population was one of the greatest challenges being a teacher.

Survey

On October 24th, I had all of the pre-calculus students answer a few questions at the beginning of class instead of having them work on a do now. I have not started teaching the advanced algebra students yet. Students were not required to write their names on the survey, but they were required to hand them in. The questions were:

- Do you have a computer at home?
- Do you have internet access at home?
- Have you ever stayed after school for help for this class?
- How much time do you spend on homework for this class?
- Is the homework easy, challenging, or frustratingly difficult for this class?

Every single student answered yes to the first two questions, except for one student that said that internet service at his or her home was down but would be back up soon. This was a great surprise to me. I decided to collect everyone’s email addresses so that I could email them homework assignments and lessons. I have also mentioned a few websites that the students may benefit visiting that was related to the course material.

The answers for the third question were also interesting. Here are the responses from each class:

| Do Pre-Calculus Students Stay After School for Help? |
|-----------------|-----|-----|
| Yes             | 3   | 17  |
| No              | 8   | 8   |
| Period 1        |     |     |
| Period 3        | 8   | 8   |
| Period 6        | 8   | 9   |

For periods 3 and 6, an exactly half of the students have stayed after school for extra help. However, only a measly 18% of the students from period 1 had gone after school for extra help! You would probably expect that students in period 1 do not do as well as the other two classes because they are not staying after school. However, the reverse is true. From the example test in Chapter 3, the grades for period 1 averaged higher than the other two classes. This paradox can easily be explained, however. Students from period 1 begin the day with Mrs. O’Leary and me. When the bell rings at the beginning of the school day, class starts. So those students were in our “homeroom”. (At Doherty there is not a dedicated homeroom period, but students come early before class starts.) Therefore, students in period 1 have an opportunity to ask Mrs. O’Leary and I questions regarding pre-calculus before class starts, so they do not need to stay after school. The students from the other two periods probably do the same thing with their period 1 classes; hence they cannot come for help before school. Those students must come after school instead.
I was impressed that so many students actually stayed after school for help. When I went to high school, I never stayed after for help because getting a ride home was always an ideal (this was before my family had cell phones). Perhaps cell phones are the very reason why students can easily stay after school these days. Cell phones allow students to instantly call their parents and tell them that they will be staying after. In my high school days, I would have to call home on some random classroom phone, hoping that the caller ID would not identify me as “Unknown Caller” so that my parents would actually pick up the phone and hope that they are home in the first place. If they did not answer the phone, then I would just have to take the city bus. Sometimes, I would have to wait an hour for the bus. I would also have to walk almost a mile because there were not any bus stops close to my house. With cell phones, parents are far more likely to answer calls and are able to pick up their kids after school. It is ironic that one of the objects most hated by teachers could be responsible for such a wonderful advantage.

Of course, Mrs. O’Leary and I had always encouraged students to stay after school for help if they struggled in the class. It seemed to be the case, however, that the same students willfully came after school, while that ones that could have benefited from it the most did not. In general, the students that should have stayed after school for help but did not had poor attendance. If those students simply attended class on a regular basis, then they probably would not need to stay after school. Dealing with poor attendance will be discussed later in this chapter.

The fourth question on the survey asked students how much time they spent doing homework on a daily basis. All three classes had the same homework assignments. Here are the results:

<table>
<thead>
<tr>
<th>How Much Time Pre-Calculus Students Spent on Homework</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Less than 20 minutes</td>
</tr>
<tr>
<td>20 minutes</td>
</tr>
<tr>
<td>30 minutes</td>
</tr>
<tr>
<td>40 minutes</td>
</tr>
<tr>
<td>50 minutes</td>
</tr>
<tr>
<td>60 minutes</td>
</tr>
<tr>
<td>More than 60 minutes</td>
</tr>
</tbody>
</table>

I have tried to keep homework assignments as consistent as possible so that they took the same amount of time to complete. Ideally I would have liked students to complete homework assignments in about 30 minutes each day. Of course, having too little or too much homework is a bad thing. Most students have five or six academic classes. So if students received 30 minutes of homework from each class, that means that they would get between 2.5 and 3 hours of homework each day. Three hours is almost half of the school day (lunch not included), so that is actually a hefty amount. If students received one hour of homework from each class, then they are getting a total of 6 hours of homework each day, which is too much! From my own high school experience, most of my teachers told students that they should spend an hour each day on homework, but in reality I only spent around thirty minutes each day for each academic class. Even as a college student, I continue to spend about thirty minutes a day on homework (with the exception of my IQP report!). I also did exceptionally well in high school, so an average student might have taken more time to complete homework.

I acknowledged the fact that the students that I taught varied in ability. Some students did not learn as quickly as others, and some students took more time doing homework than other students. Although it would be great if students spent around 30 minutes doing homework, reality is that many students spent less than that and many spent more than that. If I gave assignments that on average took one hour to complete instead of just 30 minutes to complete, then some students would be spending two hours completing homework, which would be absurd. On the other hand, if I chose to give less homework than I did, then some students would only spend 10 minutes doing it, which would also be silly. Therefore, I believe that the amount of homework that I gave was reasonable.

Although at the time the survey took place school has been in session for two months, some students in the pre-calculus classes still dropped out later on. Most of the students that dropped pre-calculus did so because they either...
did not have the proper foundation to take the class or because they could not keep up with the pace. Most of those students, I imagine, were the ones that reported spending more than an hour on homework. If those students were consistently taking twice the amount of time completing homework than their peers, then perhaps the class was indeed too demanding for them.

The final question on the survey was how challenging the homework was. Many students responded “sometimes easy, sometimes challenging”. In that case, I put them under the “Moderate” category. Here are the student responses:

<table>
<thead>
<tr>
<th>How Challenging Pre-Calculus Homework Was</th>
</tr>
</thead>
<tbody>
<tr>
<td>Easy</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>Moderate</td>
</tr>
<tr>
<td>Challenging</td>
</tr>
<tr>
<td>Difficult</td>
</tr>
</tbody>
</table>

Ideally, homework should be between moderate and challenging, especially for an honors level class. Obviously, easy homework assignments are not very effective because concepts are not being reinforced as well. On the other end of the spectrum, difficult homework assignments are discouraging and students become confused doing homework and concepts are not reinforced. The table above shows that the large majority of the students believe that the homework was either moderate or challenging, just like homework should be. Again, students have dropped out later in the course, so the number of students that though that the homework was difficult probably decreased.

**Learning Styles**

At my time at Doherty, I have discovered that students had a wide variety of learning styles. The most common type of learner that I have encountered in the classes that I taught was visual learners. I could tell that students were visual learns even though I did not ask them, because they would always demand a pictorial representation of something whenever possible. Recall from Chapter 3 that most of the period 1 students got the question wrong regarding the domain of the function \( f(x) = \frac{3x}{\sqrt{x^2 - 9}} \). The students were forced to solve this problem algebraically because graphing the function would have been too time consuming. Since the students were not able to see the result graphically, many of them got the question wrong. Most students, when attempting to solve for \( x \) when the denominator was zero got \( x = 3 \), and they believed that the domain was \( x \geq 3 \). They did not identify that \( x = -3 \) was also a solution and that \( x \leq -3 \) was also part of the domain. When I reviewed the solutions to the test with them I showed the students the graph of the function, and they identified the domain far more easily.

The obvious manner to deal with visual learners is to accommodate everything with a picture. This was quite simple for me to do because most of the topics that I taught were about graphing functions, their limits, and their derivatives. Other topics on the other hand, were not easily associated with a pictorial representation, such as matrices and system of equations in 3-dimensions. Although I did show students a 3D graph that showed a solution to a system of equations, some students got confused because they have never saw a 3D graph before and they did not know how to interpret it. Despite of this, many students appreciated the fact that a visual interpretation of a system of equations with three equations existed. It is a shame that the 3D coordinate plane is not in the curriculum, because that may have really helped reinforcing the concepts involved with systems of equations with three equations.

Another prevalent learning style that many of the students had was auditory. When students were asked to work on problems, many of the students preferred to talk about their work with each other. Also, some students would ask me questions even though the answers to their questions were written on the board. I always wrote the homework assignment on the board, but nevertheless some students still ask me if there is any homework! Sometimes, I write directions on the board, and students do not follow them because I did not orally explain the directions to them. I
have quickly learned to talk about everything; otherwise, auditory learners would miss out on important information. I also had to write down important information for visual learners.

A third type of learner was the kinesthetic one. Kinesthetic learners can be identified because they express themselves with bodily movements and use hand gestures when they talk. They are also more likely to be involved in sports. For kinesthetic learners, math classes can be quite challenging because mathematics is not exactly related to body movements. However, kinesthetic learners can adapt to such classes if they write plenty of notes. A kinesthetic learner may remember writing something down instead of seeing something written on the board. All of my students take notes (unless they are in a very bad mood). I have learned in psychology that taking notes improves the reinforcement of concepts and the ability to retain information cognitively.

**Behavioral Issues**

Most of the students that I had the privilege teaching were very well behaved. The most common behavioral issue in all the classes was socializing during inappropriate times. Most adolescents are very sociable, and they will chat at any opportunity they see fit. When I was talking to the students, they were very respectful and did not interrupt me. However, as soon as I stopped talking some students would start socializing. This was especially prevalent when I gave the students class work. I do not mind a little chatter as long as students get their work done and are not disruptive. I understand that there are several friendships in a classroom and the desire to chat is a very tempting one. However, when students abandoned their work or were so loud that they disrupted other students it become problematic. There was on girl in one of the pre-calculus classes that would talk obsessively to a boy that sat behind her. Sometimes, she would distract everyone around her. (To be honest, I found much of her conversation very interesting, and I secretly wished that I did not have to stop her.) Disciplining her was a challenge because she was a very bright girl and she was always very attentive to my instruction and she asked great questions. The boy was not nearly as problematic, because when I tell him to stop talking he did. Every time the girl became too disruptive I would ask her to quit it and concentrate on her work, but moments later she would start talking again. She talked even when no one responded back! When I reminded her that she had work to do she would apologize profusely every time but she could not help herself. I realized that I had to try something different because this was not working, so I finally threatened to change her seat. After a few times warning her that she was getting closer and closer to a permanent seat change, she quickly learned to control herself. She became far less disruptive and by the end of my teaching experience at Doherty she became one of my favorite students. I dealt with other students in a similar way, but I never had to change students’ seats.

Other serious behavioral issues developed from one advanced algebra student. The student seemed to be bipolar (I never knew if he actually had any psychological disorder). He did not want to be integrated with the rest of the class and he isolated himself in a chair in the back of the room. When I asked him why he did this, he told me that he hated being around people. I did not want to aggravate him because he was very hot-tempered, so I allowed him to sit where he pleased. I had to be very careful how I talked to him because I was very afraid that one day he would throw a chair across the room. Some days he would sit in the back of the room doing nothing, while other days he would participate like a star student. On the days when he chose to do nothing I would ask him if he understood the day’s lesson, and sometimes he would say yes and other days he would say no. I told him on a daily basis that I would help him if he would allow me to, and at first he seemed to be very annoyed by my interpositions, but over time he actually began thanking me for being so concerned. After building a shaky relationship with him (which took a few months), he even told me the things in particular that he did not understand, and I tried the best that I could to clearly anything. On the days in which he actually participated, I commented positively on his behavior and he was always appreciated my complements.

I quickly discovered that he hardly did his homework, which would have partially explained his minimal understanding of the course material. I felt like I had little choice but to give him a detention for not submitting any homework three times in a row. I told him quite clearly that if he missed the next homework assignment, he would certainly get detention. He did not do the homework assignment, and I gave him a detention but he did not show up. As a result, he had to serve several detentions with one of the assistant principles. After that incident, he must have
figured that it would just be easier to do homework rather than put up with his assistant principle so he started to do his homework. I complimented him on a daily basis on his improvement, and from that point onward I have seen substantial improvement in the quality and completeness of his work. He also participated in class more frequently because he actually understood what was going on. I asked him what his plans were in the future and he responded that he wanted to go to the military. If that is the case, then he really needs to learn how to obey authority. Nevertheless, he has improved greatly in the short period of time that he was my pupil.

Other behavioral issues were far less common, and mostly occurred sporadically depending on the students’ moods. Sometimes, although it is rare, a student would sleep during class. In those situations, I would tap on his or her shoulder and ask if he or she feels well. More than not, the student becomes too embarrassed to sleep in class again. Some students were also rude to either their pairs or to me. If students were rude to each other, I would tell them that their behavior was inappropriate and immature. The students usually reconcile and they explain that they were “just kidding”. Never were these two behavioral issues so serious that I had to go beyond verbal discipline to solve them. One of the greatest advantages teaching honors upperclassmen is that they are generally well behaved and responsible, and behavior is not normally an issue. Many of those students, after all, are nearly ready for college.

Absence and Tardiness

Absence was very problematic for some students, while other students were never absent. When a student was absent, I always allowed the student a few days to catch up with homework assignments. The challenge with absent students, however, is the class lessons and work that they missed. When students return to school, they may be confused about what is going on, and instead of missing out on just one day of school they are actually missing out on several. I was never absent in high school, so I do not know how difficult it is to actually catch up. However, I was dismissed several times and I know how hard it can be to make up work for just a few classes. I imagine missing a whole day of school would be three times as bad as being dismissed.

Recall from Chapter 3 that I email students lesson plans and homework assignments. So if a student was absent, he or she would have access to some of the information that was discussed during class. I encouraged absent students to continue checking their email for these updates. This had been extremely successful in cases where students were conscientious enough to take my advice. Of course, there are students that procrastinate, so if they miss school they rather deal with the consequences later. Those students that did not check their email for course updates were confused and anxious when they returned. Dealing with these students was very challenging because I had to continue that day's lesson and try to get the absent student(s) to understand things. Usually, the do nows were related to the material from the previous day, so when the students that were absent try to do the do now they are not able to. So, while the rest of the class settles in and completes the do now, I identify those absent students and try to give them the most critical information that I can in five minutes. Of course, that is not enough time to go over everything that they missed, but at least I can point out the theorems and main concepts that were practiced. I usually go over the do now with the students, so the students that were absent get some exposure to what happened when they were gone. When I taught the day’s lesson, I would always encourage people to ask questions, including the ones that missed class. Of course, if a student has no idea what is going on, then it is hard for that student to ask anything because he or she does not even know what to ask. In this situation (which is the worst possible scenario), I could only hope that that student goes home and studies what he or she missed. Unfortunately, the textbooks were not the highest quality, but fortunately every one of my students had internet access at home. I encouraged students to use the internet as an educational tool, and I occasionally listed useful websites on the whiteboard. I also allow absent students to copy notes from another student, preferably not during class.

There was one particular student that had very bad attendance. It was so bad that he had no chance to do well in the course. He got suspended for smoking in a bathroom, which prevented him from attending class in over a week. The days that he was in class, I talked to him about the seriousness of his absence, but he simply apologized and swore that he would improve. On the contrary, he only got worse until he never showed up to class again. I felt horrible about this situation, but I realized that this student was choosing to miss class on his own free will, and I
could not force him to attend class. Unfortunately, there are situations that are beyond a teacher’s control, and that was one of those cases. He will most likely be required to repeat a grade due to his absenteeism. I have discovered that he is also failing nearly every class. He does, however, plan on attending an Ivy League school, which would be impossible in his current circumstances. He has an extreme case of egotism in which he believes he is perfect in every way, when in fact he is not. He requires more help than I could ever give him.

Tardiness was also an issue among some students. Tardiness is not as serious as being absent because a tardy student usually does not miss the lesson. Recall that the do now takes place in the beginning of class, so a tardy student would have to be more than five minutes late to have missed anything new. There was one particular student that consistently arrived late. She claims that she was late because she come from the “100’s” which is on the completely opposite side of the school and she had to go to her locker. Doherty is a very large school, although not nearly as enormous as Lowell High School. When I went to Lowell High School, students were only allotted five minutes between classes. The school compromises of three separate buildings that add up to eleven floors in total. It would seem nearly impossible to get to class on time, but it is possible because I have done it for four years.

I had one English teacher that was so strict that anyone tardy on the day of the quarterly exam would receive a zero on it, and the quarterly exam was worth 25% of the term grade. Her own nephew showed up late, and she gave him a zero and he nearly failed her class! The reason why I mention this anecdote is because although I would not do anything as crazy as my English teacher, I do believe that late students need to be disciplined. I realize that Doherty is a large school, but if I made it to classes on time and my school was bigger, than the students at Doherty should also be able to make it to class on time. Therefore, the student that was constantly late to class was given a warning that if she was late again she would receive a detention. Sure enough she was late again, and she got a detention. Fortunately, she was struggling with the course material so during detention she was able to study pre-calculus with Mrs. O’Leary. (The reason why she was struggling, by the way, was not due to her tardiness. She is just a very distracted person that did not grasp new material as easily as the rest of the students.) Her understanding of what was going on in class grew, and her tardiness improved slightly. I could tell that she was really trying to be on time, so no further disciplinary perused. I have however, encouraged her to stay after school for additional help due to her slower comprehension of the material, and she had done so.

\[ 72 \] I have also monitored her very carefully, making sure that she remained on task. Often times, she would simply stare into space and I would remind her of the work at hand. I would ask her to answer at least one question a day so that I knew that she understood what was going on.
New Students

Doherty has a high mobility rate of 14.6% (see table 11, Chapter 1), which means that almost 15% of students are transferred in or out of the school. This does not account for the number of students that are transferred in and out of individual classes within the school. In all of the classes that I taught, there have been a few students that transferred to a lower level. However, I have not received any new students, so I never had to consider what to do in that situation.

Handicaps

Almost all of the students that I taught had no medical or physical handicaps that hindered their ability to work and to learn. There was one student, however, that only had three fingers and so his penmanship was messy. I did not realize this when I started teaching and when I graded in his first homework assignment I wrote a comment on his paper explaining that he should write neater. He got a ✓ on the assignment, not due to the messiness (which I always give students a warning before I deduct points for), for the numerous mistakes that he made. The homework assignments that he produced in the future were much neater, but were still a bit messy. I did not however, take off points because it was an improvement compared what he handed in before. He had never told me of his physical handicap so I continued writing comments on his assignments demanding neatness. When I finally realized that he was missing fingers I felt really bad and I apologized to him if I had upset him in any way. He said that he understood and that I have not offended him. I have granted him extra time on tests because he wrote slower than other students and I made sure he finished copying things down from the board before I erased. To my surprise, his penmanship continued to improve, so much in fact that it was indistinguishable to the other students’ work. I was extremely impressed and I praised the neatness of his writing. It is very remarkable that a student that had a valid excuse not to perform as well as the other students did not use it. Instead, he defeated his handicap and managed to produce work that was just as good as everyone else’s. Yet, students without valid excuses use them all the time, and they allow their flaws to ruin their work. Everyone handicapped or not could really learn from what this student has done.

I had another student that got a concussion playing soccer and was absent a few days. She arrived back to school with a medical note from her doctor explaining that she may have difficulty recalling and remembering information. However, in the classroom her concussion did not seem to have much effect at all, and she was able to work normally. In fact, her absenteeism was more problematic than her memory loss (if she had any).

Parental Issues

I have never dealt with students’ parents directly so I do not know how annoying they can be personally. Mrs. O’Leary, however, shares with me such experiences. Parents this particular school year are not bad in general, but there was one case in which the parents were very unrealistic. The student that got suspended for smoking in the bathroom and that had horrible attendance and wants to go to an Ivy League college has parents that are extremely optimistic about their son. The guidance counselors at Doherty would not allow the student to take some honors and AP level classes that he wanted to take because of low grades from the previous school year. His parents thought this was an insult and went to City Hall to complain about this “injustice”. City Hall required the school to allow the student to take whatever classes he wanted to take. So, he ends up in Mrs. O’Leary’s honors pre-calculus class even though he nearly failed advanced algebra. The student failed pre-calculus the first two terms, and his parents are very concerned about their son’s future, but they think that their son should just be allowed to stay in all of his advanced classes. When the student got suspended, the parents got even more worried, but they were certain that their son would improve. I learned that the student’s father is an engineer, and he wants his eldest son to be as successful as he was and he wants his son to take AP calculus. Unfortunately, his parents are putting their son inside a self-destructive trap, and if they do not place their son into the proper classes, he may end up being a high school dropout. I am not saying that he will become a dropout, but students that constantly fail classes are truants, and that get suspended are at a far greater risk of that fate. He and his parents have to understand that he cannot continue making the mistakes that they are making. Unfortunately, there is really nothing I can do about what his parents
think. (What parent would listen to a childless nineteen year old?) I can only hope that the student gets back on track.

**Teaching Method**

What I have learned about my students has influenced the way that I taught. I realized that my students were very social, so a lecture would not be the best teaching strategy. (I am currently am in a lecture based class in which the professor mumbles to the board for an hour and it is extremely boring.) A totally open discussion, on the other hand, would not be very good either because students would become too distracted or would begin chit-chatting about random things. What had worked the best was something in between a lecture and an open discussion, in which I taught students new concepts while allowing breaks in between for questions and comments. After I have finished teaching, I would give the students class work to practice what they just learned. I have allowed them to work together if they wished. As I have previously stated in this chapter, students generally stayed on task, but sometimes students would talk instead of doing their class work. Threatening seat changes and adding class work to the homework assignments have been effective in eliminating most of these distractions.

I have also learned the interest that many of the students had. Several students, especially my pre-calculus ones, were concurrently taking AP biology, physics, English, and Spanish. Many students were involved in sports, including soccer, football, and basketball. Some students were “hardcore gamers” while others were only casual gamers. (The day *Call of Duty: Black Ops* was released, that was all students talked about. It was even on the school news.) I also had students that loved nature (thus taking AP biology). Later on in my teaching experience, I have tried to acknowledge students’ interests by creating word problems that were about these topics. In order to satisfy everyone, I had to make several problems, and I allowed students to choose the ones that interested them the most. Students seemed to enjoy solving problems when they could relate to them. Students were also less likely to diverge from their work. This was, however, a lot more work for me because I had to make and solve the problems myself, so I have not done this as much as I probably could have. If there was one thing that I could do to improve my teaching, it would be to relate the subject matter to the students’ interest better.

**Conclusion**

Teaching would be a difficult task unless the teacher knows his or her students well. A teacher must learn what types of learning styles are prevalent in the classroom. If a teacher talks without writing anything down, for example, and the students are visual learners and not auditory learners, then teaching would not be very effective. A teacher should also learn the students’ interest so that he or she can relate course material with topics that the students like. A teacher should also know which students have behavioral or attendance problems and how to solve them, if possible. A teacher must address every student, not only those that are doing well in the course. In fact, the brightest students could probably succeed without a teacher at all.
Chapter 5: Assessment

Introduction:

Without assessments, a teacher would not have any idea if his or her students have mastered the objectives that they should have. Teachers are able to monitor and evaluate students’ progress by giving students class work, tests, quizzes, projects, and homework assignments. That is why those four things are so prevalent in every academic subject. Assessments are also important because they may predict whether or not students will do well on statewide exams, such as the MCAS, especially if the assessments are mapped directly to the state frameworks. Since I only taught students that have already passed, or will most certainly pass the MCAS, I was not concerned about this. Rather, I was concerned about whether or not students will have a strong foundation to have successful futures. For most of the students that I taught, their future will be college. The state frameworks do not only exist so that teachers know what students have to master to pass the MCAS. Their main purpose is to prepare students for their futures after high school. Thus, even though I was not worried about the MCAS, I was still concerned about my students fulfilling the state standards. Assessments were the greatest tools to find out what standards students have become proficient in, and which standards students still need to master. Assessments also revealed what methods students preferred to solve problems with, and how students think (whether the logic was correct or not). Once a teacher has discovered the students’ weaknesses and strengths, the teacher can then utilize the students’ strengths to improve their weaknesses.

Do Now

The do nows that I had the students do in the beginning of each class were one form of assessment. The purpose of the do nows was to have the students start thinking about math before actually starting the lesson and to review what they have learned while allowing the teacher a few extra minutes to prepare for class. While the students worked on the do now I had to deal with absent students, pass back graded assignments, set up the ELMO or write things on the board, take attendance when Mrs. O’Leary was not around, and answer students’ questions. Because of all of these duties, however, I did not have much time to go around the room and see how the students were doing the problems. I tried my best to see students’ work, but it was not always possible. What was more revealing was when I reviewed the do now and asked students how they did the problems. I rarely did out the do now on the board myself, because I was more interested in how the students’ approached the problems. Occasionally, students would have more than one way of doing the same thing, in which case I would inquire whether or not one way was “better” than the other. For example, one advanced algebra student used substitution to solve a system of equations, and another student used elimination. For that particular problem, \[ \begin{cases} 3x = y + 2 \\ -3x = y - 2 \end{cases} \] elimination would be considerably easier, and thus “better”. I took that opportunity to discuss that analyzing the problem before trying to solve it will often lead to simpler computations, even though it may seem like analyzing the problem is just an extra step. During do now reviews, a student would occasionally say the wrong answer or write the wrong answer on the board, in which case I would have the students (not me!) pinpoint the student’s error. This helped me determine where students may incorrectly do something, and I could teach them to avoid common mistakes. Finally, students often ask questions, which help me assess their understanding and curiosity of the subject matter. If possible, I have students try to answer each other’s questions; otherwise, I will. The reason why I have students do some of the teaching is because that way I can evaluate whether or not students truly understand what is going on. A common cliché is “you don’t really understand a subject until you can teach it.”

Class work

After I went over the day’s lesson, I usually gave students class work. When students were doing class work, I always went around the room and looked at how they were solving the problems. Since I usually did not have a chance to see every student’s work on the do now, class work has been a very good assessment tool. Class work was usually the first opportunity that students got to try out what they have just learned. Therefore, it was typical for students to make numerous errors and to ask many questions when they were doing class work. There are two types
of mistakes that students make in class work, which are errors related to new concepts and errors related to old concepts. As a teacher, it was very important for me to distinguish between the two types of errors. The first type of error is a natural occurrence, since making mistakes is part of learning something new. The later type of error, on the other hand, is more alarming because it indicates that a student did not master a concept that he or she should have previously mastered. In addition, making these types of mistakes usually makes learning the new concept harder because the student is concentrating on mastering older concepts instead.

As an example, consider two advanced algebra students. Students should have learned how to factor quadratics, and in a particular lesson, students were “taught” how solve quadratic equations by factoring. (The reason why I put the word “taught” in quotes will become apparent momentarily.) After the lesson, I gave students class work on solving quadratic equations. One student factored expressions correctly, but did not make sure that one side of the equation was equal to zero first. This student was making errors related to the new concept, and so this was not a very serious problem. In fact, it was beneficial that the student made the mistake, realized it, fixed it, and learned from it. Another student made many mistakes factoring. This student was making mistakes related to an old concept that should have already been mastered. This was a more serious problem because the student was struggling on an older concept instead of focusing on the newer concept of actually solving equations. Once I detected such problems, it was important that I corrected them as soon as possible; otherwise, the student may not be able to do more advanced things that depend on previous knowledge. Issues like these usually happened because students did not do their homework or when they were absent, and so they did not practice applying what they were shown in class. Thus, they actually did not learn the concept, they have just seen it. That is why I put “taught” in quotes. Just because I have shown students how to do something, that does not mean that they have learned how to do it.

Class work was also helpful to me because it revealed how students were doing problems. More often than not, students would execute problems exactly how I showed them how to do them. Sometimes, students would be daring enough to tackle the problems their own way. This would often lead to a monstrosity of errors, but sometimes their alternative methods have actually worked. Sometimes, especially in the advanced algebra class where things were more procedural, I would list the steps that students can take to solve a problem. Most students have followed the steps, while others would not. The ones that did not follow the steps typically made more mistakes, but the more clever students did not. I do not force students to follow the steps that I gave them; they were only suggestions on how to approach problems. I did not mind witty students “cutting corners” to arrive at solutions as long as their procedures were correct because in order to cut corners, one has to know the topic thoroughly before being able to recognize that a shortcut exist. One instance in which students discovered a shortcut to problems was when I introduced the pre-calculus students to the derivatives of various polynomials using the limit definition of the derivative. Eventually, some students discovered the power rule before I formally taught it, and would compute derivatives without taking limits. When students discovered this, I encouraged them to check their answers using the power rule. I still required them to evaluate the limit, however, because limits are extremely important in calculus. (They would also have finished their work twenty times faster than the rest of the students, which would have been problematic.) When students tried to use incorrect shortcuts, on the other hand, their work became a disaster, and I had to show the students why they are wrong so they will not be tempted to do it again. Famous “shortcuts” that many of my students have used were $\sqrt{a^2 + b^2} = a + b$, $(a + b)^2 = a^2 + b^2$, and $\frac{1}{a+b} = \frac{1}{a} + \frac{1}{b}$.

Homework

Homework was one of the most important forms of assessment because I had as much time as I needed to examine students’ work. Homework is usually not the first time students actually attempt to practice concepts because they are usually given related class work first. In theory, homework should have been cleaner and contain fewer errors than class work if students have actually learned anything working on class work. Also, because homework assignments were graded, I expected students to put more effort into them. Some students have always done wonderful on homework, while others have not. I have discussed in Chapter 4 how I managed to get students to improve the quality of their homework assignments.
Some homework assignments were collected, while other assignments were only checked off for completeness. It would have been too tedious for me (and probably for the students) to collect all homework assignments. The homework assignments that I collected proved to be a very valuable tool to measure students’ progress. When I graded homework assignments, I did not simply check the final answers. What was more important to me was how the students arrived at their solutions, regardless if the solution was correct or not. When students made a mistake, I always circled what they did wrong and wrote a brief comment explaining why. I really hate it when I get an assignment back, and there are question marks all over my paper. What does a question mark mean, and what exactly did I do wrong? If a professor wrote a comment in Chinese it would have been more helpful for me than a question mark! When a particular mistake was common in several homework assignments, I made sure that I brought it up when I handed back the assignments. I always asked students if there were any questions on the homework after I collect it and after I return it. Just like the questions on class work, these questions were a good indication of whether or not students actually understood what was on the homework. If I got a ton of questions after I collected the homework assignment, then that was an indication that students struggled doing the assignment and that they needed more practice. If I got no questions, on the other hand, then that either meant that the students understood what they were asked to do or they were so lost that they did not even know what to ask. I found out which case it was simply if students complained about the assignment or not.

Homework was an indicator of whether or not to go onto the next topic or not. If students generally did poorly on an assignment, then they obviously needed to spend more time developing the relevant concepts. In the contrary, if students done universally well on a homework assignment, then they are most likely to be ready to move on. Many teachers are stressed about all of the standards that they need to address throughout the school year, and so they jump from one topic to the next regardless of whether or not students have actually learned the subject matter. The students unfortunate enough to have such teachers end up learning very little throughout the year because they seen a million things all too fast. To make matters worse, more advanced material often depends on previous material, so a teacher that rushed through an elementary topic will have to backtrack because the students did not learn it the first time around. This actually waste more time than it saves. It would actually be better if those teachers went at the students’ pace addressing half of the standards rather than rushing through all of them. That way, students actually learn something. Of course, it would be ideal if students were capable of learning ultra fast and if all the standards can be fulfilled. However, the classroom environment is far from ideal, and so that is not always possible. The best a teacher can do is to carefully monitor students’ progress and decide which standards can be realistically achieved. Homework assignments are one of the best forms of assessments to do that, because they are done on a regular basis and teachers are able to take the time examining them carefully at home.

Quizzes

Quizzes were given between tests and covered only one or two sections from the curriculum. I always told students when a quiz was coming up. To me, it does not make sense to give pop quizzes because there are already plenty of forms of daily assessment and pop quizzes typically destroy student-teacher relationships. Because I thought that all the other forms of assessment were proficient in evaluating students’ progress and mastery of the course standards, quizzes were given sporadically. In fact, I did not even give advanced algebra students quizzes at all. Generally, quizzes were suppose to be short and take no more than fifteen minutes, however, because I overestimated the simplicity of some of them, they took much longer for students to complete. I have tried to give students quizzes when I believed that they knew the material sufficiently well and when I was concluding one section and beginning another. Quizzes just had short answer questions or graphing questions. When I graded quizzes, I always granted partial credit for whatever students done correctly. I also carefully scanned through students work so that I could pinpoint what exactly they done wrong, and I always wrote a comment on the quizzes explaining where students went wrong. If students did poorly on quizzes then I did not go on to the next section. Instead I reviewed whatever students were struggling with before moving onto the next section. In general, students have done well on quizzes. I projected a few quizzes using the ELMO in order to save paper; however I stopped doing that to decrease the probability of cheating. Quizzes were treated very much like tests, so most of the information regarding tests also applied for quizzes. One notable advantage of quizzes, however, was that I could give them any day of the week, whereas tests I had to give on Wednesdays.
Pre-Calculus Quizzes

Symmetry Quiz (Period 1)

1) Complete the graph below under the following conditions. The coordinates (13, 3) and (5, 1) are on the graph. Make sure you label the corresponding points of these coordinates for the different types of symmetry. (20 points each)
   a) The graph is symmetric with respect to the y-axis.
   b) The graph has rotational symmetry with respect to the origin.
   c) The graph is symmetric with respect to the line $y = x$.

2) Prove or disprove the following statements (10 points each):
   a) $y = 3x - 2$ is symmetric with respect to the x-axis. \[ \text{No} \]
   b) $y = 7x^4 + 3x^2 - 5$ is symmetric with respect to the y-axis. \[ \text{Yes} \]
   c) $y = \sqrt{x^2 + 4}$ is symmetric with respect to the y-axis. \[ \text{Yes} \]
   d) $y = \frac{1}{3x}$ is symmetric with respect to the line $y = -x$. \[ \text{Yes} \]

3) BONUS (5 points each)
   a) Prove or disprove: $y = \sin(x)$ is odd.
   b) Give an example of a single function that is symmetric with respect to the y-axis, the x-axis, the line $y = -x$, the line $y = x$, and the origin.

Symmetry Quiz (Period 3)

1) Complete the graph below under the following conditions. The coordinates (13, 3) and (5, 1) are on the graph. Make sure you label the corresponding points of these coordinates for the different types of symmetry. (20 points each)
   d) The graph is symmetric with respect to the x-axis.
   e) The graph has rotational symmetry with respect to the origin.
   f) The graph is symmetric with respect to the line $y = -x$. 
2) Prove or disprove the following statements (10 points each):

e) $y = 3x - 2$ is symmetric with respect to the line $y = x$.

f) $y = 7x^4 + 3x^2 - 5$ is symmetric with respect to the y-axis.

g) $y = \sqrt{x^2 + 4}$ is symmetric with respect to the origin.

h) $y = \frac{1}{3x}$ is symmetric with respect to the x-axis.

3) BONUS (5 points each)

a) Prove or disprove: $y = \cos(x)$ is even.

b) Sketch $y = \sin(2x)$ and $y = 2\sin(x)$.

Makeup Symmetry Quiz

1) Complete the graph below under the following conditions. The coordinates (2, 0) and (4, 12) are on the graph. Make sure you label the corresponding points of these coordinates for the different types of symmetry. (20 points each)

g) The graph is symmetric with respect to the x-axis.

h) The graph has rotational symmetry with respect to the y-axis.

i) The graph is symmetric with respect to the line $y = x$. 

2) Prove or disprove the following statements (10 points each):
   i) \( y = 3x - 2 \) is symmetric with respect to the origin.
   j) \( y = 7x^4 + 3x^2 - 5 \) is symmetric with respect to the y-axis.
   k) \( y = \sqrt{x^2 + 4} \) is symmetric with respect to the x-axis.
   l) \( y = \frac{1}{3x} \) is symmetric with respect to the line \( y = x \).

3) BONUS (5 points each)
   c) Prove or disprove: \( y = [x] \) is odd.
   d) Identify a function that is both even and odd.

Translation Quiz (Period 1)

1. Graph \( y = |x - 3| + 7 \)
2. Graph \( y = 2(x + 1)^2 \)
3. Graph \( f^{-1}(x) \) if \( f(x) = 2x - 2 \)
4. Graph \( f^{-1}(x) \) if \( f(x) = \sqrt{x} \)

Translation Quiz (Period 3)

1. Graph \( y = |x + 2| - 1 \)
2. Graph \( y = -(x)^2 + 2 \)
3. Graph \( f^{-1}(x) \) if \( f(x) = -2x - 3 \)
4. Graph \( f^{-1}(x) \) if \( f(x) = \sqrt{x + 2} \)

Tests

Another very important form of assessment is tests. If tests are carefully constructed to reflect the objectives of the course, then were the ultimate indication of whether or not students have mastered the content that they should have. I have created tests with care, making sure that they represented the skills that I wanted the students to have. I also made sure that the tests contained variety, in both the type of problem (multiple choice, short answer, word problems, graphing, etc.) and the method behind solving problems (recalling information or recognizing information, algebraic or pictorial, theory or calculation, etc.) Variation allowed me to assess the students’ ability to solve a wide range of problems in different forms. In addition, every student has his or her preference for what type of problem that they get on tests. Likewise, students have their unique dislikes. In order to balance the personal taste, it was necessary to have such variation.

Multiple choice problems tests students’ ability to recognize information among a list of items. Since the answers to these questions were right in front of the students’, many of them liked them and were not so stressed when they took tests. On the other hand, several students easily got confused between the correct answers and really “good” wrong answers and often got multiple choice questions incorrect. These students disliked multiple choice questions because they thought they were being tricked. In the table from Chapter 4 that list each question on a test and how many students got each question wrong, the reader will notice that many students got multiple choice questions wrong. Some multiple choice questions were much more specific than others. Some multiple choice questions demanded students to recall something that I have said in class only a few times, so only careful listeners that attended class regularly would know (as opposed to guess) the answer. I always put one or two of such questions on tests to reward and encourage students to be prudent and punctual. The downside of having such questions, however, is that their scope was narrow, and only tested a very small segment of what I taught. Therefore, the large majority of the questions that I put on tests required a broader knowledge of concepts. Multiple choice questions are important because virtually all standardized test are primarily in this form, and students have to learn how to answer such questions. Therefore, whether students love them or hate them, multiple choice questions may very well determine what college they will go to (SAT) and how much college credit they will receive from high school (AP).
An added bonus to multiple choice questions is that they were easy to grade. However, students rarely showed their work that lead them to their answers, so I could not evaluate their logic or algebra.

Short answers were also easy to grade, and they were better assessment tools because students showed how they arrived at their answers. Because of these two benefits, the majority of the questions on the tests that I made were in this form. Most short answers required students to perform an algebraic procedure, such as solving a system of equations and filling in the coordinate pair or triple. Students that felt like they were being tricked with the multiple choice questions did not feel this way with the short answer format because these problems did not list convincing incorrect answers. On the other hand, other students did not like short answer because the correct answer is not contained in a list. In other words, short answer questions tested students’ ability to derive or recall answers, rather than simply recognize answers. When I graded short answers, I granted partial credit for students that showed their work and who did something right, but I did not grant partial credit for multiple choice questions. Interestingly, students typically did better on the short answer questions than the multiple choice questions. Perhaps this is because students were more use to having short answer questions as opposed to multiple choice questions on math tests, and because I rarely gave students multiple choice questions on any other form of assessment. In retrospect, I probably should have gave students some multiple choice questions in homework and class work so that they would be more comfortable answering those types of questions.

Every test also contained at least one word problem. Word problems are very important because mathematics was primarily developed to solve real world issues. Word problems were also powerful assessment tools because they required the most out of students. In order to answer a word problem correctly, students had to convert the problem into a mathematical representation, solve the mathematical form of the problem, then convert the mathematical solution back into a meaningful form that solves the word problem. A student that leaves his final answer as “5” will not receive full credit for the problem, because “5” is meaningless in a physical context without units. “5” could mean five school buses, five sandwiches, five days, five o’clock, etc. Since word problems require multiple steps, I always gave students partial credit representative of what they have done correctly. Because of the amount of work required to complete word problems and because of the wider range of skills that they assess, they were worth a large portion of the test score. Also, since word problems are more complex than the other types of questions on tests, I limited the number of them on exams; otherwise students would not have time to finish the tests. (I also limited the number of word problems on test because they were the most time consuming to grade.)

The final form of question that appeared on test was graphing problems. Sometimes, an entire question would require students to graph, other times, graphing is only part of a multi-part problem or a word problem. (Multi-part problems were just several short answer and graphing questions related to each other.) Graphing is an essential skill to have, especially when one wants to approximate solutions to a system of (in)equalities. A great deal of advanced algebra is also learning how to graph various functions, so it would be ludicrous not to have graphing questions on advanced algebra tests. Graphing is also essential for linear programming, which was taught in pre-calculus. Partial credit was earned for graphing problems. For example, when I asked students to graph the feasible region for a system of inequalities, I gave partial credit for correct intercepts, correct slopes, correct line type (solid or dashed), and correct shading (above or below), and deducted points if any one of these things were incorrect.

I tried my best to give tests only when I felt that students were ready to demonstrate their mastery of the material. However, this proved to be fairly difficult because I was only allowed to give tests on Wednesdays. If I felt that students were slightly underprepared, I had to make the tough decision of whether or not to postpone a test for a whole week. I tried to alleviate some of this drama by providing students a review day before the exam, so they have a chance to ask questions and work on problems that may appear on the tests. I also told students which concepts in particular may appear on the tests. (Of course, I did not outright tell students precisely what was going to be on the tests.)

On all of the tests that I gave, I never allowed students to use a calculator. If I allow only scientific calculators, then students that owned only graphing calculators would have been at a disadvantage. Therefore, if I did allow students to use a calculator, then I would have to give everyone a graphing calculator to be fair. However, there are graphing problems on tests, and letting students use a graphing calculator would defeat the purpose of having such questions
on the tests in the first place. I am aware that calculators are allowed on standardized test, but in my opinion, this introduces a gross disadvantage for low-income students and a huge advantage for everyone else. To me, it is obvious that someone that uses a TI-Nspire CX CAS (retailed at ≈$150) on the SAT will outperform someone that uses a one dollar calculator. In fact, there is a positive correlation between parent’s income and students’ mean SAT scores, in which the College Board even admits. There can be, however, many other reasons for this correlation, and discussing them is beyond the scope of this paper.

**Family Income vs. Median SAT Scores**

![Graph showing Family Income vs. Median SAT Scores](http://www.danpink.com/archives/2012/02/how-to-predict-a-students-sat-score-look-at-the-parents-tax-return)

On a few of the pre-calculus tests, I allowed students to use a “cheat sheet”, a piece of paper that contains students’ notes. I allowed students to use cheat sheets for the exam on linear programming and the exam on reflections. I required all students to hand in their cheat sheets along with their exams so that they would not be able to share them with students in other classes. I allowed students to use cheat sheets for a few reasons. One reason was because my mentor warned me that particular procedures, such as linear programming are typically difficult for students to memorize, and she made the suggestion that I allow students to use cheat sheets. She occasionally allows her students to use cheat sheets on exams. The other reason why I allowed students to use cheat sheets is because I know that many professors allow open book and open notes exams. I even know professors that have “open Internet” exams! The SAT and other math standardized tests also have a section with important equations. Thirdly, I thought that if I allowed students to make cheat sheets for exams, that would encourage them to study. However, I felt that the use of cheat sheets was conflicting with the purpose of the exams themselves. The exams were suppose to demonstrate students’ mastery of the subject matter and their ability to apply what they learned, but the use of cheat sheets introduced a questionable factor of whether or not students knew what they were doing or where just copying information onto their tests. Since I gave partial credit to many tests questions, it would have been easy for students to find something on their cheat sheet and write it on their test without actually knowing how to do the problem and still earn partial credit. I have also noticed that some students would rely on their cheat sheets as if they were a divine gift from God, and not study for a test. Review sessions were also not very productive because students knew that they could just write things down onto their cheat sheets that they were too lazy to study. For these reasons, I decided that I would not allow students to use cheat sheets for the remaining tests.

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2.4-2.6 Pre-Calculus Test (Period 1)

1. (5 points) What is the domain of \( f(x) = \frac{3x}{\sqrt{x^2-9}} \)?

2. (5 points) If \( f(x) = 3x - 2 \) and \( g(x) = 2x^2 - 3 \), then what is \( f(g(x)) \)?

3. (5 points) Which of the following are not all convex polygons?
   a. All regular polygons
   b. All quadrilaterals
   c. All triangles
   d. All polygons in which all interior angles are less than or equal to 180°

4. (5 points) A system of equations with four variables and four equations can never be solved with which method:
   a. Elimination
   b. Substitution
   c. Graphing
   d. Guess and check

5. (5 points) A feasible region is:
   a. The set of all the vertices of a convex polygon.
   b. The function that is to be maximized or minimized in a linear programming problem.
   c. The solution to a system of inequalities.
   d. The maximum or minimum value for the objective function.

6. (20 points) Solve the system of equations algebraically:
   \[
   \begin{align*}
   x + y + z &= 6 \\
   -x + 2y + 3z &= 12 \\
   x - 2y - z &= -6
   \end{align*}
   \]

7. (10 points) Solve the system of equations algebraically:
   \[
   \begin{align*}
   2x - y &= 9 \\
   x + 3y &= -6
   \end{align*}
   \]

8. (25 points)
   a. (10 points) Graph the system of inequalities below.
   b. (5 points) Determine if the feasible region is a convex polygon.
   c. (5 points) Determine the vertices of the polygon.
   d. (5 points) Find the maximum of \( f(x, y) = x - y \) restricted on the polygonal set.
106

\[
\begin{align*}
0 &\leq x \leq 5 \\
0 &\leq y \leq 6 \\
2 &\leq x + y \leq 8
\end{align*}
\]

9. (20 points)
A company produces remote controls and televisions. Long-term projections indicate an expected demand of at least 100 remote controls and 200 televisions each day. Because of limitations on production capacity, no more than 200 remote controls and 300 televisions can be made each day. To satisfy a shipping contract, a total of at least 400 items must be shipped each day.

If each remote control sold results in a $5 loss, while each television sold results in a $50 profit, how many of each item should be made daily to maximize net profits?

2.4-2.6 Pre-Calculus Test (Period 3)

1. (5 points) What is the domain of \( f(x) = \frac{x}{\sqrt{x^2-4}} \)?

2. (5 points) Compute \[
\begin{vmatrix}
1 & 0 & 2 \\
2 & 1 & -1 \\
-1 & -2 & 1
\end{vmatrix}.
\]
3. (5 points) Which of the following are **not** all convex polygons?
   a. All regular polygons
   b. All triangles
   c. All quadrilaterals
   d. All polygons in which all interior angles are less than or equal to 180°

4. (5 points) A system of equations with four variables and four equations can **never** be solved with which method:
   a. Elimination
   b. Substitution
   c. Guess and check
   d. graphing

5. (5 points) A feasible region is:
   a. The solution to a system of inequalities.
   b. The function that is to be maximized or minimized in a linear programming problem.
   c. The set of all the vertices of a convex polygon.
   d. The maximum or minimum value for the objective function.

6. (20 points) Solve the system of equations algebraically:
   \[
   \begin{align*}
   x + y + z &= 12 \\
   -x + 2y - z &= 0 \\
   -x + y + z &= 8
   \end{align*}
   \]

7. (10 points) Solve the system of equations algebraically:
   \[
   \begin{align*}
   -2x - 2y &= 6 \\
   x + 2y &= -4
   \end{align*}
   \]

8. (25 points)
   a. (10 points) Graph the system of inequalities below.
   b. (5 points) Determine if the feasible region is a convex polygon.
   c. (5 points) Determine the vertices of the polygon.
   d. (5 points) Find the maximum of \( f(x, y) = x + y \) restricted on the polygonal set.
9. (20 points)
A company produces remote controls and televisions. Long-term projections indicate an expected demand of at least 150 remote controls and 250 televisions each day. Because of limitations on production capacity, no more than 200 remote controls and 350 televisions can be made each day. To satisfy a shipping contract, a total of at least 500 items must be shipped each day.

If each remote control sold results in a $5 loss, while each television sold results in a $50 profit, how many of each item should be made daily to maximize net profits?
2.4-2.6 Pre-Calculus Test (Period 6)

1. (5 points) If \( f(x) = 2x - 5 \) and \( g(x) = x^2 - 4 \), what is \( g(f(x)) \)?

2. (5 points) Compute \[
\begin{vmatrix}
1 & 0 & 2 \\
2 & 1 & -1 \\
-1 & 2 & 1
\end{vmatrix}.
\]

3. (5 points) Which of the following are not all convex polygons?
   a. All quadrilaterals
   b. All triangles
   c. All regular polygons
   d. All polygons in which all interior angles are less than or equal to 180°

4. (5 points) A system of equations with four variables and four equations can never be solved with which method:
   a. Elimination
   b. Graphing
   c. Guess and check
   d. Substitution

5. (5 points) A feasible region is:
   a. The maximum or minimum value for the objective function.
   b. The function that is to be maximized or minimized in a linear programming problem.
   c. The set of all the vertices of a convex polygon.
   d. The solution to the system of inequalities.

6. (20 points) Solve the system of equations algebraically:
\[
\begin{align*}
  x + y + z &= 12 \\
  -x + 2y - z &= 0 \\
  -x + 2y + z &= 8 
\end{align*}
\]

7. (10 points) Solve the system of equations algebraically:
\[
\begin{align*}
  2x - y &= 3 \\
  -x + 2y &= 6 
\end{align*}
\]
8. (25 points)
a. (10 points) Graph the system of inequalities below.

\[
\begin{align*}
0 & \leq x \leq 3 \\
0 & \leq y \leq 4 \\
1 & \leq y - x \leq 2
\end{align*}
\]

b. (5 points) Determine if the feasible region is a convex polygon.

c. (5 points) Determine the vertices of the polygon.

d. (5 points) Find the maximum of \( f(x, y) = x + 2y \) restricted on the polygonal set.

9. (20 points)
A company produces remote controls and televisions. Long-term projections indicate an expected demand of at least 50 remote controls and 220 televisions each day. Because of limitations on production capacity, no more than 100 remote controls and 250 televisions can be made each day. To satisfy a shipping contract, a total of at least 300 items must be shipped each day.

If each remote control sold results in a $5 loss, while each television sold results in a $50 profit, how many of each item should be made daily to maximize net profits?
2.4-2.6 Pre-Calculus Test Results

<table>
<thead>
<tr>
<th></th>
<th>Average Grade</th>
<th>Highest Grade</th>
<th>Lowest Grade</th>
</tr>
</thead>
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<td>97</td>
<td>55</td>
</tr>
<tr>
<td>Period 3</td>
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<th>Period 1</th>
<th>Period 3</th>
<th>Period 6</th>
</tr>
</thead>
<tbody>
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<td>4</td>
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<tr>
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<td>8</td>
<td>6</td>
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<tr>
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<td>8</td>
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<tr>
<td># of students that got #5 incorrect</td>
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<td>4</td>
</tr>
<tr>
<td># of students that got #6 incorrect</td>
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<td>2</td>
<td>5</td>
</tr>
<tr>
<td># of students that got #7 incorrect</td>
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<td>0</td>
<td>3</td>
</tr>
<tr>
<td># of students that got #8a incorrect</td>
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<td>7</td>
</tr>
<tr>
<td># of students that got #8b incorrect</td>
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<td>1</td>
<td>2</td>
</tr>
<tr>
<td># of students that got #8c incorrect</td>
<td>2</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td># of students that got #8d incorrect</td>
<td>5</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td># of students that got #9 incorrect</td>
<td>6</td>
<td>10</td>
<td>14</td>
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</tbody>
</table>

Generally, students did pretty well on this exam. Surprisingly, the question the most students got wrong was the question asking the domain of f(x). This was supposed to be an easy review question. Many students knew that they had to solve the equation when the denominator of the fraction equaled zero, but some students only determined one instead of two solutions, while other students incorrectly wrote the domain as a<x<b or a>x<b instead of a<x<b or a>x>b. Question 3 was also challenging for students, but for a different reason - it required students to think about something that I did not explicitly say in class. Instead, students had to come up with a shape that was not convex and was on the list of shapes. Hardly anyone chose d because that was the definition of convex polygons that I gave the students. Likewise, question 4 was challenging for students because I did not go in depth with systems with four or more variables. I did, however, mention in class that it not possible to graph an equation in more than three dimensions (i.e. having more than three variables), so students that listened carefully got this question correct. In fact, the very reason why I gave that question on the test in the first place was so that I could determine who and how many students would remember what I said.

Students did a very good job solving the systems. By and large, the students that got the questions wrong understood the process of elimination or substitution, but made simple algebraic mistakes that jeopardized their “solution”. I gave students partial credit if it was apparent that the student understood the concept of elimination or substitution, even if they forgot a sign or whatnot.

Students also did great on problem 8. The only trouble was when students made a mistake in part and the mistake traveled though the entire problem. In this situation, I gave students partial credit for each part of the question. I did not think it made sense for students to lose 20 points for a graphing mistake, especially since the other three parts tested skills other than graphing.

The most common error for question 9 was graphing. Students knew how to translate the word problem into a system of inequalities, but they created messy graphs and so their vertices were messed up. When they evaluated the objective function at these vertices, they usually got the wrong answer (sometimes they were just lucky). I think that some students were running out of time doing this problem and they rushed through it.
3.1-3.4 Pre-Calculus Test (Period 1)

1. All of the following are equivalent except: (5 points)
   a. Switching the x and y values of a function
   b. Finding the inverse of a function
   c. Setting the denominator of a function to zero
   d. Reflecting a function along the line y = x

2. Which of the following is an even function? (5 points)
   a. \( y = x^2 \)
   b. \( y = \frac{1}{x} \)
   c. \( y = \sqrt{x} \)
   d. \( y = x^3 \)

3. Reflecting along the line \( y = -x \) is the same as: (5 points)
   a. Switching all of the x and y values
   b. Switching all of the x and y values, and changing both of their signs
   c. Switching all of the x and y values, and changing the sign of the x values only
   d. Switching all of the x and y values, and changing the sign of the y values only

4. Which of the following statements is true? (5 points)
   a. A rational function with a vertical asymptote is always undefined on the vertical asymptote.
   b. All rational functions have asymptotes.
   c. A function can have at most one vertical asymptote.
   d. If a function is undefined for some value of x, then it cannot be rational.

5. Graph the following functions: (15 points each)
   a. \( y = 2x^2 + 2 \)
   b. \( y = -|x + 2| - 4 \)

6. Find any asymptotes, then graph: (15 points each)
   a. \( y = \frac{1}{x^2 - 2} \)
   b. \( y = \frac{x}{(x-1)(x+1)} \)

7. Graph the inverse of 5a and 5b. If the graphs of 5a and 5b are incorrect, but you correctly graph their inverse, then you may get full credit for this problem. (10 points)

8. Prove or disprove: \( y = |x| + \sqrt{2} \) is symmetric with respect to the y-axis. (10 points)
3.1-3.4 Pre-Calculus Test (Period 3)

1. All of the following are equivalent except: (5 points)
   a. Switching the x and y values of a function
   b. Setting the denominator of a function to zero
   c. Finding the inverse of a function
   d. Reflecting a function along the line y = x

2. Which of the following is an odd function? (5 points)
   a. \( y = \sqrt{x} \)
   b. \( y = \frac{1}{x} \)
   c. \( y = x^2 \)
   d. \( y = x^3 \)

3. Reflecting along the line \( y = -x \) is the same as: (5 points)
   a. Switching all of the x and y values
   b. Switching all of the x and y values, and changing the sign of the x values only
   c. Switching all of the x and y values, and changing the sign of the y values only
   d. Switching all of the x and y values, and changing both of their signs

4. Which of the following statements is true? (5 points)
   a. All rational functions have asymptotes.
   b. A rational function with a vertical asymptote is always undefined on a vertical asymptote.
   c. A function can have at most one vertical asymptote.
   d. If a function is undefined for some value of x, then it cannot be rational.

5. Graph the following functions: (15 points each)
   a. \( y = 0.5x^2 - 1 \)
   b. \( y = -\sqrt{x - 2} + 3 \)

6. Find any asymptotes, then graph: (15 points each)
   a. \( y = \frac{x}{x^2 + 1} \)
   b. \( y = \frac{1}{(x+2)(x-3)} \)

7. Graph the inverse of 5a and 5b. If the graphs of 5a and 5b are incorrect, but you correctly graph their inverse, then you may get full credit for this problem. (10 points)

8. Prove or disprove: \( y = 2x^4 + \pi \) is symmetric with respect to the y-axis. (10 points)
3.1-3.4 Pre-Calculus Test Results

<table>
<thead>
<tr>
<th></th>
<th>Average Grade</th>
<th>Highest Grade</th>
<th>Lowest Grade</th>
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</thead>
<tbody>
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<td>100</td>
<td>50</td>
</tr>
<tr>
<td>Period 3</td>
<td>79.8</td>
<td>97</td>
<td>43</td>
</tr>
</tbody>
</table>

The first two questions were extremely easy for students, and almost all of them got those ones correct. Question 3 was slightly harder, but still the vast majority got this one correct. Question 4, on the other hand was challenging for students. Many students answered “If a function is undefined for some value of x, then it cannot be rational”. This answer sounds like a correct answer. (An undefined number certainly cannot be rational, so why not a function? The flaw of this argument, of course, is that a rational number and a rational function are two entirely different things.) Most students did fine on the fifth question, but some students still had trouble determining the effect that the different terms in the equation had on the graph.

By far the most challenging problem on the exam was question 6. All students received some partial credit, but few got full credit. Most students had no difficulty determining all the asymptotes of the functions, but the difficulty lied in determining if the function was going to infinity or negative infinity. A more terrifying error that students committed was factoring the denominators incorrectly, so they got absolutely wrong graphs. I had a hard time sleeping after I discovered that some pre-calculus students didn’t know how to factor. I ended up spending a whole day reviewing factoring with the students.

Most students did well on question 7. However, those that could not answer #1 did a horrible job (no wonder…). In general, students did well on the last question, except a few of them that did not know what to do about the irrational number.

3.6-3.8 Pre-Calculus Test (Period 1)

- This is a closed book and closed notes exam.
- No calculators are allowed.
- A pencil is required. If you do not have a pencil or if you use pen, 5 points will be deducted from your test.
- \( \sqrt{\frac{1}{2}} \approx 0.707107 \)

1. Use the limit definition of the derivative to find the derivative of \( y = x^2 - 4 \). You will not receive credit for using the power rule, but you may check your answer using the power rule.

2. Using any method, find the derivative of the following:
   a. \( y = 3x^2 - 4x + 5 \)
   b. \( y = \sqrt{x} + 3 \)
   c. \( y = 2x - 4 \)
   d. \( y = 30x^3 - 3x^2 - 2\sqrt{x^3} \)
3. Identify all of the critical points of the following:
   a. \( y = x^3 - 1 \)
   b. \( y = 2x^2 - 3x \)
   c. \( y = 2x^4 - 2x^2 \)

4. Suppose the position function of a moving object is \( y = 3x^2 - 2x + 5 \) for \( 0 \leq x \leq 10 \). What is the minimum velocity of this object?

5. Are the following continuous or discontinuous? If discontinuous, identify the type of discontinuity.
   a. \( y = 3x^4 - 3x^2 + 3 \)
   b. \( y = \frac{x}{x-4} \)
   c. \( y = [x] - 1 \)
   d. \( y = \frac{x^2 - 16}{x-4} \)

6. Find:
   a. \( \lim_{x \to \infty} \frac{1}{x+1} \)
   b. \( \lim_{x \to -\infty} -3x^2 - 2x + 5 \)
   c. \( \lim_{x \to \infty} \frac{x}{2x-4} \)

3.6-3.8 Pre-Calculus Test Results

<table>
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<tr>
<th>Period</th>
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<th>Lowest Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>71.0</td>
<td>100</td>
<td>50</td>
</tr>
</tbody>
</table>

Students did very well on the first question, which was a nice surprise because I expected it to be more challenging than it actually was. This was probably due to the fact that I told the students such a question would appear on the test, so they knew it was coming. Students also did a great job on number 2, which was essential skill to have to pass the exam in the first place. A few students did not know what to do with the exponent inside of the square root (something that I later reviewed in a do-now.) Question 3 was so-so. Students knew how to take the first and second derivatives of the functions, but finding the zeros of them was another matter. Part c was especially difficult for students because they did not know what to do with the cubic first derivative (factoring didn’t come to their mind.)

Question 4 was a disaster. Students knew that they had to take the derivative, but was it the first, second, or third that they had to equate to zero? Velocity is the first derivative of the position function, so the second derivative must equal zero to locate the minimum. Half of the students could not make it this far. The second battle was checking the endpoints, which many students ignored, therefore getting the answer wrong. After the exam, I thoroughly went over this problem, and I explained why velocity was the first derivative, and why the minimum of velocity was when the second derivative was zero or at an endpoint.

Question 5 was easy for students. Almost everyone correctly identified discontinuous functions, but there were a few that confused the different types of discontinuity. The final question on the exam was quite challenging for students probably because I went over end behavior the Monday and Tuesday before the exam. I seriously
questioned whether or not I should have postponed the test one more week or not. I really hated that policy were math exams had to be given on Wednesdays, that is for sure!

Advanced Algebra Chapter 3 Test

1. Solve the following by graphing the system of linear equations: (20 points)
   a. \[ \begin{cases} \frac{1}{2}x - 3y = 10 \\ \frac{1}{4}x + 2y = -2 \end{cases} \]
   b. \[ \begin{cases} y = 2x - 1 \\ -6x + 3y = -3 \end{cases} \]

2. Solve one of the following systems of equations algebraically: (20 points)
   a. \[ \begin{cases} \frac{2}{3}x + 3y = -34 \\ x - \frac{1}{2}y = -1 \end{cases} \]
   b. \[ \begin{cases} 7x + 5y = -12 \\ 3x - 4y = 1 \end{cases} \]

3. Solve one of the following systems of equations algebraically: (30 points)
   a. \[ \begin{cases} 2x - y + 4z = 19 \\ -x + 3y - 2z = -7 \\ 4x + 2y + 3z = 37 \end{cases} \]
   b. \[ \begin{cases} x + y + z = 3 \\ 3x - 4y + 2z = -28 \\ -x + 5y + z = 23 \end{cases} \]

4. Show the solution of the following systems of inequalities by graphing: (10 pts)
   a. \[ \begin{cases} 3x - y > 12 \\ -x + 8y > -4 \end{cases} \]
   b. \[ \begin{cases} x - 4y \leq -10 \\ y \leq 3|x - 1| \end{cases} \]

5. Choose one word problem to solve: (20 points)
   a. Darlene is making a quilt that has alternating stripes of regular quilting fabric and sateen fabric. She spends $76 on a total of 17 yards of the two fabrics at a fabric store. Sateen fabric costs $5 per yard and quilting fabric costs $3 per yard. How many yards of each type of fabric did she buy?

   b. Many businesses pay website hosting companies to store and maintain the computer files that make up their websites. Internet service providers also offer website hosting. The costs for website hosting offered by a website hosting company and an Internet service provider are shown in the table. Find the number of months after which the total cost for website hosting will be the same for both companies.

<table>
<thead>
<tr>
<th>Company</th>
<th>Set-up fee (dollars)</th>
<th>Cost per month (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internet service provider</td>
<td>10</td>
<td>21.45</td>
</tr>
<tr>
<td>Website hosting company</td>
<td>None</td>
<td>21.85</td>
</tr>
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</table>
Bonus: Write a system of linear inequalities for the shaded region:

Advanced Algebra Chapter 3 Test Results

<table>
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Students did pretty well on the first three questions. Just like the pre-calculus students, most advanced algebra students understood the process of solving a system of equations, but made algebraic mistakes that messed up the whole problem. Still, some students had no idea what they were doing. Needless to say, these students didn’t bother doing homework either.

Question 4 was a bit harder for students because there were many little things to mess up on, such as choosing a solid or dashed line, where to shade, the slopes, and the y-intercepts.

Question 5 was interesting, because the second word problem was much easier to solve that the first, yet barely anyone chose to solve the second problem. In fact, the second word problem only requires one equation! I purposely did this just to see if anyone would cleverly chose whichever problem would be easier to solve. Perhaps many students were scared when they saw the decimals in the second problem. Another theory is that students attempted the first word problem and only worked on the second word problem only if they got stuck, which I think was the most likely case. Students did okay on this problem. Most students were able to translate the word problem into a system of equations, but like problems 1 and 2, students performed various algebraic mistakes solving for the answer.

The bonus was extremely easy for students that bothered doing it. Most students decided that they didn’t have time for a very easy question. I guess the word “bonus” disguised the problem as being much harder than it actually was. Oh well…

Notice the huge difference between the highest score and the lowest score. This represents the tremendous range of ability that students have in the honors advanced algebra class. Clearly, there are students that do not belong in the honors level. Some students belong in level 1, while others belong in a more advanced class. Unfortunately though, Doherty does not offer high honors classes (like Lowell High School), so challenging the brightest students and at the same time ensuring that the slowest students had an idea what was going on was an ever present issue.
Extra Credit

The final form of assessment that I used when I was teaching at Doherty were extra credit assignments. I only gave two extra credit assignments to my pre-calculus classes. The purpose of extra credit was to challenge the brightest students and to see how well students are able to apply what they have learned. There were students that have managed to get perfect scores on both extra credit assignments; however, most students did not attempt the linear programming extra credit assignment. Note that the syllabus for pre-calculus explicitly says that no extra credit would be given, however, my mentor allowed me to bend this rule. I graded the extra credit assignments just like any other homework assignment, granting partial credit for what was done correctly. I allowed students to work on extra credit together. Calculators and any other resources were also allowed. The linear programming extra credit assignment was actually a homework problem that I was given in MA 2071.

Extra Credit Assignment 1

For this assignment, students were asked to write a system of inequalities that can be used to solve the following linear programming problem:

1.2 A small airline, Ivy Air, flies between three cities: Ithaca, Newark, and Boston. They offer several flights but, for this problem, let us focus on the Friday afternoon flight that departs from Ithaca, stops in Newark, and continues to Boston. There are three types of passengers:
   (a) Those traveling from Ithaca to Newark.
   (b) Those traveling from Newark to Boston.
   (c) Those traveling from Ithaca to Boston.

   The aircraft is a small commuter plane that seats 30 passengers. The airline offers three fare classes:
   (a) Y class: full coach.
   (b) B class: nonrefundable.
   (c) M class: nonrefundable, 3-week advanced purchase.

   Ticket prices, which are largely determined by external influences (i.e., competitors), have been set and advertised as follows:

<table>
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<tr>
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<th>Ithaca–Newark</th>
<th>Newark–Boston</th>
<th>Ithaca–Boston</th>
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<td>300</td>
<td>160</td>
<td>360</td>
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<tr>
<td>B</td>
<td>220</td>
<td>130</td>
<td>280</td>
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<tr>
<td>M</td>
<td>100</td>
<td>80</td>
<td>140</td>
</tr>
</tbody>
</table>

   Based on past experience, demand forecasters at Ivy Air have determined the following upper bounds on the number of potential customers in each of the 9 possible origin-destination/fare-class combinations:

<table>
<thead>
<tr>
<th></th>
<th>Ithaca–Newark</th>
<th>Newark–Boston</th>
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<td>Y</td>
<td>4</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>B</td>
<td>8</td>
<td>13</td>
<td>10</td>
</tr>
<tr>
<td>M</td>
<td>22</td>
<td>20</td>
<td>18</td>
</tr>
</tbody>
</table>

   The goal is to decide how many tickets from each of the 9 origin/destination/fare-class combinations to sell. The constraints are that the plane cannot be overbooked on either of the two legs of the flight and that the number of tickets made available cannot exceed the forecasted maximum demand. The objective is to maximize the revenue. Formulate this problem as a linear programming problem.
Extra Credit Assignment 2

Students were asked to find the equation of the line tangent to the given curve at the given values of x. Students had to approximate the slope by calculating the secant slope of two points close to the given values of x. This proved to be very challenging for students even though they were allowed to use graphing calculators because many of them were unfamiliar with some of the functions, such as $y = \ln(x)$.

Period 1:

1. $y = 3^x$ when $x = 2$
2. $y = (x + 2)(x - 2)$ when $x = -2$
3. $y = \sin(x)$ when $x = \pi$ (in radians)
4. $y = \ln(x)$ when $x = 10$
5. $y = \frac{x}{\sin(x)}$ when $x = \pi/2$ (in radians)
6.

Period 3:

1. $y = -\sqrt{x}$ when $x = 1$
2. $y = 2x^2$ when $x = -2$
3. $y = 1/x$ when $x = 1$
4. $y = -x^3$ when $x = -3$
5. $y = \sin(x)$ when $x = \pi/2$ (in radians)

Conclusion

Assessment is essential for all teachers because instructors need to know if their students are improving and becoming proficient in course objectives. Since different forms of assessments have various advantages and disadvantages, a variety of them must be developed. In order for assessments to be as useful as possible for teachers, they must accurately reflect the courses goals and expected outcomes. In recent years, assessment has become ever more important due to statewide testing and increasingly demanding standards. Assessment is also important because most students require satisfactory college entrance exam scores to go to college. Furthermore, as a nation we have been outperformed by several European and Asian countries in terms of academic performance, and without carefully assessing students’ progress and achievement we cannot hope to improve our global ranking. Therefore, it is every teacher’s responsibility to ensure that their students are receiving a quality education, and that they are making steady progress to be competitive members of global society in the future.
Closing Remarks

Teaching at Doherty was one of the most noteworthy experiences that I had in my life. I went into that school thinking that teaching would be a highly fulfilling accomplishment, and it was. I thought that it would improve the way I communicate with others and the way I present myself, and it did. I thought that the experience would make me more confident with myself and be more relaxed speaking in public, and it did. What struck me more, however, was what I thought that it would be but was not. I believed that it would be simple to teach people things that I already knew, but it definitely was not. I believed that simply observing teachers do their job for years would be enough experience to be an effective teacher, but it was not. I believed that I would rarely make mistakes, but I made mistakes constantly. The amount that I grew due to this experience was due to these unexpected hurdles. Are not the greatest achievements of mankind the result of unexpected drawbacks?

Teaching is a beautiful affair, one like no other. Our brains are most precious to us, and their contents most revered. To spread knowledge, an artifact of truth, and give it to a future generation, so that they may utilize it, treasure it, extend it, and do what they wish with it, is an intimate thing. Teachers hope that that artifact grows into something greater, something modern and advanced, so that the future society, the society that we may never realize, will flourish off of it. Teachers care for the future, and remain selfless and generous, so much unlike the temptations of human nature to be vain and narcissistic.

I did not know these things before I taught at Doherty, but my experience there made me realize these wonderful things. I am not blind of the faults that exist in the education system today, but I can understand the riches of it as well. Yes, there are unmotivated students, awful textbooks, and corrupt politics choking the public educational system. Everyone knows that. What is constantly being overlooked, however, are its treasures. We have a free educational system in which everyone has the right and the freedom to receive. Is not that a tremendous gift alone? Children of all cultures, races, creeds, appearances, and aspirations go to school under one roof and collectively work together, learn together, eat together, and laugh together. The world of discrimination, prejudice, and persecution is fading rapidly, and once gone imagine the results of the cooperation of one seamless united human race!

I refuse to call this section a conclusion, because it most certainly is not. Instead it is the beginning of my journey. I may have finished teaching a Doherty, but I will continue educating, and I will continue growing. Currently, I am a peer learning assistant (PLA) at WPI for differential equations. Next term, I will continue being PLA, but for calculus II. In the future, I want to tutor children so that I can better understand how they think and how they make errors, so that when I seek a permanent teaching career, I will be able to predict where students may go astray before they actually do. Teaching is a never ending profession, and it takes a great deal of effort and experience to excel in it. The demands for teachers are constantly increasing as more stringent standards are made and the distractions of our modern world increase. However, it is also an extremely fulfilling career because teachers know that what they do truly matters. I experienced this feeling teaching at Doherty, and I know that educating is what I truly want to continue doing.
Overview

Number and Quantity
The Complex Number System
• Perform arithmetic operations with complex numbers.
• Use complex numbers in polynomial identities and equations.
Vector and Matrix Quantities
• Represent and model with vector quantities.
• Perform operations on matrices and use matrices in applications.

Algebra
Seeing Structure in Expressions
• Interpret the structure of expressions.
• Write expressions in equivalent forms to solve problems.
Arithmetic with Polynomials and Rational Expressions
• Perform arithmetic operations on polynomials.
• Understand the relationship between zeros and factors of polynomials.
• Use polynomial identities to solve problems.
• Rewrite rational expressions.
Creating Equations
• Create equations that describe numbers or relationships.
Reasoning with Equations and Inequalities
• Understand solving equations as a process of reasoning and explain the reasoning.
• Represent and solve equations and inequalities graphically.

Functions
Interpreting Functions
• Interpret functions that arise in applications in terms of the context.
• Analyze functions using different representations.
Building Functions
• Build a function that models a relationship between two quantities.
• Build new functions from existing functions.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for an express regularity in repeated reasoning.

Functions (cont’d.)
Linear, Quadratic, and Exponential Models
• Construct and compare linear, quadratic, and exponential models and solve problems.
Trigonometric Functions
• Extend the domain of trigonometric functions using the unit circle.
• Model periodic phenomena with trigonometric functions.
• Prove and apply trigonometric identities.
Statistics and Probability
Interpreting Categorical and Quantitative Data
• Summarize, represent and interpret data on a single count or measurement variable.
Making Inferences and Justifying Conclusions
• Understand and evaluate random processes underlying statistical experiments.
• Make inferences and justify conclusions from sample surveys, experiments and observational studies.
Using Probability to Make Decisions
• Use probability to evaluate outcomes of decisions.
Appendix A: 2011 Massachusetts Curriculum Frameworks for Algebra 2

Number and Quantity
The Complex Number System
N-CN
Perform arithmetic operations with complex numbers.
1. Know there is a complex number $i$ such that $i^2 = -1$, and every complex number has the form $a + bi$ with $a$ and $b$ real.
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Use complex numbers in polynomial identities and equations.
7. Solve quadratic equations with real coefficients that have complex solutions.
8. (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities
N-VM
Represent and model with vector quantities.
1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\mathbf{v}$, $|\mathbf{v}|$, $||\mathbf{v}||$, $\mathbf{v}$).
3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on matrices and use matrices in applications.
6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
12. (+) Work with $2 \times 2$ matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Algebra
Seeing Structure in Expressions
A-SSE
Interpret the structure of expressions.
1. Interpret expressions that represent a quantity in terms of its context. *
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of $P$ and a factor not depending on $P$.
2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems.
4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. *

Arithmetic with Polynomials and Rational Expressions
A-APR
Perform arithmetic operations on polynomials.
1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
   MA.1.a. Divide polynomials.

* indicates Modeling standard.
(+) indicates standard beyond College and Career Ready.
Appendix A: 2011 Massachusetts Curriculum Frameworks for Algebra 2

Understand the relationship between zeros and factors of polynomials.

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

4. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of $x$ and $y$ for a positive integer $n$, where $x$ and $y$ are any numbers, with coefficients determined for example by Pascal’s Triangle.

Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Creating Equations

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law $V = IR$ to highlight resistance $R$.

Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning.

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Represent and solve equations and inequalities graphically.

11. Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

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74 The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

* indicates Modeling standard.

(+) indicates standard beyond College and Career Ready.

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Appendix A: 2011 Massachusetts Curriculum Frameworks for Algebra 2

Functions

Interpreting Functions

Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function \( h(n) \) gives the number of person-hours it takes to assemble \( n \) engines in a factory, then the positive integers would be an appropriate domain for the function.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
   b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
   c. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   MA.8.c. Translate among different representations of functions and relations: graphs, equations, point sets, and tables.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.
   b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Build new functions from existing functions.

3. Identify the effect on the graph of replacing \( f(x) \) by \( f(x) + k \), \( kf(x) \), \( f(kx) \), and \( f(x + k) \) for specific values of \( k \) (both positive and negative); find the value of \( k \) given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

4. Find inverse functions.
   a. Solve an equation of the form \( f(x) = c \) for a simple function \( f \) that has an inverse and write an expression for the inverse. For example, \( f(x) = 2x^3 \) or \( f(x) = (x + 1)/(x - 1) \) for \( x \neq 1 \).

Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models and solve problems.

4. For exponential models, express as a logarithm the solution to \( ab^ct = d \) where \( a, c, \) and \( d \) are numbers and the base \( b \) is 2, 10, or \( e \); evaluate the logarithm using technology.

* indicates Modeling standard.
Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.
1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Model periodic phenomena with trigonometric functions.
5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. *

Prove and apply trigonometric identities.
8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant.

Statistics and Probability

Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable.
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. *

Making Inferences and Justifying Conclusions

Understand and evaluate random processes underlying statistical experiments.
1. Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that population. *
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? *

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. *
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. *
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. *
6. Evaluate reports based on data. *

Using Probability to Make Decisions

Use probability to evaluate outcomes of decisions.
6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). *
7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).75 *

* indicates Modeling standard.

75 Replacing the hockey goalie with an extra skater.

(+) indicates standard beyond College and Career Ready.
Appendix B: 2011 Massachusetts Curriculum Frameworks for Pre-Calculus

Introduction

Precalculus combines the trigonometric, geometric, and algebraic techniques needed to prepare students for the study of calculus, and strengthens students' conceptual understanding of problems and mathematical reasoning in solving problems. Facility with these topics is especially important for students intending to study calculus, physics, and other sciences, and/or engineering in college. Because the standards for this course are (+) standards, students selecting this Model Precalculus course should have met the college and career ready standards.

For the high school Model Precalculus course, instructional time should focus on four critical areas: (1) extend work with complex numbers; (2) expand understanding of logarithms and exponential functions; (3) use characteristics of polynomial and rational functions to sketch graphs of those functions; and (4) perform operations with vectors.

(1) Students continue their work with complex numbers. They perform arithmetic operations with complex numbers and represent them and the operations on the complex plane. Students investigate and identify the characteristics of the graphs of polar equations, using graphing tools. This includes classification of polar equations, the effects of changes in the parameters in polar equations, conversion of complex numbers from rectangular form to polar form and vice versa, and the intersection of the graphs of polar equations.

(2) Students expand their understanding of functions to include logarithmic and trigonometric functions. They investigate and identify the characteristics of exponential and logarithmic functions in order to graph these functions and solve equations and practical problems. This includes the role of \( e \), natural and common logarithms, laws of exponents and logarithms, and the solutions of logarithmic and exponential equations. Students model periodic phenomena with trigonometric functions and prove trigonometric identities. Other trigonometric topics include reviewing unit circle trigonometry, proving trigonometric identities, solving trigonometric equations, and graphing trigonometric functions.

(3) Students investigate and identify the characteristics of polynomial and rational functions and use these to sketch the graphs of the functions. They determine zeros, upper and lower bounds, \( y \)-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. Students translate between the geometric description and equation of conic sections. They deepen their understanding of the Fundamental Theorem of Algebra.

(4) Students perform operations with vectors in the coordinate plane and solve practical problems using vectors. This includes the following topics: operations of addition, subtraction, scalar multiplication, and inner (dot) product; norm of a vector; unit vector; graphing; properties; simple proofs; complex numbers (as vectors); and perpendicular components.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.
Overview

Number and Quantity

The Complex Number System
- Perform arithmetic operations with complex numbers.
- Represent complex numbers and their operations on the complex plane.
- Use complex numbers in polynomial identities and equations.

Vector and Matrix Quantities
- Represent and model with vector quantities.
- Perform operations on vectors.
- Perform operations on matrices and use matrices in applications.

Algebra

Arithmetic with Polynomials and Rational Expressions
- Use polynomial identities to solve problems
- Rewrite rational expressions.

Reasoning with Equations and Inequalities
- Solve systems of equations.

Functions

Interpreting Functions
- Analyze functions using different representations.

Building Functions
- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

Trigonometric Functions
- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric functions.
- Prove and apply trigonometric identities.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for an express regularity in repeated reasoning.

Geometry

Similarity, Right Triangles, and Trigonometry
- Apply trigonometry to general triangles.

Circles
- Understand and apply theorems about circles.

Expressing Geometric Properties with Equations
- Translate between the geometric description and the equation for a conic section.

Geometric Measurement and Dimension
- Explain volume formulas and use them to solve problems.
- Visualize relationships between two-dimensional and three-dimensional objects.
Appendix B: 2011 Massachusetts Curriculum Frameworks for Pre-Calculus

Content Standards

Number and Quantity

The Complex Number System

Perform arithmetic operations with complex numbers.

3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, \((-1 + \sqrt{3}i)^3 = 8\) because \((-1 + \sqrt{3}i)\) has modulus 2 and argument 120°.

6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.

8. (+) Extend polynomial identities to the complex numbers. For example, rewrite \(x^2 + 4\) as \((x + 2i)(x - 2i)\).

9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities

Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., \(\mathbf{v}\), \(|\mathbf{v}|\), \(\mathbf{v}\)).

2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

4. (+) Add and subtract vectors.
   a. (+) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
   b. (+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
   c. (+) Understand vector subtraction \(\mathbf{v} - \mathbf{w}\) as \(\mathbf{v} + (-\mathbf{w})\), where \(-\mathbf{w}\) is the additive inverse of \(\mathbf{w}\), with the same magnitude as \(\mathbf{w}\) and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

5. (+) Multiply a vector by a scalar.
   a. (+) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as \(c(\mathbf{v}_x, \mathbf{v}_y) = (cv_x, cv_y)\).
   b. (+) Compute the magnitude of a scalar multiple \(c\mathbf{v}\) using \(||c\mathbf{v}|| = |c||\mathbf{v}|\). Compute the direction of \(c\mathbf{v}\) knowing that when \(|c|\mathbf{v} \neq 0\), the direction of \(c\mathbf{v}\) is either along \(\mathbf{v}\) (for \(c > 0\)) or against \(\mathbf{v}\) (for \(c < 0\)).

(+) indicates standard beyond College and Career Ready.
Appendix B: 2011 Massachusetts Curriculum Frameworks for Pre-Calculus

Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
12. (+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Algebra

Arithmetic with Polynomials and Rational Expressions A-APR

Use polynomial identities to solve problems.

5. (+) Know and apply the Binomial Theorem for the expansion of \((x + y)^n\) in powers of \(x\) and \(y\) for a positive integer \(n\), where \(x\) and \(y\) are any numbers, with coefficients determined for example by Pascal's Triangle.\(^76\)

Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; write \(a(x)/b(x)\) in the form \(q(x) + r(x)/b(x)\), where \(a(x), b(x), q(x), \) and \(r(x)\) are polynomials with the degree of \(r(x)\) less than the degree of \(b(x)\), using inspection, long division, or, for the more complicated examples, a computer algebra system.
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Reasoning with Equations and Inequalities A-REI

Solve systems of equations.

8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.
9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Functions

Interpreting Functions F-IF

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *
   d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. *

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\(^76\) The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument. (+) indicates standard beyond College and Career Ready.

* indicates Modeling standard.
### Building Functions  

**F-BF**

**Build a function that models a relationship between two quantities.**

1. Write a function that describes a relationship between two quantities. *
   c. (+) Compose functions. For example, if \( T(y) \) is the temperature in the atmosphere as a function of height, and \( h(t) \) is the height of a weather balloon as a function of time, then \( T(h(t)) \) is the temperature at the location of the weather balloon as a function of time. *

**Build new functions from existing functions.**

4. Find inverse functions.
   b. (+) Verify by composition that one function is the inverse of another.
   c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
   d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

### Trigonometric Functions  

**F-TF**

**Extend the domain of trigonometric functions using the unit circle.**

3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for \( \pi/3 \), \( \pi/4 \) and \( \pi/6 \), and use the unit circle to express the values of sine, cosine, and tangent for \( \pi - x \), \( \pi + x \), and \( 2\pi - x \) in terms of their values for \( x \), where \( x \) is any real number.

4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

**Model periodic phenomena with trigonometric functions.**

6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. *

**Prove and apply trigonometric identities.**

9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

### Geometry  

**G-SRT**

**Apply trigonometry to general triangles.**

9. (+) Derive the formula \( A = \frac{1}{2}ab \sin(C) \) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.

11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

**Circles  

**G-C**

**Understand and apply theorems about circles.**

4. (+) Construct a tangent line from a point outside a given circle to the circle.

* indicates Modeling standard.
(+)(+) indicates standard beyond College and Career Ready.
Expressing Geometric Properties with Equations  
G-GPE

Translate between the geometric description and the equation for a conic section.

3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
   MA.3.a. (+) Use equations and graphs of conic sections to model real-world problems. *

Geometric Measurement and Dimension  
G-GMD

Explain volume formulas and use them to solve problems.

2. (+) Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures.

Visualize relationships between two-dimensional and three-dimensional objects.

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

* indicates Modeling standard.
(+) indicates standard beyond College and Career Ready.
Appendix C: 2000 Massachusetts Curriculum Frameworks for Algebra 2

Learning Standards for Algebra II

Note: The parentheses at the end of a learning standard contain the code number for the corresponding standard in the two-year grade spans.

### Number Sense and Operations

**Understand numbers**, ways of representing numbers, relationships among numbers, and number systems

**Understand meanings** of operations and how they relate to one another

**Compute fluently** and make reasonable estimates

**Students engage in problem solving, communicating, reasoning, connecting, and representing as they:**

| N.1 | Define complex numbers (e.g., $a + bi$) and operations on them, in particular, addition, subtraction, multiplication, and division. Relate the system of complex numbers to the systems of real and rational numbers. (12.N.1) |
| N.2 | Simplify numerical expressions with powers and roots, including fractional and negative exponents. (12.N.2) |

### Patterns, Relations, and Algebra

**Understand patterns**, relations, and functions

**Represent and analyze** mathematical situations and structures using algebraic symbols

**Use mathematical models** to represent and understand quantitative relationships

**Analyze change** in various contexts

**Students engage in problem solving, communicating, reasoning, connecting, and representing as they:**

| P.1 | Describe, complete, extend, analyze, generalize, and create a wide variety of patterns, including iterative and recursive patterns such as Pascal’s Triangle. (12.P.1) |
| P.2 | Identify arithmetic and geometric sequences and finite arithmetic and geometric series. Use the properties of such sequences and series to solve problems, including finding the formula for the general term and the sum, recursively and explicitly. (12.P.2) |
| P.3 | Demonstrate an understanding of the binomial theorem and use it in the solution of problems. (12.P.3) |
| P.4 | Demonstrate an understanding of the exponential and logarithmic functions. |
| P.5 | Perform operations on functions, including composition. Find inverses of functions. (12.P.5) |
| P.6 | Given algebraic, numeric and/or graphical representations, recognize functions as polynomial, rational, logarithmic, or exponential. (12.P.6) |
| P.7 | Find solutions to quadratic equations (with real coefficients and real or complex roots) and apply to the solutions of problems. (12.P.7) |
| P.8 | Solve a variety of equations and inequalities using algebraic, graphical, and numerical methods, including the quadratic formula; use technology where appropriate. Include polynomial, exponential, and logarithmic functions; expressions involving the absolute values; and simple rational expressions. (12.P.8) |
| P.9 | Use matrices to solve systems of linear equations. Apply to the solution of everyday problems. (12.P.9) |
| P.10 | Use symbolic, numeric, and graphical methods to solve systems of equations and/or inequalities involving algebraic, exponential, and logarithmic expressions. Also use technology where appropriate. Describe the relationships among the methods. (12.P.10) |
| P.11 | Solve everyday problems that can be modeled using polynomial, rational, exponential, logarithmic, and step functions, absolute values and square roots. Apply appropriate graphical, tabular, or symbolic methods to the solution. Include growth and decay; logistic growth; joint (e.g., $I = Prt$, $y = k(w_1 + w_2)$), and combined $(F = G(m_1,m_2)/d^2)$ variation. (12.P.11) |
| P.12 | Identify maximum and minimum values of functions in simple situations. Apply to the solution of problems. (12.P.12) |
| P.13 | Describe the translations and scale changes of a given function $f(x)$ resulting from substitutions for the various parameters $a$, $b$, $c$, and $d$ in $y = af(b(x + c) + d)$. In particular, describe the effect of such transformations. |
### Data Analysis, Statistics, and Probability

- **Formulate questions** that can be addressed with data and collect, organize, and display relevant data to answer them.
- **Select and use** appropriate statistical methods to analyze data.
- **Develop and evaluate** inferences and predictions that are based on data.
- **Understand and apply** basic concepts of probability.

*Students engage in problem solving, communicating, reasoning, connecting, and representing as they:*

- **D.1** Select an appropriate graphical representation for a set of data and use appropriate statistics (e.g., quartile or percentile distribution) to communicate information about the data. (12.D.2)
- **D.2** Use combinatorics (e.g., “fundamental counting principle,” permutations, and combinations) to solve problems, in particular, to compute probabilities of compound events. Use technology as appropriate. (12.D.6)
# Learning Standards for Precalculus

Note: The parentheses at the end of a learning standard contain the code number for the corresponding standard in the two-year grade spans.

## Learning Standards for Number Sense and Operations

- **Understand numbers**, ways of representing numbers, relationships among numbers, and number systems
- **Understand meanings** of operations and how they relate to one another
- **Compute fluently** and make reasonable estimates

Students engage in problem solving, communicating, reasoning, connecting, and representing as they:

**N.1** Plot complex numbers using both rectangular and polar coordinates systems. Represent complex numbers using polar coordinates, i.e., \(a + bi = r(\cos \theta + i\sin \theta)\). Apply DeMoivre’s theorem to multiply, take roots, and raise complex numbers to a power.

## Patterns, Relations, and Algebra

- **Understand patterns**, relations, and functions
- **Represent and analyze** mathematical situations and structures using algebraic symbols
- **Use mathematical models** to represent and understand quantitative relationships
- **Analyze change** in various contexts

Students engage in problem solving, communicating, reasoning, connecting, and representing as they:

**P.1** Use mathematical induction to prove theorems and verify summation formulas, e.g., verify \[
\sum_{k=1}^{n} k^2 = \frac{n(n + 1)(2n + 1)}{6}.
\]

**P.2** Relate the number of roots of a polynomial to its degree. Solve quadratic equations with complex coefficients.

**P.3** Demonstrate an understanding of the trigonometric functions (sine, cosine, tangent, cosecant, secant, and cotangent). Relate the functions to their geometric definitions.

**P.4** Explain the identity \(\sin^2 \theta + \cos^2 \theta = 1\). Relate the identity to the Pythagorean theorem.

**P.5** Demonstrate an understanding of the formulas for the sine and cosine of the sum or the difference of two angles. Relate the formulas to DeMoivre’s theorem and use them to prove other trigonometric identities. Apply to the solution of problems.

**P.6** Understand, predict, and interpret the effects of the parameters \(a\), \(\omega\), \(b\), and \(c\) on the graph of \(y = a\sin(\omega(x - b)) + c\); similarly for the cosine and tangent. Use to model periodic processes. (12.P.13)

**P.7** Translate between geometric, algebraic, and parametric representations of curves. Apply to the solution of problems.

**P.8** Identify and discuss features of conic sections: axes, foci, asymptotes, and tangents. Convert between different algebraic representations of conic sections.

**P.9** Relate the slope of a tangent line at a specific point on a curve to the instantaneous rate of change. Explain the significance of a horizontal tangent line. Apply these concepts to the solution of problems.
Appendix D: 2000 Massachusetts Curriculum Frameworks for Precalculus

**Geometry**

Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

Specify locations and describe spatial relationships using coordinate geometry and other representational systems

Apply transformations and use symmetry to analyze mathematical situations

Use visualization, spatial reasoning, and geometric modeling to solve problems

**Students engage in problem solving, communicating, reasoning, connecting, and representing as they:**

G.1 Demonstrate an understanding of the laws of sines and cosines. Use the laws to solve for the unknown sides or angles in triangles. Determine the area of a triangle given the length of two adjacent sides and the measure of the included angle. (12.G.2)

G.2 Use the notion of vectors to solve problems. Describe addition of vectors, multiplication of a vector by a scalar, and the dot product of two vectors, both symbolically and geometrically. Use vector methods to obtain geometric results. (12.G.3)

G.3 Apply properties of angles, parallel lines, arcs, radii, chords, tangents, and secants to solve problems. (12.G.5)

**Measurement**

Understand measurable attributes of objects and the units, systems, and processes of measurement

Apply appropriate techniques, tools, and formulas to determine measurements

**Students engage in problem solving, communicating, reasoning, connecting, and representing as they:**

M.1 Describe the relationship between degree and radian measures, and use radian measure in the solution of problems, in particular problems involving angular velocity and acceleration. (12.M.1)

M.2 Use dimensional analysis for unit conversion and to confirm that expressions and equations make sense. (12.M.2)

**Data Analysis, Statistics, and Probability**

Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them

Select and use appropriate statistical methods to analyze data

Develop and evaluate inferences and predictions that are based on data

Understand and apply basic concepts of probability

**Students engage in problem solving, communicating, reasoning, connecting, and representing as they:**

D.1 Design surveys and apply random sampling techniques to avoid bias in the data collection. (12.D.1)

D.2 Apply regression results and curve fitting to make predictions from data. (12.D.3)

D.3 Apply uniform, normal, and binomial distributions to the solutions of problems. (12.D.4)

D.4 Describe a set of frequency distribution data by spread (variance and standard deviation), skewness, symmetry, number of modes, or other characteristics. Use these concepts in everyday applications. (12.D.5)

D.5 Compare the results of simulations (e.g., random number tables, random functions, and area models) with predicted probabilities. (12.D.7)
Appendix E: NCTM Curriculum Standards for Grades 9-12

NCTM Curriculum Standards for Grades 9-12

From the website of the National Council of Teachers of Mathematics (http://www.nctm.org).

Standard 1: Mathematics as Problem Solving.
In grades 9-12, the mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can--

- use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content;
- apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics;
- recognize and formulate problems from situations within and outside mathematics;
- apply the process of mathematical modeling to real-world problem situations.

Standard 2: Mathematics as Communication.
In grades 9-12, the mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can--

- reflect upon and clarify their thinking about mathematical ideas and relationships;
- formulate mathematical definitions and express generalizations discovered through investigations;
- express mathematical ideas orally and in writing;
- read written presentations of mathematics with understanding;
- ask clarifying and extending questions related to mathematics they have read or heard about;
- appreciate the economy, power, and elegance of mathematical notation and its role in the development of mathematical ideas.

Standard 3: Mathematics as Reasoning.
In grades 9-12, the mathematics curriculum should include numerous and varied experiences that reinforce and extend logical reasoning skills so that all students can--

- make and test conjectures;
- formulate counterexamples;
- follow logical arguments;
- judge the validity of arguments;
- construct simple valid arguments;

and so that, in addition, college-intending students can--

- construct proofs for mathematical assertions, including indirect proofs and proofs by mathematical induction.

Standard 4: Mathematical Connections.
In grades 9-12, the mathematics curriculum should include investigation of the connections and interplay among various mathematical topics and their applications so that all students can--

- recognize equivalent representations of the same concept;
- relate procedures in one representation to procedures in an equivalent representation;

http://www.ms.uky.edu/~lee/ma310/anec/node2.html
Appendix E: NCTM Curriculum Standards for Grades 9-12

- use and value the connections among mathematical topics;
- use and value the connections between mathematics and other disciplines.

**Standard 5: Algebra.**
In grades 9-12, the mathematics curriculum should include the continued study of algebraic concepts and methods so that all students can--

- represent situations that involve variable quantities with expressions, equations, inequalities, and matrices;
- use tables and graphs as tools to interpret expressions, equations, and inequalities;
- operate on expressions and matrices, and solve equations and inequalities;
- appreciate the power of mathematical abstraction and symbolism;

and so that, in addition, college-intending students can-

- use matrices to solve linear systems;
- demonstrate technical facility with algebraic transformations, including techniques based on the theory of equations.

**Standard 6: Functions.**
In grades 9-12, the mathematics curriculum should include the continued study of functions so that all students can--

- model real-world phenomena with a variety of functions;
- represent and analyze relationships using tables, verbal rules, equations, and graphs;
- translate among tabular, symbolic, and graphical representations of functions;
- recognize that a variety of problem situations can be modeled by the same type of function;
- analyze the effects of parameter changes on the graphs of functions;

and so that, in addition, college-intending students can--

- understand operations on, and the general properties and behavior of, classes of functions.

**Standard 7: Geometry From a Synthetic Perspective.**
In grades 9-12, the mathematics curriculum should include the continued study of the geometry of two and three dimensions so that all students can--

- interpret and draw three-dimensional objects;
- represent problem situations with geometric models and apply properties of figures;
- classify figures in terms of congruence and similarity and apply these relationships;
- deduce properties of, and relationships between, figures from given assumptions;

and so that, in addition, college-intending students can-

- develop an understanding of an axiomatic system through investigating and comparing various geometries.

**Standard 8: Geometry From an Algebraic Perspective.**
In grades 9-12, the mathematics curriculum should include the study of the geometry of two and three dimensions from an algebraic point of view so that all students can--

- translate between synthetic and coordinate representations;
Appendix E: NCTM Curriculum Standards for Grades 9-12

- deduce properties of figures using transformations and using coordinates;
- identify congruent and similar figures using transformations;
- analyze properties of Euclidean transformations and relate translations to vectors;

and so that, in addition, college-intending students can-

- deduce properties of figures using vectors;
- apply transformations, coordinates, and vectors in problem solving.

**Standard 9: Trigonometry.**

In grades 9-12, the mathematics curriculum should include the study of trigonometry so that all students can--

- apply trigonometry to problem situations involving triangles;
- explore periodic real-world phenomena using the sine and cosine functions;

and so that, in addition, college-intending students can--

- understand the connection between trigonometric and circular functions;
- use circular functions to model periodic real-world phenomena;
- apply general graphing techniques to trigonometric functions;
- solve trigonometric equations and verify trigonometric identities;
- understand the connections between trigonometric functions and polar coordinates, complex numbers, and series.

**Standard 10: Statistics.**

In grades 9-12, the mathematics curriculum should include the continued study of data analysis and statistics so that all students can--

- construct and draw inferences from charts, tables, and graphs that summarize data from real-world situations;
- use curve fitting to predict from data;
- understand and apply measures of central tendency, variability, and correlation;
- understand sampling and recognize its role in statistical claims;
- design a statistical experiment to study a problem, conduct the experiment, and interpret and communicate the outcomes;
- analyze the effects of data transformations on measures of central tendency and variability;

and so that, in addition, college-intending students can-

- transform data to aid in data interpretation and prediction;
- test hypotheses using appropriate statistics.

**Standard 11: Probability.**

In grades 9-12, the mathematics curriculum should include the continued study of probability so that all students can--

- use experimental or theoretical probability, as appropriate, to represent and solve problems involving uncertainty;
- use simulations to estimate probabilities;
- understand the concept of a random variable;
- create and interpret discrete probability distributions;
Appendix E: NCTM Curriculum Standards for Grades 9-12

- describe, in general terms, the normal curve and use its properties to answer questions about sets of data that are assumed to be normally distributed;

and so that, in addition, college-intending students can--

- apply the concept of a random variable to generate and interpret probability distributions including binomial, uniform, normal, and chi square.

**Standard 12: Discrete Mathematics.**
In grades 9-12, the mathematics curriculum should include topics from discrete mathematics so that all students can--

- represent problem situations using discrete structures such as finite graphs, matrices, sequences, and recurrence relations;
- represent and analyze finite graphs using matrices;
- develop and analyze algorithms;
- solve enumeration and finite probability problems;

and so that, in addition, college-intending students can--

- represent and solve problems using linear programming and difference equations;
- investigate problem situations that arise in connection with computer validation and the application of algorithms.

**Standard 13: Conceptual Underpinnings of Calculus.**
In grades 9-12, the mathematics curriculum should include the informal exploration of calculus concepts from both a graphical and a numerical perspective so that all students can--

- determine maximum and minimum points of a graph and interpret the results in problem situations;
- investigate limiting processes by examining infinite sequences and series and areas under curves;

and so that, in addition, college-intending students can--

- understand the conceptual foundations of limit, the area under a curve, the rate of change, and the slope of a tangent line, and their applications in other disciplines;
- analyze the graphs of polynomial, rational, radical, and transcendental functions.

**Standard 14: Mathematical Structure.**
In grades 9-12, the mathematics curriculum should include the study of mathematical structure so that all students can--

- compare and contrast the real number system and its various subsystems with regard to their structural characteristics;
- understand the logic of algebraic procedures;
- appreciate that seemingly different mathematical systems may be essentially the same;

and so that, in addition, college-intending students can--

- develop the complex number system and demonstrate facility with its operations;
- prove elementary theorems within various mathematical structures, such as groups and fields;
- develop an understanding of the nature and purpose of axiomatic systems.
## Appendix F: Course Outline for Algebra 1

**Worcester Public Schools**

**Course Syllabus – Part II, Academic Content for the First Semester**

**Algebra I**

<table>
<thead>
<tr>
<th>Content/Topics –</th>
<th>Skills</th>
<th>Required Papers/Projects, Readings, and Final Assessment/Test</th>
<th>Academic Standards (Worcester Benchmarks and State Frameworks)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Properties of Real Numbers</td>
<td>Solve using operations of real numbers and the order of operations Use the distributive property Combine like terms Solve absolute value problems</td>
<td></td>
<td>AIN.1 Identify and use the properties of operations on real numbers.</td>
</tr>
<tr>
<td>Solving Linear Equations</td>
<td>Solve one-step and multi-step equations Evaluate equations with variables on both sides Solve problems using ratios, rates, and percents</td>
<td></td>
<td>AIP.2 Use properties of the real number system to judge the validity of equations and inequalities.</td>
</tr>
<tr>
<td>Graphing Linear Equations</td>
<td>Describe the coordinate plane Graph lines including horizontal and vertical lines Find intercepts and slope Identify direct variation Utilize the slope-intercept form</td>
<td></td>
<td>AIP.5 Demonstrate an understanding of the relationship between various representations of a line.</td>
</tr>
<tr>
<td>Writing Linear Equations</td>
<td>Use the slope-intercept and point-slope forms of a line Find linear equations given two</td>
<td></td>
<td>AIP.6 Find linear equations that represent lines either perpendicular or parallel to a given line and through a point.</td>
</tr>
</tbody>
</table>
## Appendix F: Course Outline for Algebra 1

<table>
<thead>
<tr>
<th>Solving and Graphing Linear Inequalities</th>
<th>Points</th>
<th>Use the standard form of a line</th>
<th>Identify perpendicular lines</th>
<th>Solve one-step and multi-step inequalities</th>
<th>Find compound inequalities</th>
<th>Solve absolute value inequalities</th>
<th>Analyze linear inequalities in two variables</th>
<th>A.I.P.10 Solve equations and inequalities including those involving absolute value of linear expressions.</th>
</tr>
</thead>
</table>


## Appendix F: Course Outline for Algebra 1

Worcester Public Schools

Course Syllabus – Part II, Academic Content for the Second Semester

### Algebra 1

<table>
<thead>
<tr>
<th>Content/Topics –</th>
<th>Skills</th>
<th>Required Papers/Projects, Readings, and Final Assessment/Test</th>
<th>Academic Standards (Worcester Benchmarks and State Frameworks)</th>
</tr>
</thead>
</table>
| Systems of Linear Equations and Inequalities | Graph linear systems  
Solve systems by substitution and combinations  
Use systems of linear equations and inequalities to solve problems | | AIP.12 Solve everyday problems that can be modeled using systems of linear equations or inequalities. |
| Exponents and Exponential Functions | Use properties of exponents  
Graph exponential functions  
Use scientific notation  
Use exponential growth and decay in the solution of problems | | AIP.11 Solve everyday problems that can be modeled using exponential functions. |
| Quadratic Equations and Functions | Solve square roots  
Simplify radicals  
Graph quadratics  
Solve quadratics  
Use the quadratic formula  
Find the discriminant  
Graph quadratic inequalities | | AIP.9 Find solutions to quadratic equations (with real roots) by factoring, completing the square, or using the quadratic formula. |
| Polynomials and Factoring | Solve operations with polynomials  
Solve quadratics in factored form  
Factor quadratics  
Factor cubics | | AIP.7 Add, subtract, and multiply polynomials. Divide polynomials by monomials. |
## Rational Expressions and Equations

- Use proportions, direct and inverse variation to solve problems
- Simplify rational expressions
- Use operations with rational expressions
- Solve rational equations

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>A1.P.8</strong> Demonstrate facility in symbolic manipulation of polynomial and rational expressions.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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### Appendix G: Course Outline for Geometry

Worcester Public Schools  
High School Curriculum  

**Course Syllabus—Part II, Academic Content for the First Semester**  
**Geometry**

<table>
<thead>
<tr>
<th>Content/Topics –</th>
<th>Skills</th>
<th>Required Papers/Projects, Readings, and Final Assessment/Test</th>
<th>Academic Standards (Worcester Benchmarks and State Frameworks)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segments and Angles</td>
<td>Find segment and angle bisectors Identify complementary and supplementary angles Use vertical angles in the solution to problems Utilize deductive reasoning</td>
<td></td>
<td>10.G.3 Recognize and solve problems involving angles formed by transversals of coplanar lines.</td>
</tr>
<tr>
<td></td>
<td>Use perpendicular theorems Identify angles formed by transversals Demonstrate proofs using parallel lines and transversals Use perpendicular and parallel lines in problems</td>
<td></td>
<td>10.G.8 Find linear equations that represent lines either perpendicular or parallel to a given line and through a point.</td>
</tr>
<tr>
<td>Parallel and Perpendicular Lines</td>
<td>Classify triangles Identify isosceles and equilateral triangles Use the Pythagorean theorem Prove triangle inequalities</td>
<td></td>
<td>10.G.5 Solve simple triangle problems using the triangle angle sum property and /or the Pythagorean theorem.</td>
</tr>
<tr>
<td>Triangle Relationships</td>
<td>Prove triangles are congruent Use hypotenuse – leg congruence to solve problems Use congruent triangles Identify reflections and symmetry</td>
<td></td>
<td>10.G.4 Apply congruence and similarity correspondences and properties of the figures to find missing parts of geometric figures, and provide logical justification.</td>
</tr>
<tr>
<td>Congruent triangles</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Appendix G: Course Outline for Geometry

Worcester Public Schools  
High School Curriculum  
Course Syllabus – Part II, Academic Content for the Second Semester  
Geometry

<table>
<thead>
<tr>
<th>Content/Topics –</th>
<th>Skills</th>
<th>Required Papers/Projects, Readings, and Final Assessment/Test</th>
<th>Academic Standards (Worcester Benchmarks and State Frameworks)</th>
</tr>
</thead>
</table>
| Quadrilaterals   | Identify properties of parallelograms, rhombuses, rectangles, and squares  
Identify trapezoids  
Find ratios and proportions  
Identify similar polygons  
Prove angle, angle, angle similarity  
Prove side, side, side and side, angle, side congruence  
Use proportions and similar triangles in solutions to problems  
Find angles in polygons  
Compute the area of quadrilaterals  
Find the area and circumference of circles |  | 10.G.1 Identify figures using properties of sides, angles, and diagonals.  
10.G.4 Apply congruence and similarity correspondences and properties of the figures to find missing parts of geometric figures. |
| Similarity       | Polygons and Area | SA of prisms and cylinders  
SA of pyramids and cones  
Volume of prisms, cylinders  
Volume of pyramids, cones  
SA and volume of spheres | 10.M.1 Calculate perimeter, circumference, and area of common geometric figures.  
10.M.2 Given the formula, find the lateral area, surface area, and volume of prisms, pyramids, spheres, cylinders, and cones. |
## Appendix G: Course Outline for Geometry

| Right Triangles and Trigonometry | Simplify square roots  
Solve problems using 45-45-90 and  
30-60-90 triangles  
Find tangent ratios  
Identify sine and cosine ratios | 10.G.6 Use the properties of special  
triangles to solve problems. |
Appendix H: Pre-Calculus Lesson Plans

September 28th, 2011

Overview:

Students were recently introduced to matrices. They were shown how to add matrices and multiply matrices to scalars and other matrices. Today, students will be introduced to determinants, and how to find the inverse of a matrix. Later on, students will be able to solve $Ax=b$ by finding the inverse of a matrix.

Massachusetts Curriculum Frameworks Addressed in this Lesson:

N-VM 9  Understand that, unlike matrix multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

N-VM 10  The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
Determinants of $3 \times 3$ matrices

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

= minors

Example:

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1 \begin{vmatrix} 5 & 6 \\ 7 & 9 \end{vmatrix} - 2 \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3 \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix}$$

$$= 1[5 \cdot 9 - 6 \cdot 7] - 2[4 \cdot 9 - 6 \cdot 7] + 3[4 \cdot 8 - 5 \cdot 7]$$

$$= 9 - 12 + 9 = 6$$

Have students work on these two examples:

$$\begin{vmatrix} 1 & 4 & 2 \\ 2 & 5 & -3 \\ 3 & 4 & 1 \end{vmatrix} = 1 \begin{vmatrix} 5 & -3 \\ 4 & 1 \end{vmatrix} - 4 \begin{vmatrix} 2 & -3 \\ 3 & 1 \end{vmatrix} + 2 \begin{vmatrix} 2 & 5 \\ 3 & 4 \end{vmatrix}$$

$$= 1[5 \cdot 1 - (-3) \cdot 4] - 4[2 \cdot 1 - (-3) \cdot 3] + 2[2 \cdot 4 - 5 \cdot 3]$$

$$= 5 + 12 - 8 + 18 = 25$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} - 0 \begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix} + 0 \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}$$

$$= 1[1 \cdot 1 - 0 \cdot 0] - 0[0 \cdot 0 - 1 \cdot 0] + 0[0 \cdot 0 - 0 \cdot 0]$$

$$= 1$$

An $n \times n$ matrix that looks like $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is called the identity matrix. The identity matrix is labeled $I$.

Inverses of $2 \times 2$ Matrices (AKA multiplicative inverse)

$S^{-1}$ is the inverse of $S$, which is $\frac{1}{S}$. $S^{-1} \cdot S = S \cdot S^{-1} = I$.

Similarly for matrices, $A^{-1}A = AA^{-1} = I$.

Notice that in this special case, multiplication is commutative.
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Let's try to calculate the elements in $A^{-1}$.

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$.

Then,

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Therefore,

1. $ae+bg = 1$
2. $ce+dg = 0$
3. $af+bh = 0$
4. $cf+dh = 1$

Isolate $g$ in (1)

$g = \frac{1-ae}{b}$

Isolate $f$ in (1)

$f = \frac{-bh}{a}$

Substitute $g$ in (2)

$ce + d\left(\frac{1-ae}{b}\right) = 0$

$ce + \left(\frac{1-ae}{b}\right) = 0$

Solve for $e$

$e = \frac{d}{ad-bc}$

Substitute $f$ in (3)

$c\left(\frac{-bh}{a}\right) + dh = 1$

Solve for $h$

$h = \frac{-a}{ad-bc}$

Substitute $e$ in (1)

$a\left(\frac{d}{ad-bc}\right) + bg = 1$

Solve for $g$

$g = \frac{-c}{ad-bc}$

Substitute $h$ in (1)

$af + b\left(\frac{a}{ad-bc}\right) = 0$

Solve for $f$

$f = \frac{-b}{ad-bc}$
Appendix H: Pre-Calculus Lesson Plans

Notice that the denominator on all of the elements is the determinant of A. Therefore,

\[
A^{-1} = \frac{1}{\text{det}(A)} \begin{bmatrix}
  d & -b \\
  -c & a
\end{bmatrix}
\]

Note that if \( \text{det}(A) = 0 \), \( A^{-1} \) does not exist because we cannot divide by zero.

Ex: Find \( \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}^{-1} \)

\[
= \frac{1}{(3)(1) - (3)(4)} 
= \frac{-1}{-10}
= \frac{1}{10}
\]

\[
\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{10} & \frac{3}{10} \\ \frac{-1}{10} & \frac{2}{10} \end{bmatrix}
\]

Properties of the Identity Matrix
- \( \text{det}(I) = 1 \)
- \( A^{-1}A = AA^{-1} = I \)
- \( IA = AI = A \)
- \( I^{-1} = I \)
Appendix H: Pre-Calculus Lesson Plans

Honors Pre Calculus

October 11-13\textsuperscript{th}, 2011

Solving Systems of Inequalities

Overview:

Previously in this course, students have graphed inequalities and solved systems of equations graphically. Now, it is time to combine these two skills together in order to solve systems of inequalities graphically. This lesson corresponds to section 2-5 in the textbook Advanced Mathematical Concepts. In this lesson, students will be introduced to a word problem regarding profit in which revenue must exceed cost, and items cannot be priced too high. The profitable region and a break-even point can be obtained by graphing a system of inequalities. Students will also see situations where systems of inequalities have no solution. Finally, students will use the vertex theorem to find the maxima and minima of functions restricted on a polygonal convex set. These skills are necessary to solve problems using linear programming.

Massachusetts Curriculum Frameworks Addressed in this Lesson:

A-CED-3:

Represent constraints by inequalities, and by systems inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

A-REI-12:

Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Do-Now:

10/11:

Graph the inequalities $y \geq 0.5x + 2000$ and $y \leq 2x$ on the same graph in the first quadrant. The x and y axis should extend into the thousands.
Graph the following system of inequalities: $y \leq 3x - 2, y \geq 3x + 3$. Does a solution exist? How about $y \geq 3x - 2 \text{ and } y \leq 3x + 3$? Answer: Yes; no.
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10/13

Do problem 21 on page 90:

Bob chase intends to major in medicine at the Harvard Medical School. He had been told that he must have two outstanding admissions scores in order to be admitted to a pre-med program. He needs an ACT score of at least 30 and a SAT score of at least 1200. Write four inequalities to represent this situation, and graph the inequalities to show the solution.

Let y be Bob’s ACT score, and x his SAT score. Then:

\[ 30 \leq y \leq 36 \]  
His ACT score must be between the min allowed and the max possible.

\[ 1200 \leq x \leq 1600 \]  
His SAT score must be between the min allowed and the max possible.

I also ask the students what each vertex means in terms of the word problem. For example, (1600, 30) is the maximum possible SAT score and the minimum ACT score acceptable.

Review Homework:

Homework is collected before it is reviewed. The methodology behind this is that homework can be a vital means of feedback, so that the teacher knows if the students understand the material. If the homework is reviewed then collected, many students will erase their work and replace their answers with the ones written on the board. Therefore, using homework as a means of feedback may be compromised. If students had trouble with a particular problem, that problem should be memorable, and since all the problems were from the textbook, students can refer to it if they did not remember the exact problem. Therefore, having their homework is not necessary to review it. Yesterday’s homework problems were #12, 14, 18, and 23 on page 82-83. If no student has any questions, then I will write the equations for #23 on
Appendix H: Pre-Calculus Lesson Plans

the board, solve for s, and then show them that it is indeed possible to solve this problem graphically using a computer, since many students are curious about this:

\[
\begin{align*}
3b + 2s + 4j &= 292 \\
4b + s + 3j &= 252 \\
-s + j &= 4
\end{align*}
\]

\[
\begin{align*}
s &= 146 - \frac{3}{2}b - 2j \\
s &= 252 - 4b - 3j \\
s &= -4 + j
\end{align*}
\]

\[
\begin{align*}
z &= 146 - \frac{3}{2}x - 2y \\
z &= 252 - 4x - 3y \\
z &= -4 + y
\end{align*}
\]

Replace s with z, b with x, and j with y, then graph the planes using 3D graphing software. The intersection is the solution to the problem: (28, 36, 32).

If students do have questions on the homework, I will answer those questions instead, and reserve this exercise until the end of class if time permits.

Lesson:

Because of events that I was unaware of before I wrote this lesson plan, such as PSATs and goal setting, this took three days to go over this material. Wednesday was the day of the PSAT. Since the class was smaller, I was able to breeze through the vertex theorem. I let them do their homework afterwards because I did not want them to be too far ahead.

I introduce the following word problem to the class:

A culinary professional formulated a recipe for a new soft drink and would like to manufacture and sell his product. He figures that his starting cost will be $2000, and that it would cost 50 cents to produce each drink. He also wants to keep the price no higher than $2 per drink to remain competitive. How many drinks must he sell to break even, and under what circumstances will be turn a profit?

I go over this problem on the board, asking students to produce that answers for each step:

1. Identify the variables. The variables are the number of drinks sold, and the income.
2. Assign algebraic variables. Let x be the # of drinks sold, and y be the income.
3. Determine inequalities. \( x \geq 0, \ y \geq 0, \ y \geq 0.5x + 2000, y \leq 2x \)
4. Graph the inequalities on the same graph. They already did this completing the do-now.
5. Find the breakeven point. The breakeven point is the point of intersection between the two lines, which is (1333.33..., 2666.66...). However, the number of drinks is not a whole number. So the number of drinks sold to break even is 1334, with an income of $2667.
6. Determine the feasible solution (profitable region). The feasible solution is where all the shaded regions intersect.

I will then have students graph the following system of inequalities individually (if they happen to help each other out, that is acceptable, but each student should produce a graph):
Appendix H: Pre-Calculus Lesson Plans

Since there is no region in which the shaded regions intersect, there is no solution.

Next, students will be introduced to concave and convex polygons. This is important for the vertex theorem. A convex polygon is a polygon in which all interior angles are less than 180 degrees. A polygon that is not convex is a concave polygon. All regular polygons are convex. A star is an example of a concave polygon.

Now, the vertex theorem can be introduced. Suppose that I wanted to maximize the function \( f(x, y) = 3x - 2y \), but the x and y values are restricted to a convex polygon. In this situation, I can use the vertex theorem to find the maximum of such a function.

**Vertex Theorem:**

*The maximum or minimum value of* \( f(x, y) = ax + by + c \) *on a polygonal convex set occurs at a vertex of the polygonal boundary.*

A concrete example is presented. Suppose our function \( f(x, y) = 3x - 2y \) is restricted on:

\[
\begin{align*}
0 &\leq y \leq 3x + 4 \\
0 &\leq x \leq 5
\end{align*}
\]

I ask the students to graph the system individually:
Appendix H: Pre-Calculus Lesson Plans

Note that the shaded region is a convex polygon. Therefore, we can use the vertex theorem to compute the maximum of the function. The vertices of this polygon are (0,0), (5,0), (5,19), and (0,4), so I ask the students to compute f(x, y) at these values:

\[ f(0,0) = 0 \quad f(5,0) = 15 \quad f(5,19) = -23 \quad f(0,4) = -8 \]

So the maximum is f(5,0)= 15. The minimum can also be determined, which is f(5,19)= -23.

If time permits, I will have the students work on other problems using the vertex theorem from the book.

Homework:

10/11
Pages 89-90, #11-14, 22 (Just graph. Do not find the max or min of the objective function)

10/12
Pages 89-90, Finish #11-14.

10/13
Pages 89-90, Finish #11-14. Do 15, 16
Appendix H: Pre-Calculus Lesson Plans

October 14-18

Massachusetts Curriculum Frameworks Addressed in this Lesson:

A-CED-2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axis with labels and scales.

A-CED-3 Represent constraints by equations or inequalities, and by system of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

A-REI-12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

F-IF-4 For a function that models a relationship between two quantities, interpret key features of a graph and tables of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F-BF-1 Write a function that describes a relationship between two quantities.

Do Now:

10/14

Students are told to clear everything off of their desk. They are instructed to answer the following questions by themselves:

1) What is a convex polygon?
2) What does the vertex theorem state?
3) What is the set of all the points in which a solution of a system of inequalities called?

This is an ungraded and anonymous assignment to be collected. The purposes of these questions are to determine what students recall from yesterday’s lesson, and what type of learner they are. For example, if they write a sentence answering the first question, then they are verbal learners. On the other hand, if they draw pictures of convex polygons, then they are visual learners.

After I have done this exercise, I realized that the vast majority gave a written explanation for 1, although a few students still drew pictures. Most students were able to state part of the vertex theorem, but very few were able to correctly state the whole thing. About half of the students were able to answer 3 correctly (feasible solution).

10/17
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I write the steps to solve the linear programming problem on the board. Then, I ask them to try to do the problem regarding graphing and scientific calculators below (in green). After they attempt the problem, I will work out the problem with them.

10/18

I ask students to finish the linear programming problem in purple in groups of two.

Homework Review:

I ask if students have any questions on the homework. If they do, I will answer them. I will go around, making sure that they completed the assignment.

10/14: Students had many questions regarding homework, mostly on graphing. Reviewing homework took up most of the time.

10/17: I projected homework solutions on the board. Reviewing homework in this way eliminated the time wasted writing everything on the board.

Lesson:

10/14

Alternative method to determine the concavity of a polygon:

Choose any two points on the polygonal boundary, and draw a line segment with those two points as endpoints. If the line segment does not lie entirely inside of the polygon, then the polygon is concave. Examples are drawn on the board.

A linear programming is a branch of mathematics that involves optimizing an objective function, given constraints usually in the form of a system of inequalities. The objective function is the function that we seek to maximize or minimize that is restricted on a convex polygonal set.

Ex:

A calculator company produces a scientific calculator and a graphing calculator. Long-term projections indicate an expected demand of at least 100 scientific and 80 graphing calculators each day. Because of limitations on production capacity, no more than 200 scientific and 170 graphing calculators can be made daily. To satisfy a shipping contract, a total of at least 200 calculators much be shipped each day.

If each scientific calculator sold results in a $2 loss, but each graphing calculator produces a $5 profit, how many of each type should be made daily to maximize net profits?

(Source: http://www.purplemath.com/modules/linprog3.htm)

10/17
Appendix H: Pre-Calculus Lesson Plans

Just like the word problems that we had earlier, our first few steps are the same.

*Step 1: Determine variables:*

The variables are the number of scientific calculators produced and the number of graphing calculators produced.

*Step 2: Assign letters to the variables:*

Let $x$ be the number of scientific calculators produced, and let $y$ be the number of graphing calculators produced.

*Step 3: Determine the system of inequalities:*

\[
\begin{align*}
    x &\geq 100 & \text{The number of scientific calculators produced must be at least 100} \\
    y &\geq 80 & \text{The number of graphing calculators produced must be at least 80} \\
    x &\leq 200 & \text{The number of scientific calculators produced may not exceed 200} \\
    y &\leq 170 & \text{The number of graphing calculators produced may not exceed 170} \\
    x + y &\geq 200 & \text{The total number of calculators produced must be at least 200}
\end{align*}
\]

Notice that the inequalities $x \geq 0, y \geq 0$ that would normally restrict the number of calculators produced to be nonnegative are redundant, since we already are restricting them to a minimum of 100 and 80, respectively. The variables in the system of inequalities are called **decision variables**, because they “decide” how the objective function is constrained.

*Step 4: Determine the objective function (the function to be maximized or minimized)*

\[ R = -2x + 5y \]

*Step 5: Graph the system of inequalities:*
From this point on, it is a matter of applying the vertex theorem:

**Step 6: Determine if the feasible region a convex polygon.**

The feasible region is a convex polygon.

**Step 7: Determine the vertices of the polygon:**

(100, 170), (200, 170), (200, 80), (120, 80), and (100, 100) are vertices of the polygon.

**Step 8: Substitute the coordinate pairs into the objective function:**

\[ R(100, 170) = 650 \]
\[ R(200, 170) = 450 \]
\[ R(200, 80) = 0 \]
\[ R(120, 80) = 160 \]
\[ R(100, 100) = 300 \]

**Step 9: Determine what the optimal solution of the objective function is:**

Producing 100 scientific calculators and 170 graphing calculators will yield the maximum profit of $650.

I will then pick a random point located in the feasible region. Since we want to maximize \( R \), we should be interested in making \( x \) as small as possible and \( y \) as large as possible. So, in terms of a graph, we want to move to the left and upwards as much as possible. Thus, moving the point toward the upper left hand corner will “corner” us so that we can no longer move further up or to the left. This is where the optimal solution exists. This is also why the optimal solution is always at a vertex.
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Class work:

10/18

I then ask students to work on the following problem in groups of two:

In order to ensure optimal health (and thus accurate test results), a lab technician needs to feed the rabbits a daily diet containing a minimum of 24 grams (g) of fat, 36 g of carbohydrates, and 4 g of protein. But the rabbits should be fed no more than five ounces of food a day.

Rather than order rabbit food that is custom-blended, it is cheaper to order Food X and Food Y, and blend them for an optimal mix. Food X contains 8 g of fat, 12 g of carbohydrates, and 2 g of protein per ounce, and costs $0.20 per ounce. Food Y contains 12 g of fat, 12 g of carbohydrates, and 1 g of protein per ounce, at a cost of $0.30 per ounce.

What is the optimal blend?

Source: http://www.purplemath.com/modules/linprog4.htm

As students work on this problem, I go around the room answering questions and helping students. Before the bell rings, I ask students to do each of the nine steps:

Step 1: Determine variables:

The variables are the number of ounces of Food X and the number of ounces of Food Y.

Step 2: Assign letters to the variables:

Let x be the number of ounces of Food X, and y be the number of ounces of Food Y.

Step 3: Determine the system of inequalities:

\[
\begin{align*}
  x &\geq 0 & \text{The number of ounces of Food X cannot be negative.} \\
  y &\geq 0 & \text{The number of ounces of Food Y cannot be negative.} \\
  8x + 12y &\geq 24 & \text{The total fat must be at least 24g.} \\
  12x + 12y &\geq 36 & \text{The total carbs must be at least 36g.} \\
  2x + y &\geq 4 & \text{The total protein must be at least 4g.} \\
  x + y &\leq 5 & \text{The total amount of food must not exceed 5oz.}
\end{align*}
\]

Step 4: Determine the objective function (the function to be maximized or minimized)

\[
C = 0.2x + 0.3y
\]

Step 5: Graph the system of inequalities:
Step 6: Determine if the feasible region a convex polygon.

The feasible region is a convex polygon.

Step 7: Determine the vertices of the polygon:

(0,4), (0,5), (3,0), (5,0), and (1,2) are vertices of the polygon.

Step 8: Substitute the coordinate pairs into the objective function:

C (0, 4) = 1.20
C (0, 5) = 1.50
C (3, 0) = 0.60
C (5, 0) = 1.00
C (1, 2) = 0.80

Step 9: Determine what the optimal solution of the objective function is:

Producing a mixture containing only 3oz of Food X will yield the cheapest solution at $0.60.

If there is time, I will go over situations that are unbounded, infeasible, and contain alternate optimal solutions.

Homework:

10/14
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p.99 #18-21. Solve the system of inequalities algebraically (not with matrices). This assignment is to be collected. Work on extra credit, due Wednesday. Test on Wednesday, covering 2-4 thru 2-6.

10/17

p. 94 #4-11 (this is actually one problem divided into parts)

p. 95 #15, 17

10/18

Study for Wednesday’s test. Finish extra credit assignment (worth 10% on the test).
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October 20-24

Massachusetts Curriculum Frameworks Addressed in this Lesson:

F-BF-3 Recognize even and odd functions from their graphs and algebraic expressions for them.

G-CO-4 Develop definitions of rotation and reflections.

Do now:

10/20

Are the following shapes symmetric? If so, how are they symmetric?

Yes, vertically  Yes, horizontally  No  Yes, rotational

10/21

Sketch a function that is symmetric with respect to the line y= -x.

10/24

Students anonymously answer the following questions on paper that is to be collected:

- Do you have a computer at home?
- Do you have internet access at home?
- Have you ever stayed after school for help for this class?
- How much time do you spend on homework for this class?
- Is the homework easy, challenging, or frustratingly difficult in this class?

Lesson:

Below is a list of functions, their graphs, and the type of symmetry they have. Students should be familiar with the graphs of some functions, such as \( y = x^2 \), while other functions such as that of an ellipse are not so familiar. I could have students graph most of the functions using a graphing calculator, but I believed that that would be very time consuming, and so it is better project the graphs on the board. For the similar graphs that students should be familiar with, I asked them if they could explain what they would look like before showing them the functions. For the more complicated functions I did not do this. What was more important was whether or not students could determine symmetry by looking at the graph, not necessarily the function. Later on, students should be able to determine symmetry by the function, but not at this point. Also, students will have plenty of practice graphing functions with the calculator later on in the course.
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10/20

Types of symmetry:
Along the y-axis:

\[ y = x^2 \qquad y = \cos(x) + 1 \]

Along the x-axis:

\[ y = \pm \sqrt{x} \qquad y = \pm|x + 2| \]

Along the line \( y = x \):
Along the line \( y = -x \):

\[ 29x^2 + 24xy + 36y^2 - 5995 = 0 \]

\((0, 15.49)\) and \((-15.49, 0)\) are on the graph.

Around the origin:

\[ y = \begin{cases} 
\pm \sqrt{4 - x^2} & \text{if } -2 \leq x \leq 0 \\
(x + 2) & \text{if } x < -2 \text{ and } x > 0
\end{cases} \]
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\[ y = x^3 \quad \text{and} \quad y = \tan(x) \quad -\frac{4\pi}{3} < x < \frac{4\pi}{3} \]

Rotational symmetry around a point:

\[ y = \pm \sqrt{4 - (x + 2)^2} + 2 \quad \text{and} \quad y = \begin{cases} 3 & -1 \leq x \leq 1 \\ 2 & -2 \leq x \leq 0 \\ x + 2 & 0 \leq x \leq 1 \\ x + 4 & -2 \leq x \leq -1 \end{cases} \]
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10/21

I explain that the symbol \( \in \) means “belong to”. With the information above, I have the students complete the table individually:

<table>
<thead>
<tr>
<th>Symmetric with Respect to</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>x-axis</td>
<td>((a, -b) \in S \text{ if and only if } (a, b) \in S)</td>
</tr>
<tr>
<td>y-axis</td>
<td>((-a, b) \in S \text{ if and only if } (a, b) \in S)</td>
</tr>
<tr>
<td>(y = x)</td>
<td>((b, a) \in S \text{ if and only if } (a, b) \in S)</td>
</tr>
<tr>
<td>(y = -x)</td>
<td>((-b, -a) \in S \text{ if and only if } (a, b) \in S)</td>
</tr>
<tr>
<td>The origin (0,0)</td>
<td>((-a, -b) \in S \text{ if and only if } (a, b) \in S)</td>
</tr>
<tr>
<td>The point (c, d)</td>
<td>((-a + c, -b + d) \in S \text{ if and only if } (a + c, b + d) \in S)</td>
</tr>
</tbody>
</table>

Prove that \(y = x^4 + 2x^2\) is symmetric with respect to the y-axis.

A picture is not a proof! Use the definition instead:

If \((a, b)\) is on the graph, then \((-a, b)\) is on the graph.

In other words, \(x\) changes sign, and \(y\) remains the same.

So, substituting \(-x\) for \(x\) in our equation should be equivalent to what we have above.

\[
y = (-x)^4 + 2(-x)^2 = x^4 + 2x^2
\]

Prove or disprove: \(y = \frac{1}{x} + 2\) is symmetric along the origin.

From the graph, this does not appear to be true.

Disprove it with the definition:

\((a, b)\) is on the graph if and only if \((-a, -b)\) is on the graph.

We know that the point \((1, 3)\) is a solution to the function.
If the function is symmetric along the origin, then \((-1, -3)\) must also be a solution of the function.

However, \((-1, -3)\) is not a solution of the function.
Therefore, the function is not symmetric along the origin.
This type of proof is called a **proof by contradiction**.
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10/23

I have students try the following, or I do some problems with them, depending on how well they grasp the new material:

- Prove or disprove: \( y = x^2 + 2 \) is symmetric along the y-axis.
- Prove or disprove: \( y = 3x \) is symmetric along the origin.
- Prove or disprove: \( y = 2x + 1 \) is symmetric along the x-axis.
- Prove or disprove: \( y = \cos(x) \) is symmetric along the y-axis.
- Prove or disprove: \( y = 2/x \) is symmetric along the line \( y = -x \).

An **even** function is a function that is symmetric along the y-axis.

An **odd** function is a function that is symmetric with respect to the origin.

Determine if the following functions are even, odd, or neither:

\[
\begin{align*}
y &= \sin(x) \\
y &= \cos(x) - 2 \\
x^2y &= 1 \\
y &= 3(x - 1)^2 \\
y &= 3x^2 - 2x
\end{align*}
\]

Note that if a polynomial has all odd powers of \( x \), then it is an odd function.

Note that if a polynomial has all even powers of \( x \), then it is an even function.

Note that if a function is even (symmetric along the y-axis), then \( f(x) = f(-x) \).

Note that if a function is odd (symmetric along the origin), then \(-f(x) = f(-x)\).

**Homework:**

10/20 p. 113 #11-13 p.114 #20-22

10/21 p. 114 #32-37

10/24 p.114 #26-30 even. Also **prove** the claims that you made on 32-36 even. (To be collected). Quiz on symmetry Wednesday.

**October 25-26**
Families of Graphs

Massachusetts Curriculum Frameworks Addressed in this Lesson:

G-CO-3 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure.

F-BF-3 Identify the effect on the graph of replacing f(x) by f(x)+k, kf(x), f(kx), and f(x+k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Do Now:

Complete the graph in the following cases:

a) It is symmetric with respect to the y-axis.

b) It is symmetric with respect to the origin.

c) It is symmetric with respect to the line y=x.

Vocabulary:

Parent Graph: The simplest graph that represents a family of graphs.

Parent function: The simplest function that represents a family of functions.

Greatest integer function (also called the floor function): A function in which all x values are rounded to the greatest integer not greater than x, symbolically written as $y = \lfloor x \rfloor$ or equivalently $y = [x]$.

Examples:

Parent of $y = a(bx + c)^{2} + d$  

Parent of $y = a(bx + c)^{3} + d$
Appendix H: Pre-Calculus Lesson Plans

\[ y = x^2 \]

\[ y = x^3 \]

\[
\begin{align*}
\text{Parent of } y &= a(bx + c)^4 + d \\
\text{Parent of } y &= x^4 \\
\end{align*}
\]

\[
\begin{align*}
\text{Parent of } y &= a(bx + c)^5 + d \\
\text{Parent of } y &= x^5 \\
\end{align*}
\]

\[
\begin{align*}
\text{Parent of } y &= a|bx + c| + d \\
\text{Parent of } y &= |x| \\
\end{align*}
\]

\[
\begin{align*}
\text{Parent of } y &= a\sqrt{bx + c} + d \\
\text{Parent of } y &= \sqrt{x} \\
\end{align*}
\]

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Parent of \( y = a[bx + c] + d \)

\[
y = [x]
\]

Parent of \( y = a\left(\frac{1}{bx} + c\right) + d \)

\[
y = \frac{1}{x}
\]

Translations:
In the examples above, the numbers \( c \) and \( d \) result in translations of the parent graph.

<table>
<thead>
<tr>
<th>C and D Values</th>
<th>Translation</th>
</tr>
</thead>
</table>

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| Positive c | Move to the left c units |
| Negative c | Move to the right c units |
| Positive d | Move up d units |
| Negative d | Move down d units |

Examples:

Parent function:
\[ y = x^2 \]

Child functions:
\[ y = x^2 + 2 \]
\[ y = (x + 3)^2 \]
\[ y = (x - 1)^2 - 4 \]

Parent function:
\[ y = |x| \]

Child functions:
\[ y = |x + 2| - 5 \]
\[ y = |x - 3| + 2 \]
\[ y = |x + 3| - 2 \]

Horizontal Stretching and Shrinking:
The value of \( b \) determines how much a parent graph stretches or shrinks horizontally.

<table>
<thead>
<tr>
<th>B Value</th>
<th>Horizontal Stretch/Shrink</th>
</tr>
</thead>
</table>
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<table>
<thead>
<tr>
<th>b &gt; 1</th>
<th>Shrink the graph horizontally</th>
</tr>
</thead>
<tbody>
<tr>
<td>b &gt; 0 and b &lt; 1</td>
<td>Stretch the graph horizontally</td>
</tr>
</tbody>
</table>

Example:

**Parent function:**
\[ y = \frac{1}{x} \]

**Child Functions:**
- \[ y = (2x)^{-1} \]
- \[ y = (0.5x)^{-1} \]
- \[ y = (3x)^{-1} \]

**Vertical Stretching and Shrinking:**
The value of \( a \) determines how much a parent graph stretches or shrinks vertically.

<table>
<thead>
<tr>
<th>A Value</th>
<th>Vertical Stretch/Shrink</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a &gt; 1 )</td>
<td>Stretch the graph vertically</td>
</tr>
<tr>
<td>( a &gt; 0 ) and ( a &lt; 1 )</td>
<td>Shrink the graph vertically</td>
</tr>
</tbody>
</table>

Example:

**Parent function:**

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\[ y = x^3 \]

Child functions:

\[ y = 2x^3 \]
\[ y = 0.5x^3 \]
\[ y = 1.5x^3 \]

Reflections:
If \( a \) is negative, a reflection occurs on a horizontal line.
The horizontal line of reflection is determined by the value of \( d : y = d \).
Reflections should be the last transformations that you perform.

<table>
<thead>
<tr>
<th>( a ) Value</th>
<th>Type of Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a \leq -1 )</td>
<td>Reflect along the line ( y = d )</td>
</tr>
<tr>
<td>( a &lt; -1 )</td>
<td>Stretch the graph vertically, then reflect along ( y = d )</td>
</tr>
<tr>
<td>( a &lt; 0 ) and ( a &gt; -1 )</td>
<td>Shrink the graph vertically, then reflect along ( y = d )</td>
</tr>
</tbody>
</table>

Example:

Parent function:
\[ y = \sqrt{x} \quad \text{(Notice} \ d = 0) \]

Child functions:
\[ y = -\sqrt{x} \]
\[ y = -0.5\sqrt{x} \]
\[ y = -2\sqrt{x} \]

Finally, if \( b \) is negative, reflect the graph with respect to the y-axis.
Remember to keep reflections for last!

<table>
<thead>
<tr>
<th>( B ) Value</th>
<th>Type of Transformation</th>
</tr>
</thead>
</table>

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<table>
<thead>
<tr>
<th>Condition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b = -1 )</td>
<td>Reflect along the y-axis</td>
</tr>
<tr>
<td>( b &lt; -1 )</td>
<td>Shrink the graph horizontally, then reflect along the y-axis</td>
</tr>
<tr>
<td>( b &lt; 0 ) and ( b &gt; -1 )</td>
<td>Stretch the graph horizontally, then reflect along the y-axis.</td>
</tr>
</tbody>
</table>

Example:

**Parent function:**
\[ y = |x| \]

**Child functions:**
- \( y = |x - 2| + 1 \)
- \( y = |-0.5x - 2| + 1 \)
- \( y = |−2x - 2| + 1 \)

Alternative to vertical reflection:
If a function has a nonzero value for \( c \) and a value of \( b \) other than 1, then the function does not behave as expected:

Example:
\[ y = (2x + 3)^2 \]

In this situation, we have \( b = 2 \) and \( c = 3 \), so we expect the parent function to move 2 units to the left, and to stretch vertically. However, the parent function is actually shifted 1.5 units to the left. No reflection along the y-axis is necessary.

So, when values of \( b \) and \( c \) are not trivial, shift \( c / b \) units.

Homework:
p. 123 Graph 22(all) and 24
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October 27

Massachusetts Curriculum Frameworks Addressed in this Lesson:

G-CO-3 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure.

F-BF-3 Identify the effect on the graph of replacing \( f(x) \) by \( f(x)+k \), \( kf(x) \), \( f(kx) \), and \( f(x+k) \) for specific values of \( k \) (both positive and negative); find the value of \( k \) given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Do Now:

Students are asked to sketch the following functions, each on a separate graph. A good domain is \(-5<x<5\) and a good range is \(-5<y<15\).

1. \( y = 2|x - 3| + 5 \)
2. \( y = (2x - 2)^2 - 2 \)
3. \( y = [x + 2] + 3 \)

Solution:

Homework Review:

I give the students a matching exercise in which the solutions to the homework problems are projected on the board, and they are to match the correct graph with its corresponding function. Then I randomly give each student a paper to correct. Students that had done the homework would avoid the embarrassment of having another student see their (better) work. I hope this encourages students to do homework.
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Class work:

For the remainder of the class time, students graph (manually) as many of these problems as possible:

\[ y = (2x + 1)^2 \]
\[ y = 2(2x + 1)^2 \]
\[ y = \sqrt{x - 4} + 5 \]
\[ y = -2\sqrt{2x - 2} \]
\[ y = (-2x + 2)^{-1} \]
\[ y = 2(2x - 2)^{-1} + 2 \]
Homework:
Draw a child graph of six parent functions. At least three of the graphs must have a translation, at least three of the graphs must have a stretch or shrink, and at least two graphs must have a reflection. Also, write function that is represented by the new graph. These will be presented on Friday.
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October 28

Massachusetts Curriculum Frameworks Addressed in this Lesson

G-CO-3 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure.

F-BF-3 Identify the effect on the graph of replacing f(x) by f(x)+k, kf(x), f(kx), and f(x+k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

F-BF 4 Find inverse functions. (Homework. This should be mostly a review, except that students learn that an inverse of a function is a reflection along the line y = x.)

Do Now:

These problems are just a review of what students have done previously in the course. I believe that it is good to review old material on occasion because it improves retention.

1. Compute \[
\begin{vmatrix}
2 & -3 \\
4 & 1 \\
\end{vmatrix}
\]

2. Solve \[
\begin{align*}
2y + 3x &= 5 \\
-y + x &= 4
\end{align*}
\]

3. Graph \(y = -2|x - 1| + 1\)

Homework Review:

I ask students to present graphs of their work to the class, and I ask the rest of the students to figure out the function for the graphs. Then I ask other students to present their functions, and I ask the rest of the students to figure out the graphs.

Game:

I set up four stations, each one with a set of cards. Each card has a function on one side and a graph on the other. The first player simply asks who has the graph for the function listed on his or her card. The player with the corresponding graph then says that he or she does, and then ask who has the graph for the function listed on his or her card. This continues until each card is used. Students rotate to the different stations so that they have the opportunity to play each game.

Review:

If there is time left, I will have students work on graphing various functions involving transforming parent functions.
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Homework:

Review finding the inverse of a function. Then do the following problems:

1. If $f(x) = 3x - 2$, find $f^{-1}(x)$.
2. If $f(x) = 3x^2 - 2x + 5$, find $f^{-1}(x)$.
3. If $f^{-1}(x) = \sqrt{2x - 1}$, find $f(x)$. (This is not a typo)

When you find the inverse of a function, you switch its $x$ and $y$ values. Graphically, this means the function is reflected on the line $y = x$. Use this fact to do the following:

1. Graph $f^{-1}(x)$ when $f(x) = -x + 4$.
2. Graph $f^{-1}(x)$ when $f(x) = x^2 + 2x - 3$.
3. Prove or disprove: The inverse of $y = \frac{1}{2x}$ is symmetric with respect to the line $y = x$. 
Overview:
Previously, students have learned how to find the inverse of a function algebraically and graphically. Students also know how to graph simple functions that are translations of their parent functions. Within the next few weeks, students will be introduced to the concepts of limits and derivatives. Students will learn the concept of a limit by graphing rational functions. By the end of today, students will be able to identify rational functions and to find vertical and horizontal asymptotes algebraically.

Massachusetts Curriculum Frameworks Addressed in this Lesson
F-BF 4  Find inverse functions. (Do-Now)
F-IF 7d  Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

Do Now:
Without doing any algebra, graph the inverse of the following functions:

\[ y = x^2 \]
\[ y = 3x - 2 \]
\[ y = |x - 2| \]

Vocabulary:
A rational function is a function in the form \( y = \frac{a(x)}{b(x)} \) where \( b \neq 0 \) and \( a(x) \) and \( b(x) \) are polynomials in terms of \( x \). \( y = 1/x \) is the parent graph of rational functions.
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Example:

\[ y = \frac{3x^2}{x(x-2)} \] is a rational function because it can be written in the form \( y = \frac{a(x)}{b(x)} \).

\[ y = \frac{\sqrt{x}}{x+1} \] is not a rational function because \( \sqrt{x} \) is not a polynomial.

An asymptote is a line in which a graph of a rational function approaches as \( x \) or \( y \) gets very large or very small.

Example:

The graph of \( y = \frac{1}{x} \) has two asymptotes. One is the line \( y = 0 \), and the other is the line \( x = 0 \).

A vertical asymptote is a vertical line in which a graph of a rational function approaches, but does not intersect with. The line \( x = a \) is a vertical asymptote of \( f(x) \) if \( f(x) \) approaches \( \infty \) or \( -\infty \) as \( x \) approaches \( a \) from either the left or from the right. A vertical asymptote occurs when the denominator of a rational function in terms of \( x \) “tries” to be zero.
Example:

\[ y = \frac{2x}{x-3} \] has a vertical asymptote at \( x=3 \) because that is when the denominator is zero. Notice that the rational function is in terms of \( x \).

A horizontal asymptote is a horizontal line in which a graph of a rational function approaches. It is possible for a graph to intersect the horizontal asymptote. The line \( y = b \) is a vertical asymptote of \( f(x) \) if \( f(x) \) approaches \( b \) as \( x \) approaches \( \infty \) or \( -\infty \). A horizontal asymptote occurs when the denominator of a rational function in terms of \( y \) “tries” to be zero.

Example:

\[ x = \frac{1}{y+2} \] has a horizontal asymptote at \( y = -2 \) because that is when the denominator is zero. Notice that the rational function is in terms of \( y \).
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A **slant asymptote** is an asymptote that is *neither* horizontal nor vertical. The line \( l \) is a slant asymptote of \( f(x) \) if \( f(x) \) approaches \( l \) as \( x \) approaches \( \infty \) or \( -\infty \). A slant asymptote occurs when the degree of the numerator is exactly one greater than the degree of the denominator.

Example:

\[ y = \frac{x^3}{x(x-1)} \] has a slant asymptote because the degree of the numerator, which is three, is exactly one greater than the degree of the denominator, which is two. The slant asymptote has the equation \( y = x + 1 \). The vertical asymptote \( x = 1 \) is not a slant asymptote.

![Graph of function](image)

You should be able to recognize when slant asymptotes occur, but I will not test you on your ability to derive the equation of such asymptotes (it involves long division of polynomials).

**Rational Functions without Asymptotes**

Not all rational functions have asymptotes. If the denominator is one, then the rational function is simply a polynomial function that is completely smooth. If you can **factor** the numerator and denominator and cancel out terms so that it becomes a polynomial function, then it has **no asymptotes**. However, the function will still have **holes** where the denominator could not be zero.

Example:

\[ y = \frac{x^2 + x - 6}{x + 3} = \frac{(x - 2)(x + 3)}{x + 3} = x - 2, \; x \neq -3 \]

Notice that when the \( x + 3 \) terms cancel, the restriction that \( x \neq -3 \) is added, since the denominator cannot be zero. Therefore, the graph will look like the line \( y = x - 2 \), except it has a **hole** when \( x = -3 \).
When you have a rational function, always try to factor the numerator and denominator and cancel out terms. If you can and you are left with a polynomial function, then the function has no asymptotes, but it may have holes. If you cannot cancel out terms, and the denominator is not one, then it does have asymptotes.

Class work:

Find any asymptotes and holes.

\[
\begin{align*}
y &= \frac{x}{x - 1} \\
y &= \frac{x^2}{x^2 - 1} \\
y &= \frac{x^2}{x^2 + 1} \\
y &= \frac{x^2 + x - 12}{x + 4} \\
y &= \frac{1}{x - 2}
\end{align*}
\]

Homework:

Page 140, Guided Practice 11-14, Exercises 15-18. If a slant asymptote exists, explain why. It is not necessary to find the equation of slant asymptotes. To be collected.
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November 3

Massachusetts Curriculum Frameworks Addressed in this Section:

F-IF 7d  Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

Do Now:

No class finished the problems from the previous lesson plan, so the do-now will be a few of those.

I will also ask them if it is possible for a rational function to have more than one horizontal asymptote, and if it is possible for a rational function to have more than one vertical asymptote. (Yes to both.)

Homework Review:

I will go over any questions the students have, then I will collect it. I require that students do not write on their homework as we go over it. If students do not have any questions, I will have a few students show and explain their work to the class.

Alternative Method to Determine Horizontal Asymptotes:

I have the students attempt to find the horizontal asymptote of \( y = \frac{x^2-4}{x^2-4x} \). Doing so requires them to solve \( x \) in terms of \( y \) using the quadratic formula. Doing so results in:

\[
\frac{2(y + \sqrt{y^2 - y + 1})}{y - 1}, \quad \frac{2(-y + \sqrt{y^2 - y + 1})}{y - 1}
\]

I expect most students to struggle trying to solve for \( x \). But those that are successful will realize that \( y = 1 \) is a horizontal asymptote for the function, as shown below:

![Graph of a rational function with horizontal asymptote at y = 1](image)
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Next, I show the students a better method to find horizontal asymptotes if solving for x is too difficult:

When we have a rational function \( f(x) = \frac{a_1x^n + a_2x^{n-1} + \ldots}{b_1x^m + b_2x^{m-1} + \ldots} \)

- If \( n < m \) then the x-axis is the horizontal asymptote.
- If \( n = m \) then the line \( y = \frac{a_1}{b_1} \) is the horizontal asymptote.
- If \( n > m \) there will be no horizontal asymptote.

So, in the case that we have above, \( n = m = 2 \), so the horizontal asymptote is \( y = \frac{1}{1} = 1 \).

**Graphing Rational Functions**

\( y = \frac{1}{x} \) is the parent function of rational functions. You can use the translations that we learned to graph simple rational functions with one horizontal and vertical asymptote.

**Examples**

1. \( y = \frac{1}{2(x+1)} \)
   
   This function is just a vertical stretch of \( y = \frac{1}{x} \) shifted one unit to the left.
   
   Notice that the vertical asymptote is \( x = -1 \) and that the horizontal asymptote is \( y = 0 \).
3. \( y = \frac{1}{(x-2)} - 3 \)

This function is the graph of \( y = \frac{1}{x} \) shifted two units to the right and three units down. The horizontal asymptote is \( y=0 \) and the vertical asymptote is \( x=2 \):

![Graph of the function]

When our rational function is not in the form \( y = \frac{1}{a(bx+c)} + d \), then it gets more complicated:

**Examples**

1. \( f(x) = \frac{x}{3x+2} \)

The vertical asymptote is \( x = -2/3 \) and the horizontal asymptote is \( y = 1/3 \). However several graphs have these two properties (I will draw them on the board). So, which one is it? We will need to test what happens when the function approaches \( x = -2/3 \) from the right and from the left. This is because for our vertical asymptote, our function either approaches \( \infty \) or \( -\infty \), and we need to figure out which one it is.

We know that \(-0.667 < -\frac{2}{3} < -0.666\), so we will evaluate our function for -0.666 and -0.667.

\[
\begin{align*}
f(-0.667) &= 667 \\
f(-0.666) &= -333
\end{align*}
\]
What happens if we pick x values even closer to -2/3?

\[ f(-0.6667) = 6667 \]
\[ f(-0.666) = -3333 \]

So as we approach x=-2/3 from the left, the function approaches \( \infty \), and as our function approaches x = -2/3 from the right, the function approaches \(-\infty\). Our graph is finally:

2. \( f(x) = \frac{x}{x^2 - 4} \)

The horizontal asymptote is y=1, and the vertical asymptotes are x = 2 and x = -2.

We need to know what happens when we approach x = 2 and x = -2 from both the left and right:

<table>
<thead>
<tr>
<th>For x = -2:</th>
<th>For x = 2:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(-1.9) = 4.871794872 )</td>
<td>( f(1.9) = -4.871794872 )</td>
</tr>
<tr>
<td>( f(-1.99) = 49.87468672 )</td>
<td>( f(1.99) = -49.87468672 )</td>
</tr>
<tr>
<td>( f(-1.999) = 499.8749687 )</td>
<td>( f(1.999) = -499.8749687 )</td>
</tr>
<tr>
<td>( f(-2) =? )</td>
<td>( f(2) =? )</td>
</tr>
<tr>
<td>( f(-2.001) = -500.1249688 )</td>
<td>( f(2.001) = 500.1249688 )</td>
</tr>
<tr>
<td>( f(-2.01) = 50.12468828 )</td>
<td>( f(2.01) = 50.12468828 )</td>
</tr>
<tr>
<td>( f(-2.1) = -5.121951220 )</td>
<td>( f(2.1) = 5.121951220 )</td>
</tr>
</tbody>
</table>
We can see from this information that as $x$ approaches $-2$ from the left and when $x$ approaches 2 from the left, our function approaches $-\infty$, while as $x$ approaches $-2$ from the right and as $x$ approaches 2 from the right, our function approaches $\infty$. So, the graph of our function is:

Some other functions and their graphs:

$$y = \frac{x}{(x - 2)^2}$$
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\[ y = \frac{x}{2x + 1} \]

\[ y = \frac{1}{(x - 1)(x + 1)(x - 2)} \]
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\[ y = \frac{2x}{4x - 3} \]

\[ y = \frac{2}{x(x - 3)} + 2 \]

**Homework**

Pages 140-141, #20, 27, 31
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November 14

Massachusetts Curriculum Frameworks Addressed in this Lesson:
A-REI-4b Solve quadratic equations by... factoring.
A-SSE-2 Use the structure of an expression to identify ways to write it.

Do Now:
Many students could not factor the denominators on the rational functions on the test, so I think that it is necessary to review factoring.

Factor:

\[ x^2 - 1 \]
\[ x^2 - 2x + 1 \]
\[ x^2 - x \]
\[ x^2 - x - 6 \]
\[ x^4 - 3x^3 + 2x^2 \]
\[ x^3 + 5x^2 + 6x \]

Solve by factoring:

\[ 2x^2 + 5x - 3 = 0 \]
\[ x^3 + 2x^2 + x = 0 \]

How to factor:

- Is there a common factor among all the terms? If so, factor them out.
  - Examples:
    - \[ x^3 + 2x^2 - 2x = x(x^2 + 2x - 2) \]
    - \[ 3ab + ab^2 - 2a^2b = ab(3 + b - 2a) \]
    - \[ 45x^4 - 15x^3 + 30x^2 = 15x^2(3x^2 - x + 2) \]

- If the expression can be written in the form \( a^2 + 2ab + b^2 \) then it can be factored into \( (a + b)^2 \).
  - Examples:
    - \[ x^2 + 14x + 49 = (x + 7)^2 \]
    - \[ x^4 + 6x^2 + 9 = (x^2 + 3)^2 \]
    - \[ x^2 + 2x + 1 = (x + 1)^2 \]
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- If the expression can be written in the form $a^2 - 2ab + b^2$ then it can be factored into $(a - b)^2$.
  - Examples:
    - $x^2 - 14x + 49 = (x - 7)^2$
    - $x^2 - 4x + 4 = (x - 2)^2$
    - $x^4 - 10x^2 + 25 = (x^2 - 5)^2$

- If the expression can be written in the form $a^2 - b^2$, then it can be factored into $(a + b)(a - b)$.
  - Examples:
    - $x^2 - x^4 = (x + x^2)(x - x^2) = x(1 + x)x(1 - x) = x^2(1 - x)(1 + x)$
    - $x^2 - 9 = (x - 3)(x + 3)$
    - $x^4 - 4 = (x^2 - 4)(x^2 + 4)$

- If the expression is in the form $ax^2 + bx + c$, where $a$, $b$, and $c$ are constants, then it may or may not be factored. To factor expressions in this form, use the following method:

  Example: $5x^2 - 17x + 6$

  Want two numbers that multiply to $5 \times 6 = 30$ and add to $-17$. Choose $-15$ and $-2$.

  
  $5x^2 - (15x + 2x) + 6$
  $(5x^2 - 15x) + (-2x + 6)$
  $5x(x - 3) + (-2)(x - 3)$
  $(5x - 2)(x - 3)$

  Example: $6x^2 + 11x - 10$

  Want two numbers that multiply to $6 \times -10 = -60$ and add to $11$. Choose $-4$ and $15$.

  $6x^2 + (-4x + 15x) - 10$
  $(6x^2 - 4x) + (15x - 10)$
  $2x(3x - 2) + 5(3x - 2)$
  $(3x - 2)(2x + 5)$

  Example: $12x^2 - 42x + 18$

  Pick two numbers that multiply to $12 \times 18 = 216$ and add to $-42$. Choose $-6$ and $-36$.

  $12x^2 - (6x + 36x) + 18$
  $(12x^2 - 6x) + (-36x + 18)$
  $6x(2x - 1) - 18(2x - 1)$
  $(6x - 18)(2x - 1) = 6(x - 3)(2x - 1)$

Homework:
Factor the following expressions. If it is not factorable, write “prime”. To be graded by classmates.

1. $3x^2 + 7x + 2$
2. $x^2 - 49$
3. $x^2 + 49$
4. $3x^4 + 9x^2$
5. $x^2 + 2x - 6$
6. $x^2 - 21x - 72$
7. $7x^2 - 9x + 2$
8. $3x^2 - 2x - 5$
9. $2(x - 1)^2 - 9(x - 1) - 5$
10. $33x^2 - x - 14$
11. $21x^2 + 22x - 24$
12. $x^4 - x^2 + 4x - 4$

1. $(x + 2)(3x + 1)$
2. $(x - 7)(x + 7)$
3. prime
4. $3x^2(x^2 + 3)$
5. prime
6. $(x - 24)(x + 3)$
7. $(x - 1)(7x - 2)$
8. $(x + 1)(3x - 5)$
9. $(x - 6)(2x - 1)$
10. $(3x - 2)(11x + 7)$
11. $(3x - 2)(7x + 12)$
12. $(x - 1)(x + 2)(x^2 - x + \ldots)$
Appendix H: Pre-Calculus Lesson Plans

November 15-16

Massachusetts Curriculum Frameworks Addressed in this Lesson

In this lesson, students will learn how to approximate the slope of a function at a given point using an informal limit technique. There are no Massachusetts Curriculum Frameworks that are addressed in this lesson. However, the concepts of a limit and slope are extremely important for AP Calculus, and this is a major topic outlined in the curriculum for pre-calculus. The focus of this class for the next few weeks will not be on the frameworks, but on an intro to calculus.

Do Now:

11/15

Graph $y = x^2$ and $y = 2x + 1$ on the same plane. The line is tangent to the parabola. What is the slope of the line? What is the slope of the parabola at (1, 1)?

11/16

(from SAT) If $0 \leq x \leq 8$ and $-1 \leq y \leq 3$, which of the following gives the set of all possible value of $xy$?

a) $xy = 4$
b) $0 \leq xy \leq 24$
c) $-1 \leq xy \leq 11$
d) $-1 \leq xy \leq 24$
e) $-8 \leq xy \leq 24$

Lesson:

How do we find the slope of a curve at a specific point? Suppose that we wanted to find the slope of $y = x^2$ on (2, 4). We could estimate the slope of the curve at that point by drawing a line through the parabola and calculating its slope:
Appendix H: Pre-Calculus Lesson Plans

The line goes through the points (0, 0) and (2, 4), so it has a slope of (4-0) / (2-0) = 2.

How would we get a better estimate for the slope at (2, 4)? Instead of picking the origin, we can choose a closer point to (2, 4) for the line to go through. **We must pick points on the curve!** Let’s try (1, 1):

Now the line goes through (1, 1) and (2, 4), so it has a slope of (4-1) / (2-1) = 3.

We could get an even better approximation by choosing an even closer point to (2, 4), such as (1.5, 2.25). Remember that we only want points on the curve!

As you can see, we are starting to get very close to the actual slope. The slope of the line that passes through (1.5, 2.25) and (2, 4) is (4-2.25) / (2-1.5) = 3.5
Let’s try to get an even better approximation. This time let’s get very close to \((2, 4)\) and see what happens. What happens if we choose \((1.9, 3.61)\) instead?

The slope of this line is almost indistinguishable to the line of the curve at \((2, 4)\). The slope of the line that passes through \((2, 4)\) and \((1.9, 3.61)\) is \((4-3.61) / (2-1.9) = 3.9\)

You might predict what the slope is, but just to be sure let’s try one more point. The slope of the line that passes through \((2, 4)\) and \((1.99, 3.9601)\) is \((4-3.9601) / (2-1.99) = 3.99\)

At this point, we can safely assume that the slope of \(y = x^2\) is 4 at \((2, 4)\).

Notice that we chose \(x\) values that are less than 2. We could have just as easily picked \(x\) values that were greater than 2.

Now, try to compute the slopes of the following curves at the specific point by completing the tables below:

7. \(y = x^3\) at \((2, 8)\)

<table>
<thead>
<tr>
<th>First point</th>
<th>(2, 8)</th>
<th>(2, 8)</th>
<th>(2.8)</th>
<th>(2.8)</th>
<th>(2, 8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second point</td>
<td>(4, _ )</td>
<td>(3, _ )</td>
<td>(2.5, _ )</td>
<td>(2.1, _ )</td>
<td>(2.01, _ )</td>
</tr>
<tr>
<td>Slope</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
8. \( y = \sqrt{x} \) at (9, 3)

<table>
<thead>
<tr>
<th>First point</th>
<th>(9, 3)</th>
<th>(9, 3)</th>
<th>(9, 3)</th>
<th>(9, 3)</th>
<th>(9, 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second point</td>
<td>(11, )</td>
<td>(10, )</td>
<td>(9.5, )</td>
<td>(9.1, )</td>
<td>(9.01, )</td>
</tr>
<tr>
<td>Slope</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix H: Pre-Calculus Lesson Plans

9. \( y = \frac{1}{x} \) on (-1, -1)

<table>
<thead>
<tr>
<th>First Point</th>
<th>(-1, -1)</th>
<th>(-1, -1)</th>
<th>(-1, -1)</th>
<th>(-1, -1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second Point</td>
<td>(-0.1, )</td>
<td>(-0.5, )</td>
<td>(-0.9, )</td>
<td>(-0.99, )</td>
</tr>
<tr>
<td>Slope</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

10. \( y = \frac{1}{x+1} \) on (-2, -1)
11. \( y = x^2 \) on (3, 9)
12. \( y = -\sqrt{x} \) when \( x = 1 \)
13. \( y = 2x^2 \) when \( x = -2 \)
14. \( y = \frac{1}{x} \) when \( x = 1 \)
15. \( y = -x^3 \) when \( x = -3 \)
16. \( y = \sin(x) \) when \( x = \pi/2 \)

Homework:
Estimate the slope of each curve at the specified point using the method outlined above:
1. \( y = x^2 \) at (4, 16)
2. \( y = 2x^2 - 3x + 5 \) at (1, 4)
3. \( y = \sqrt{x} - 2 \) at (3, 1)
4. \( y = \frac{1}{x^2} \) at (2, 0.25)
5. \( y = (x - 2)(x + 3) \) at (0, -6)

Honors Pre Calculus

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Appendix H: Pre-Calculus Lesson Plans

November 17th, 2011

Limit Definition of the Derivative.

Objectives:

- Students will recognize that the word derivative is a synonym for the word slope.
- Students will recognize the difference quotient and understand its derivation.
- Students will be able to use the limit definition of the derivative to calculate the derivative of first, second, and third degree polynomials.

Do Now:

Sketch $y = -x^2$, and estimate the slope of the line tangent to the parabola when $x = -1$.

Lesson:

Students should now that in order to estimate the slope of a curve at a specified point, they choose a point on the curve very close to the given point, then calculate the slope of the line that passes through the two points. This concept is critical to understand in order to understand the difference quotient.

I will lead a class discussion on the above concept by using an applet available online. In the applet, you are able to give it any function and it will plot it. It will also plot two points, one fixed and one that the user can move. The applet draws the secant line through the two points. At the two points get closer, I will be able to show the students that the secant line becomes closer to the tangent line. I will be able to show the students the applet by projecting it on the board. The applet is available here: http://calculusapplets.com/derivpoint.html

After this presentation, I will draw the following graph on the board:

![Graph of a function and its derivative](image)

Students should be able to know that the secant line drawn above will give an estimation of the derivative at $(x, f(x))$. The approximate derivative is:
The formula above is called the **difference quotient**. Students should know that in order to approximate the derivative better, they need to pick a point closer to the specified point. We cannot choose \( x \), but we can choose \( h \). So, to bring our “movable” point closer to the “fixed” point, \( h \) must get smaller and smaller. In other words, \( h \) must approach zero. We write this as:

\[
m = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
\]

Finally, the notation for the derivative of a function is \( f'(x) \). So, we can replace \( m \) with \( f'(x) \) to get the limit definition of the derivative:

\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
\]

I will present the class with a few examples. I will discuss the fact that we don’t like the \( h \) on the denominator because \( h \) is approaching 0. So we need to try to factor out an \( h \) in the numerator, so that the \( h \) in the denominator will cancel. That way, we are able to calculate the derivative.

**Examples:**

- \( y = 2x + 1 \) We already know the slope of this line, so we can compare it to the result given by the limit definition of the derivative.
- \( y = x^2 \)
- \( y = 2x^2 + x \)
- \( y = x^3 \)

**Homework:**

Use the limit definition of the derivative to find the derivatives of the following functions:

5. \( y = x^2 + 5 \)
6. \( y = x^2 - 2x \)
7. \( y = 3x^2 - 8x + 5 \)
8. \( y = 2x^3 - 1 \)
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Honors Pre Calculus

November 18th, 2011

Constant Rule and the Power Rule

Objectives:

- Students will learn that the derivative of a constant is a zero.
- Students will discover a pattern with the derivatives of polynomials, and will learn the power rule.
- Students will learn that the derivative of \( f(x) + g(x) \) is \( f'(x) + g'(x) \)

Do Now:

Use the limit definition of the derivative to find the derivative of a constant \( c \). Does the answer surprise you?

Lesson:

The result of the do now is the constant rule:

\[ f(x) = c, \text{ then } f'(x) = 0 \]

I will draw a few graphs to convince students that this is true if they are not able to convince themselves.

Next, I will have students complete the following table. Some of these derivatives have been calculated before:

<table>
<thead>
<tr>
<th>( f(x) )</th>
<th>( x )</th>
<th>( x^2 )</th>
<th>( x^3 )</th>
<th>( x^4 )</th>
<th>( x^n )</th>
<th>( x+x^2 )</th>
<th>( x^2+x^3 )</th>
<th>( x^3+x^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f'(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Students may be able to discover two patterns. One is the power rule:

\[ f(x) = x^n, \text{ then } f'(x) = nx^{n-1} \]

The other is the sum rule:

*The derivative of \( f(x) + g(x) \) is \( f'(x) + g'(x) \)*

I will have students fill out the following table:

<table>
<thead>
<tr>
<th>( f(x) )</th>
<th>( x^2 )</th>
<th>( 2x^2 )</th>
<th>( 3x^2 )</th>
<th>( 4x^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f'(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Students may discover the following rule:

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If \( f(x) = cx^n = cnx^{n-1} \)

I will show the class a few examples on how to apply these four rules:

**Examples:**

Find the derivatives of the following functions:

1. \( f(x) = x^5 - x^4 + 4 \)
2. \( f(x) = 2x^8 + x^4 - x^3 + 5x^2 - 9 \)
3. \( f(x) = 5\sqrt{x} \)
4. \( f(x) = \frac{2}{x} + 5x^2 \)
5. \( f(x) = \frac{\sqrt[3]{9x^2}}{2x^2} - 4 \)

If there is time remaining, I will produce more functions for the class to evaluate the derivative for.

**Homework:**

**Period 1:**

p. 153 #16-26 even (use the rules for derivatives, not the limit definition)

**Period 3:**

Use the limit definition of the derivative to find the derivatives of the following functions:

9. \( y = x^2 + 5 \)
10. \( y = x^2 - 2x \)
11. \( y = 3x^2 - 8x + 5 \)
12. \( y = 2x^3 - 1 \)

Also, p.153 #16-26 even (use the rules for derivatives, not the limit definition)
Applications of the Derivative

Objectives:

- Students will be able to apply the derivative for real-world applications.

Do Now:

Find the derivative of the following functions:

1. \( f(x) = \sqrt{x} + \frac{1}{x} + x^2 \)
2. \( f(x) = x^{-15} + \pi x^2 \)
3. \( f(x) = \frac{1}{\sqrt{x}} - 9x^3 \)

Lesson:

The position of an object is given by the function \( f(t) = t^2 - 2t + 5 \), where \( f(x) \) is position in meters and \( t \) is time in seconds. The graph of the function is shown below.

The velocity of the object is its position divided by the time. So, by finding the derivative of the function, you can determine the velocity of the object. \( f'(t) = 2t - 2 \):
The acceleration of the object is the rate of change in velocity over time. So, to find the acceleration of the object, simply take the derivative again. \( f'''(t) = 2 \). The acceleration of the object is a constant 2 meters per second squared.

**Class work:**

Find the velocity and the acceleration of the following objects when \( x=3 \). In the following problems, \( f(x) \) is the position in meters, and \( x \) is the time in seconds.

- A car’s position is given by \( f(x) = 0.05x^5 - 0.5x^4 + 0.1x^3 + 0.5x^2 \)
- A jet’s position is given by \( f(x) = \sqrt{x} + 2x^3 - 3x^2 \)
- A starfish’s position is given by \( f(x) = 0.001x^2 - \frac{1}{2x} \)
- A comet’s position is given by \( f(x) = 20x^{20} - 400x^{12} + 22x^{10} \)
- A butterfly’s position is given by \( f(x) = (x - 1)(x + 2) - \frac{1}{x} \)

Any time you want to calculate the rate of change of something you want to find the derivative of it. For example, if temperature with respect to time is given by the equation \( T(x) = 3x^2 \), then the rate of change of the temperature is given by its derivative, \( T'(x) = 6x \).

**Class work:**

- The pressure inside of a pipe with respect to time is given by the formula \( P(t) = 3t^2 - 2t - 5 \). What is the rate of change of the pressure with respect to time?
- The altitude of a hot air balloon with respect to time is given by the equation \( A(t) = \frac{1}{t} + 0.5t^2 \). What is the rate of change in altitude with respect to time?
- The amount of fish in a tank with respect to the concentration of salt in the water is given by the equation \( F(s) = 2s - \sqrt{s} \). What is the rate of change of the amount of fish inside the tank with respect to the concentration of salt in the water?
Appendix H: Pre-Calculus Lesson Plans

- \( T(n) = 3n^4 - 2n + \sqrt{n} \) is the relation between temperature and the amount of Sodium in a reactive substance, where \( n \) is the amount of sodium. What is the rate of change of temperature with respect to the amount of sodium in the substance?

Homework:

None.
### November 22, Derivative Bingo

This is the day before Thanksgiving break, so I decided to something different. Students compute the derivatives on the right and mark the corresponding answer off below. This was a race to see who got BINGO first. Students that finish are asked to help other students.

<table>
<thead>
<tr>
<th>1/x + 4x^2</th>
<th>-1/x^3 + 2/x^3</th>
<th>2x</th>
<th>3x</th>
<th>-5x^4 - 2/x^3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/(x^2 + 4x^2)</td>
<td>-1/x^3 + 2/x^3</td>
<td>20.</td>
<td>13.</td>
<td>3x</td>
</tr>
<tr>
<td>9x^2 + 12x - 24</td>
<td>2x + 2</td>
<td>5x^4 - 12x^3 + 6x^2</td>
<td>12.</td>
<td>(x - 3)^2</td>
</tr>
<tr>
<td>9x^2 + 3x - 2</td>
<td>2x + 1</td>
<td>3x^2 - 4x + 5</td>
<td>8.</td>
<td>1/(2x)</td>
</tr>
<tr>
<td>3x^2 - 3</td>
<td>2x - 1/x^2</td>
<td>1/(2x) + 1/x</td>
<td>2.</td>
<td>+ 9</td>
</tr>
<tr>
<td>1/x^2 + 2x^2</td>
<td>3x^2 + 3</td>
<td>-6x + 3√x/2 - 2</td>
<td>14.</td>
<td>4x + 3</td>
</tr>
</tbody>
</table>

1. \( f(x) = x^2 + 2x + 5 \)
2. \( f(x) = \sqrt{x} - 2x^3 \)
3. \( f(x) = \frac{1}{x} + x^2 \)
4. \( f(x) = \frac{1}{x^2} - x^5 \)
5. \( f(x) = -3\sqrt{x} - x^2 + 4 \)
6. \( f(x) = (x - 2)(x + 3) \)
7. \( f(x) = x(x - 1)^2 \)
8. \( f(x) = x^3 - 2x^2 + 5x - 1 \)
9. \( f(x) = 2x + 3 - \frac{1}{x} + \sqrt{x} \)
10. \( f(x) = 3x^2 + 2x - 3\sqrt{x} \)
11. \( f(x) = (x - 3)^3 \)
12. \( f(x) = x^3(x - 2)(x - 1) \)
13. \( f(x) = \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} \)
14. \( f(x) = \frac{x^2}{\sqrt{x}} - 3x^2 - 2x + 3 \)
15. \( f(x) = 2x^2 + 3x - 5 \)
16. \( f(x) = \sqrt{xx} - x^2 \)
17. \( f(x) = x^3 - 3x \)
18. \( f(x) = 3(x - 2)(x + 4) \)
19. \( f(x) = 3x^3 + 3x - 2 \)
20. \( f(x) = (x - 4)(x + 4) \)
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Honors Pre Calculus

November 28th 2011

Critical Points

Objectives:

- Students will learn under what conditions a function has a maximum or a minimum.
- Students will identify extrema for functions with both a restricted domain and an unrestricted domain.

Do Now:

Find the derivative of:

1. \( y = 2x^2 - 4x + 3 \)
2. \( y = x^3 - 4 \)
3. \( y = 2x^2 - 2x - 2 \)
4. \( y = -x^3 + 2x^2 + 4x - 2 \)
5. \( y = x^3 + 2x^2 - 3x - 1 \)

Lesson:

A maximum and a minimum are the two types of critical points. There is a third type that we will discuss later. A local maximum is a point that has a greater y-coordinate than the neighboring points on a curve. A global maximum is a point that has a greater y-coordinate than any other point on a curve. A local and global minimum are defined similarly.

A maxima and minima may occur when the derivative of a function is zero. In other words, the curve has a slope of zero. A maximum or a minimum may also occur on the endpoint of an interval. As we will see, having a derivative of zero doesn’t mean anything unless we check neighboring points!

Example 1:

Consider #1 from the do-now. The graph of the function is:
Appendix H: Pre-Calculus Lesson Plans

Its derivative is $y' = 4x - 4$. A maximum or minimum may occur when either:

- The derivative is zero, or
- at an endpoint

For now, let’s deal with the domain of all real numbers, so that there aren’t any endpoints to test. So we just need to set the derivative to zero: $0 = 4x - 4 \rightarrow 4 = 4x \rightarrow x = 1$.

Substituting $x = 1$ into the original equation yields $y = 1$.

So we know that $(1, 1)$ is either a maximum or a minimum. From the graph above, we know that we have a minimum. Notice that for points on both sides of $(1, 1)$, the function has a greater $y$-coordinate than 1. Since this is the only minimum, $(1, 1)$ is a global minimum.

Example 2:

Find the maximum and minimum of $y = x^3 - 4$ on the interval $[-3, 3]$.

The graph of the function is:
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First, let’s make the derivative equal to zero: \( y' = 3x^2 = 0 \rightarrow x = 0 \).

Substituting \( x = 0 \) into \( y = x^3 - 4 \) gives us the y-coordinate \( y = -4 \).

So, \((0, -4)\) is a potential maximum or minimum. However, we see from the graph that it is neither a maximum nor a minimum. Notice that points to the right of \((0, -4)\) have a greater y-coordinate than \( y = -4 \), and that points to the left of \((0, -4)\) have a lesser y-coordinate than \( y = -4 \).

This time our domain is \([-3, 3]\) instead of all real numbers, so we still have to check the endpoints.

For \( x = -3 \), we get \( y = -31 \). This is a local minimum because points to the right of this point have a greater y-coordinate than \( y = -31 \). (There aren’t any points to the left to check)

For \( x = 3 \), we get \( y = 23 \). This is a local maximum because points to the left of this point have a lesser y-coordinate than \( y = 23 \). (There aren’t any points to the right to check)

In summary, we have \((-3, -31)\) as a minimum, and \((3, 23)\) as a maximum.

Example 3:

Find the maxima and minima of \( y = 2x^2 - 2x - 2 \) on the interval \([-10, 10]\).

The graph of the function is:

First let’s make the derivative equal to zero: \( y' = 4x - 2 = 0 \rightarrow 4x = 2 \rightarrow x = 1/2 \)

Substituting \( x = 1/2 \) into \( y = 2x^2 - 2x - 2 \) give us the y-coordinate \( y = -5/2 \)
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So (0.5, -2.5) is a potential maximum or minimum. From the graph above, it is apparent that the point is a minimum. Notice that points to the right of (0.5, -2.5) have a greater y-coordinate than -2.5, and that points to the left of (0.5, -2.5) have a greater y-coordinate than -2.5.

Checking the endpoints: When x=-10, y=218, and when x=10, y=178.

Points to the right of (-10, 218) have a lesser y-coordinate than 218, so it is a local maximum.

Similarly, points to the left of (10, 178) have a lesser y-coordinate than 178, so it is a local maximum.

Note that you can have several local maxima and minima, but at most one global maximum and one global minimum.

A table showing the behavior of neighboring points when you have a maximum or a minimum is shown on the bottom of page 158.

Class work:

Find all extrema (maxima and minima) of the last two do-now problems on the interval [-5, 5]. Then find the extrema of the following on [-2, 2]:

1. \( y = 2x^2 - x + 3 \)
2. \( y = x^2 - 2x + 2 \)
3. \( y = x^3 + x^2 + x + 1 \)
4. \( y = 9x - 4 \)
5. \( y = x^4 - x^2 + 4 \)

Homework:

Page 162, 10-16 even. Find maxima and minima only. The domain is all real numbers, so you don’t have to check endpoints, because there aren’t any endpoints!
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Honors Pre Calculus

November 29th 2011

Applications of Maxima and Minima

Objectives:

- To be able to graph the derivative of a function using a graphing calculator.
- To find the zeros of a derivative from its graph, and use that information to find extrema.
- Know what an increasing and a decreasing function is.
- Know that the sign of the derivative on both sides of a point can be used to determine if a point is a maximum or a minimum, or neither.
- Understand applications of maxima and minima, and apply extrema to real world applications.

Do Now:

With a graphing calculator, find the coordinate pairs of each extrema (maxima and minima) of the following. (This required me to show the students how to use the max and min functions on the calculator)

1. \( y = x^5 - 3x^4 - 11x^3 + 27x^2 + 10x - 24 \)
2. \( y = x^4 + 3x^3 - 11x^2 - 3x + 10 \)
3. \( y = x^4 - 5x^2 + 4 \)

![Graph of function 1](image1)

![Graph of function 2](image2)

![Graph of function 3](image3)

Solutions:

1. max: (-2.31234, 81.36889)
min: (-0.16890, -24.86834)
max: (1.52648, 7.05200)
min: (3.35476, -56.96311)

2. min: (-3.68668, -94.03956)
max: (-0.12986, 10.19779)
min: (1.56654, -4.13870)
Appendix H: Pre-Calculus Lesson Plans

3. \[ \text{min: } (\sqrt{10}/2, -9/4) = (1.58114, -2.25) \]
\[ \text{max: } (0, 4) \]
\[ \text{min: } (-\sqrt{10}/2, -9/4) = (-1.58114, -2.25) \]

When I review the do-now, I will mention the following:

A critical point is a minimum if the derivative is negative on the left, but positive on the right.
A critical point is a maximum if the derivative is positive on the left, but negative on the right.
A critical point is neither a max nor min if something else other than the above occurs.

**Vocabulary:**

An **increasing** function is a function that has a positive derivative.
A **decreasing** function is a function that has a negative derivative.

**Applications of Extrema:**

The temperature on a given day is \( T = -0.1x^2 + 2x + 42 \), where \( T \) is the temperature in Fahrenheit, and \( x \) is the time of day in hours. Find the maximum and the minimum temperature on that day. Note that the domain must be \([0, 24]\)

The graph of the function is:

The derivative of the function is:

\[ T' = -0.2x + 2 = 0 \]
Appendix H: Pre-Calculus Lesson Plans

\[-0.2x = -2\]
\[x = 10\]

The y-coordinate is:
\[T(10) = -0.1(10)^2 + 2(10) + 42 = 52\]

To the right of (10, 52), we see that the derivative is negative, and to the left of (10, 52), the derivative is positive. Therefore, (10, 52) is a maximum.

We must also check the endpoints:
\[T(0) = 42\]
\[T(24) = 32.4\]

To the right of (0, 42), the derivative is positive, so it is a minimum.
To the left of (24, 32.4), the derivative is negative, so it is a minimum.

For this problem, we are only interested in the global extremes, so the high temperature is 52° at hour 10, and the low temperature is 32.4° at hour 24.

Class work:
1. Same as the example, but this time the temperature is \(T = -0.1x^2 + x + 80\).
2. The position of a car is given by \(S = 2x^2 + 5x - 8\) where \(x\) is restricted from \([0, 10]\). \(x\) is in seconds and \(s\) is in KPH. What is the maximum velocity of the car during this time period?
3. The same as problem 3, except the function is now \(S = x^3 - 2x^2 + 10\) and the domain is \([0, 5]\)
4. What is the maximum and minimum acceleration of an object if its position is given by \(S = x^4 - 5x^3 + 2x^2 + x + 13\) on the interval \([0, 20]\)?
5. The altitude of a toy rocket is \(H = -3x^2 + 50x\), where \(h\) is in meters and \(x\) is in seconds. The ground has the altitude of 0m. The domain is restricted from time zero to whatever time the rocket hits the ground. What is the maximum altitude of the rocket? (The minimum is obviously 0m.)
6. The probability that a person will die at a certain time, rounded to the nearest minute, after getting bit from a rattlesnake is \(P = -0.001x^3 + 0.01x^2\), where \(P\) is the probability, and \(x\) is the time in minutes, restricted on \([0, 10]\). What is the most lethal time for a person bitten by a rattlesnake?

If there is time left, I will have students find the vertex of a parabola: \(ax^2+bx+c=0\).

Homework:

Write up a word problem involving maxima and minima, and solve it. Tomorrow, you will give another student your word problem to try!
Appendix H: Pre-Calculus Lesson Plans

Honors Pre-calculus

November 30th 2011

More Applications of Extrema

Objective:

Period 1 was long yesterday and got further than period 3, therefore I will spend another day on applications of extrema so that period 3 can catch up.

- Understand applications of maxima and minima, and apply extrema to real world applications.
- Understand what is the maximum number of extrema an \( n \)th degree polynomial can have.

Do Now:

p.160, Example 5

Homework Exercise:

Yesterday, I told students to write up a word problem involving maxima and minima themselves. I separated students that done their homework with students that haven’t done their homework. (I will also note this in my grade book.) Students that have done their homework will exchange problems and solve each others. The rest of the class can finish word problems that I provide. Many students had trouble with the homework assignment. Some students made a solvable solution, but the answer was unreasonable. For example, the solution to one student’s word problem was a minimum temperature of -42 degrees and a maximum temperature of 131 degrees in a course of ten hours!

Additional Word Problems:

1. A toy company wants to minimize the amount of material required to box their product. Since their product is 9” high, the box must be at least 10” high. Also, the volume of the box must be at least 250 cubic inches to allow enough room for cushioning. What dimensions should be used to minimize surface area?
2. The position of a subatomic particle is \( S = 3x^5 - 2x^4 - x^3 - 3x \). What are the maximum and minimum acceleration of the subatomic particle in the interval [0, 100]? 
3. Where is the vertex of \( y = 3x^2 - x + 4 \)?
4. At most how many extrema can a third degree polynomial have? What about a tenth degree polynomial? An \( n \)th degree polynomial?
5. P. 16, #33
6. A farmer wishes to build a rectangular pen for his animals out of 40 feet of fencing. What should be the length and width of pen so that its area is as great as possible?

Homework: p. 162, 11-17 odd, just find maxima and minima. Also #34.

Honors Pre-calculus

December 1st, 2011
Appendix H: Pre-Calculus Lesson Plans

Infection Points

Objectives:

- Students will recognize the shape of a curve that is concaved upward and concaved downward.
- Students will understand the relationship between the sign of the second derivative and concavity.
- Students will be able to identify points of inflection.

Do Now:

Can a function have an infinite number of extremes? If so, give an example.

Can a function have no extremes? If so, give an example.

Lesson:

A curve that looks like a smile, or part of a smile, is **concave upward**.

A curve that looks like a frown, or part of a frown, is **concave downward**.

A **point of inflection** is the point at which a curve changes concavity. A point of inflection occurs when the second derivative is zero, unless the point is an endpoint. A point of inflection is the third type of **critical point**.

Curves that are concave upward have a positive second derivative, while curves that are concave downward have a negative second derivative.

**Example:**

Find all the x-coordinates of infection for $y = 2x^5 - \frac{10}{3}x^3 - 2x + 5$.

\[
y' = 10x^4 - 10x^2 - 2
\]

\[
y'' = 40x^3 - 20x = 0
\]

\[
0 = x(40x^2 - 20)
\]

\[
x = 0 \quad \text{or} \quad 40x^2 - 20 = 0
\]

\[
40x^2 = 20
\]

\[
x = \pm\sqrt{0.5} = \pm0.8409
\]

So, there are points of inflection when $x=0, 0.8409,$ and $-0.8409$.

**Example:**

Identify all the **critical points** of $y = 3x^3 - 9x + 5$. 

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Appendix H: Pre-Calculus Lesson Plans

\[ y' = 9x^2 - 9 = 0 \]
\[ 9x^2 = 9 \]
\[ x = \pm 1 \]

If \( x > 1 \), then \( y' > 0 \), and if \( x < 1 \), then \( y' < 0 \)

So, (1, -1) is a **minimum**.

If \( x > -1 \) then \( y' < 0 \) and if \( x < -1 \), then \( y' > 0 \)

So, (-1, 11) is a **maximum**.

For the interval (-1, 1), the curve is **decreasing**, and for all other \( x \) values the curve is **increasing**.

\[ y'' = 18x = 0 \]
\[ x = 0 \]

If \( x > 0 \), \( y'' > 0 \), and if \( x < 0 \), \( y'' < 0 \).

So, (0, 5) is a **point of inflection**.

When \( x > 0 \), the curve is **concave upward**, and when \( x < 0 \), the curve is **concave downward**.

**Class work (calculators allowed):**

Identify all of the critical points of:

1. \( y = 6x^5 + 33x^4 - 30x^3 + 100 \)
2. \( y = 2x^3 + 4x - 2 \)
3. \( y = x^3 - 3x^2 + 8x \)
4. \( y = 9x^2 - 4x + 2 \) Hint: Can a parabola have a point of inflection? Why or why not?
5. \( y = 2x^4 - 5x^2 \)
6. \( y = x^3 + x - 8 \)
7. \( y = 2x^3 - 8x^2 - 4x + 3 \)
8. \( y = 6x^2 - 9x + 2 \)
9. \( y = 9x^3 - 2x - 5 \)
10. \( y = 4x^7 - 3x^5 \)

**Homework:**

p.162 #10-17, find the points of inflection only, if they exist.
Appendix H: Pre-Calculus Lesson Plans

Honors Pre-calculus

December 2nd 2011

Continuity

Objectives:

- Students will be able to recognize the difference between a continuous function and a discontinuous function.
- Students will be able to determine what type of discontinuities a function has from its graph.
- Students will know that polynomial functions are continuous.

Do Now:

Identify the critical points of \( f(x) = 3x^3 - 9x + 5 \).

Continuity:

If a function has no breaks or holes, it is called a **continuous** function. On the other hand, if a function has breaks and holes, it is **discontinuous**. Generally speaking, a function is continuous if you can draw it graphically without lifting your pencil. Although this is not a rigorous definition, this will be sufficient for our study.

Examples:

Graph the following functions to determine the following:

1. Is \( y = x^2 - 2x + 5 \) continuous?
2. Is \( y = x^3 - x + 1 \) continuous?
3. Is \( y = 1/x \) continuous?
4. Is \( y = \sin(x) \) continuous?
5. Is \( y = [x] \) continuous?
Appendix H: Pre-Calculus Lesson Plans

Types of Discontinuity

Asymptotic (or Infinite) Discontinuity  Jump Discontinuity  Point Discontinuity

Note that if a function is undefined at a point, it is discontinuous at that point.

Class work:

Can a polynomial function be discontinuous? Why or why not?

Can a trigonometric function be discontinuous? Why or why not?

Also do:

Page: 169 5-23

Homework:

Study for Wednesday’s Test on sections 3-6 and 3-8.
Appendix H: Pre-Calculus Lesson Plans

Honors Pre-calculus

December 5th, 2011

End Behavior

Objectives:

- Determine the end behavior (what happens when $x$ approaches $\infty$ and $-\infty$) of functions using graphs and tables.
- Evaluate limits involving $\infty$.
- Determine the end behavior of polynomial functions without graphing or making tables.

Do Now:

Determine if the following functions are continuous or discontinuous. Identify the type of discontinuity if the function is discontinuous.

1. $y = 3x^2 - 2x + 1$
2. $y = \frac{1}{x-2}$
3. $y = 3x - \frac{1}{x}$
4. $y = |x| - 4$
5. $y = \frac{x^2-4}{x+2}$
6. $y = \begin{cases} x + 1 & x \neq 0 \\ 2 & x = 0 \end{cases}$
7. $y = \begin{cases} x^2 & x > 0 \\ x & x \leq 0 \end{cases}$
8. $y = 3\tan(x)$

End Behavior:

End behavior is what value a function approaches as $x \to \infty$ or $x \to -\infty$.

Examples by Graphing:

$$y = -2x^2 + 5$$
As $x \to \infty, y \to -\infty$, and as $x \to -\infty, y \to -\infty$.

$$y = \frac{1}{x - 1}$$

As $x \to \infty, y \to 3$, and as $x \to -\infty, y \to 3$.

$$y = \frac{|x|}{x + 1}$$
As $x \to \infty, y \to 1$, and as $x \to -\infty, y \to -1$.

Examples with Tables

$$y = -x^2 + 4$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>10</th>
<th>100</th>
<th>1,000</th>
<th>10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-96</td>
<td>-996</td>
<td>-99996</td>
<td>-99999996</td>
</tr>
</tbody>
</table>

So as $x \to \infty, y \to -\infty$

<table>
<thead>
<tr>
<th>$x$</th>
<th>-10</th>
<th>-100</th>
<th>-1,000</th>
<th>-10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-96</td>
<td>-996</td>
<td>-99996</td>
<td>-99999996</td>
</tr>
</tbody>
</table>

So as $x \to -\infty, y \to -\infty$

Note that the two tables have the same $y$ coordinates. This is because our function is symmetric with respect to the $y$-axis.

$$y = \frac{4x}{2x - 5}$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>10</th>
<th>100</th>
<th>1,000</th>
<th>10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>2.66666667</td>
<td>2.05128205</td>
<td>2.00501253</td>
<td>2.00050013</td>
</tr>
</tbody>
</table>

So as $x \to \infty, y \to 2$

<table>
<thead>
<tr>
<th>$x$</th>
<th>-10</th>
<th>-100</th>
<th>-1,000</th>
<th>-10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>1.60000000</td>
<td>1.95121951</td>
<td>1.99501247</td>
<td>1.99950015</td>
</tr>
</tbody>
</table>

So as $x \to -\infty, y \to 2$
Appendix H: Pre-Calculus Lesson Plans

\[ y = \frac{|x|}{x} \]

<table>
<thead>
<tr>
<th>X</th>
<th>10</th>
<th>100</th>
<th>1,000</th>
<th>10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

So as \( x \to \infty, y \to 1 \)

<table>
<thead>
<tr>
<th>X</th>
<th>-10</th>
<th>-100</th>
<th>-1,000</th>
<th>-10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
</tbody>
</table>

So as \( x \to -\infty, y \to -1 \)

Note that this function has point discontinuity at \( x=0 \).

**Notation for End Behavior:**

For the previous example, we could have also written:

\[
\lim_{x \to \infty} \frac{|x|}{x} = 1 \quad \text{and} \quad \lim_{x \to -\infty} \frac{|x|}{x} = -1
\]

**End Behavior for Polynomials:**

Page 167 has a nice table for end behavior for polynomials.

Note that the largest power of a polynomial dominates as \( x \) becomes extremely small or extremely large. Therefore the polynomial \( P(x) = ax^n + bx^{n-1} + cx^{n-2} + \cdots \) exhibits the same end behavior as \( P(x) = ax^n \).

For positive \( a \):

<table>
<thead>
<tr>
<th>N</th>
<th>X</th>
<th>P(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Even</td>
<td>( \infty )</td>
<td>( \infty )</td>
</tr>
<tr>
<td>Even</td>
<td>( -\infty )</td>
<td>( \infty )</td>
</tr>
<tr>
<td>Odd</td>
<td>( \infty )</td>
<td>( \infty )</td>
</tr>
<tr>
<td>Odd</td>
<td>( -\infty )</td>
<td>( -\infty )</td>
</tr>
</tbody>
</table>

For negative \( a \):

<table>
<thead>
<tr>
<th>n</th>
<th>X</th>
<th>P(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Even</td>
<td>( \infty )</td>
<td>( -\infty )</td>
</tr>
<tr>
<td>Even</td>
<td>( -\infty )</td>
<td>( -\infty )</td>
</tr>
<tr>
<td>Odd</td>
<td>( \infty )</td>
<td>( -\infty )</td>
</tr>
<tr>
<td>Odd</td>
<td>( -\infty )</td>
<td>( \infty )</td>
</tr>
</tbody>
</table>
Appendix H: Pre-Calculus Lesson Plans

If you have trouble remembering this, just remember what the graphs of $y = x^2, y = x^3, y = -x^2, and y = -x^3$ look like!

Practice:


Also find the end behavior of 31-33 and 11-13.

Homework:

Study for test Wednesday! No cheat sheet!
Massachusetts Curriculum Frameworks Addressed in this Lesson:

Now that we are finished our introduction of calculus, our focus will return to fulfilling state standards:

N-CN-4 Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).

N-CN-8 Extend the polynomial identities to the complex numbers.

Do Now:

Solve:
1. $2x^3 - 4x = 0$
2. $18x^3 - 14x^2 = 0$

Change into exponential form:
1. $\sqrt{x} \sqrt{x^4} \sqrt{x^4}$
2. $\frac{\sqrt{x^3} 4x^{-2}}{x^2 \ 2 \ \frac{4x^{-2}}{3}}$

Lesson:

If you try to solve: $0 = x^2 + 4$, you will get the result $x = \pm \sqrt{-4}$. This is not a real number, because no real number can satisfy $x^2 = -4$. This is, however, an imaginary number. By definition:

$$i^2 = -1$$

So:

$$\sqrt{-4} = \sqrt{-1} \times 2^2 = 2i$$ and $x = \pm 2i$.

Complex numbers is the set of all real numbers and all imaginary numbers.
All complex numbers can be written in the form $a + bi$, where $a$ is the real part and $b$ is the imaginary part.

**Examples:**

1. Name all the number systems that include:
   a. $7$ whole, natural, integer, rational, real, complex
   b. $\pi$ irrational, real, complex
   c. $0$ natural, integer, rational, real, complex
   d. $7i+4$ complex

2. Identify the real and imaginary parts of:
   a. $6-4i$ real part: 6 imaginary part: -4
   b. $-2 + \sqrt{2}i$ real part: -2 imaginary part: $\sqrt{2}$
   c. $17i - 3$ real part: -3 imaginary part: 17
   d. $3.45$ real part: 3.45 imaginary part: 0

You can plot imaginary numbers in a **complex coordinate plane**. This is like the Cartesian coordinate plane, except the real part is in the ‘x’ axis and the imaginary part is in the ‘y’ axis.

**Examples:**

3. Plot the following points from example 2 in the complex plane (done on the board).
Appendix H: Pre-Calculus Lesson Plans

Properties of Complex Numbers:

\[
  i = \sqrt{-1}
\]

\[
  i^2 = \sqrt{(-1)^2} = -1
\]

\[
  i^3 = i^2i = -1i = -i
\]

\[
  i^4 = i^2i^2 = -1 \times -1 = 1
\]

\[
  i^5 = i^3i^2 = -i \times -i = i
\]

\[
  i^6 = i^3i^3 = -i \times -i = i^2 = -1
\]

...the pattern continues

If the power of \( i \) is even, then the result is always real (1 or -1)

If the power of \( i \) is odd, then the result is always imaginary (\( i \) or \( -i \))

\[
  (a + bi)(a - bi) = a^2 - (bi)^2 = a^2 - (-b^2) = a^2 + b^2
\]

Examples:

4. Simplify:
   a. \( i^{10} \quad i^{10} = i^2i^2i^2i^2i^2 = i^{25} = (-1)^5 = -1 \)
   b. \( 4i^{12} \quad 4i^{12} = 4i^{10}i^2 = 4 \times -1 \times -1 = 4 \)
   c. \( (3 - 4i)(3 + 4i) \quad 3^2 + 4^2 = 9 + 16 = 25 \)
   d. \( (7 - 3i)(7 + 3i) \quad 7^2 + 3^2 = 49 + 9 = 58 \)

Class work/ Homework:

1. Name all the number systems that include:
   a. 14
   b. 34.5
   c. 44.29
   d. \( \sqrt{3} \)
   e. \( \frac{\pi}{2} + i^2 \)
   f. 3 - 4i
   g. \( i^{20} \)
   h. \( \sqrt{\sqrt{81i^8}} \)
Appendix H: Pre-Calculus Lesson Plans

2. Identify the real and imaginary parts of the following. Then graph them in the complex coordinate plane. You may use one plane for all of the problems.

a. $3 - 4i$
b. $4 + 8i$
c. $4i^2 - 3i$
d. $(3 + 3i)(3 - 3i)$
e. $(3 - 3i)^2$
f. $(3 + 3i)^2$
g. $2i^3$
h. $2i^4 + 3i^3 - 2i^2 - 4i + 1$
i. $i(2i - 1)$
j. $(2 + i)(3 - 4i)(3 + 4i)$
Appendix I: Advanced Algebra Lesson Plans

October 27

Massachusetts Curriculum Frameworks Addressed in this Lesson:

A-REI-12  Graph solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality).

Do Now:

Graph the following functions separately:

1. \( y = 3x - 2 \)
2. \( y = -2x + 5 \)
3. \( y = 2|x - 3| + 1 \)

Lesson:

Inequalities involving lines:

I ask the students if the points \((1, 1), (2, 3), (1, 5),\) and \((-2, -1)\) are solutions to \(y > 3x - 2\).

\[
1 > 3(1) - 2 \rightarrow 1 > 1 \rightarrow false
\]
\[
3 > 3(2) - 2 \rightarrow 3 > 4 \rightarrow false
\]
\[
5 > 3(1) - 2 \rightarrow 5 > 1 \rightarrow true
\]
\[
-1 > 3(-2) - 2 \rightarrow -1 > -8 \rightarrow true
\]

Now that we tested several points of this inequality, we have an idea of where the solution exists. I tell the students two important things. Firstly, if an inequality is strictly unequal, a dashed line is used. Otherwise, a solid line is used. Secondly, you shade the region in which the solution exists. From the do-now, all we have to do is to choose the correct type of line and where to shade:

So, for \(y > 3x - 2\), we use a dashed line, because the inequality is strictly unequal. Secondly, we shade above the line because that is where the solutions exist. Notice that the true inequalities above are all in the shaded region, while the false ones are not.
I go over $y \leq -2x+5$ with them. They should already have drawn the line because of the do-now. Since it is not a strict inequality, a solid line is necessary. To determine where to shade, all we have to do is test one point that is not on the line. The simplest test would be the origin, $(0,0)$. Since $0 \leq 5$ is true, the origin is contained in the solution. Therefore, the shaded region is below the line:

So, to summarize the steps to graph an inequality:

1. Graph the line.
2. Determine if the line should be solid or dashed.
3. Test one point to determine where the shading is.
Appendix I: Advanced Algebra Lesson Plans

I now have the students try a few of these on their own:

\[ y > 2x+5 \]
\[ y \leq -2-3 \]
\[ y < 3x-4 \]
\[ y \geq 2x-2 \]

Finally, I give them the hint that if our inequality is in the form \( y \{\text{inequality}\} ax + b \), where \{\text{inequality}\} is either >, <, ≤, ≥, then they can use this simple rule: if the symbol is either > or ≥, then shade above the line, otherwise shade below.

**Inequalities involving absolute values:**

I tell them that the same rules apply when graphing inequalities with absolute values. They still need to graph the absolute value first, then determine if the lines are solid or dashed, and then determine where to shade.

The first example would be related to the do-now, \( y \geq 2|x - 3| + 1 \).

After graphing the line, we need to determine if the lines are solid or dashed. Since the inequality is also equal, we use a solid line. Then, we need to determine where to shade. An easy point to test would once again be the origin. Since 0≥7 is false, we do not shade in the region containing the origin. The resulting graph is:
Appendix I: Advanced Algebra Lesson Plans

If the students request it, I will do another example for them: \( y < |x + 2| - 1 \). Otherwise, I ask them to do this on their own, in addition to the following:

\[
\begin{align*}
    y & \geq -0.5|x - 1| + 3 \\
    y & \leq |x + 2| - 3 \\
    y & < 2|x - 4|
\end{align*}
\]

If there is time left, we will continue doing a mix of problems.

**Homework:**

Graph the following inequalities:

\[
\begin{align*}
    y & \geq 3x - 3 \\
    y & < -2x + 4 \\
    y & > 4x \\
    y & \leq 3x - 2 \\
    y & > |x - 2| + 1 \\
    y & \leq 2|x - 1| + 2
\end{align*}
\]
Appendix I: Advanced Algebra Lesson Plans

October 28

Massachusetts Curriculum Frameworks Addressed in this Lesson:

A-REI-6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Do Now:

Solve $4x - 2 = -3$.
Solve $-2x + 5 = 4 - 3x$.
Graph $y = 2x - 2$ and $y = x + 1$ on the same plane.

Lesson:

I explain that the solution to $y = 2x - 2$ is the corresponding line on the graph, and that the solution to $y = x + 1$ is the corresponding line on the graph. I also tell them that the intersection of the two lines is the solution to the system of equations. I remind them that since there is two variables, you need two equations to solve for them, unlike the other problems above that had only one variable and one equation. You can approximate the solution to any system of two linear equations by graphing both equations and finding its intersection:

So, for this particular system, the solution is about (and happens to be exactly) $x=3, y=4$.

You can check this algebraically by substitution:

$(4) = 2(3) - 2$ and $(4) = (3) + 1$.  

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Appendix I: Advanced Algebra Lesson Plans

Notice that in the above example, the lines intersect at a single point. Consider this system:

\[
\begin{align*}
    y &= -2x + 5 \\
    y &= -2x - 3
\end{align*}
\]

Graphing the system yields:

Since there is no point of intersection for these lines, there is no solution to this system of equations. Note that if the two equations are parallel, then there is no solution to the system of equations.

Now, consider the following:

\[
\begin{align*}
    y &= 3x - 2 \\
    3y + 6 &= 9x
\end{align*}
\]

When you graph the two equations, you get:
Notice that the two lines are identical. When this happens, the two equations are called dependent. Since there are an infinite number of points of intersection for these two lines, there are an infinite number of solutions. Actually, any point on the line \( y = 3x - 2 \) is a solution to this system.

Consider the next case:

\[
\begin{align*}
    y &= 3x - 5 \\
    y &= -4x + 3
\end{align*}
\]

Graphing produces the following result:

You might guess that the solution is \((1, -1.5)\), since that is what the graph suggests. However, if you substitute these values back into the system of equations, you will discover that the points do not solve either of the equations! This case demonstrates one pitfall of solving a system of equations by graphing—the results are often approximate and not exact. Despite of this, the graphing method allows a person to get an idea of what a valid solution may look like. We will learn how to solve these problems algebraically very soon. The algebraic method gives exact results.

As time permits, I tell the students to solve the following systems by graphing:

\[
\begin{align*}
    \begin{cases}
        y &= x + 3 \\
        y &= -2x + 6
    \end{cases} & \quad (1, 4) \\
    \begin{cases}
        y &= |x + 6| - 4 \\
        y &= -|x| + 4
    \end{cases} & \quad (-7, -3) \text{ and } (1, 3) \\
    \begin{cases}
        y &= -2x + 5 \\
        y &= -\sqrt{4}x + 3
    \end{cases} & \quad \text{No solution}
\end{align*}
\]
Appendix I: Advanced Algebra Lesson Plans

\[
\begin{align*}
\{ y &= -2x + 4 \\
2y &= -4x + 8 & \text{Infinite solutions}
\end{align*}
\]

\[
\begin{align*}
\{ y &= 3x - 2 \\
y &= 2x + 3 & (1, 1)
\end{align*}
\]

\[
\begin{align*}
\{ y &= |x - 2| \\
y &= -2|x| - 3 & \text{No solution}
\end{align*}
\]

Homework:

Find the solutions to the following problems graphically, if they exist.

\[
\begin{align*}
\{ y &= -x + 3 \\
y &= x - 4
\end{align*}
\]

\[
\begin{align*}
\{ y &= |x + 2| \\
y &= -|x + 2| + 2
\end{align*}
\]

\[
\begin{align*}
\{ y &= |x + 1| \\
y &= -|x + 3|
\end{align*}
\]

\[
\begin{align*}
\{ y &= 3x - 4 \\
2^2 &= \frac{15}{5} x - y
\end{align*}
\]
November 2

Massachusetts Curriculum Frameworks Addressed in this Lesson:

A-REI-6  Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Do Now:

Solve the following system of equations graphically:

3. \[
\begin{align*}
\begin{cases}
  y &= 3x - 9 \\
  y &= x + 1
\end{cases}
\end{align*}
\]  \(5,6\)

4. \[
\begin{align*}
\begin{cases}
  y &= |x - 1| \\
  y &= |x + 1|
\end{cases}
\end{align*}
\]  \(0,1\)

Substitution

Another method to solve a system of equations is by substitution. In substitution, you solve for one variable, and “plug it into” the other equation, giving an equation with just one variable. Then, that variable can be solved for. Once you have the value of one variable, you can substitute that value into either equation and get the value of the other variable.

Examples:

1. \[
\begin{align*}
\begin{cases}
  y &= 2x - 2 \\
  y &= -x + 7
\end{cases}
\end{align*}
\]
   \(2x - 2 = -x + 7 \rightarrow x = 3 \rightarrow y = 2(3) - 2 = 4\)

   In this problem, \(y\) was already solved for, so it is easily substituted into either equation. After you did this, you got one equation with just one variable, \(x\). So, you can solve for \(x\). Then, you can substitute \(x=5\) into either original equation, and get \(y=8\).

   You can check the solution to by substituting the values back into both equations.

2. \[
\begin{align*}
\begin{cases}
  3x &= 2y - 1 \\
  3 &= 2x - y
\end{cases}
\end{align*}
\]  \(7, 11\)

3. \[
\begin{align*}
\begin{cases}
  y &= 3x + 2 \\
  2y &= 6x + 4
\end{cases}
\end{align*}
\]

   In this problem, you end up with the trivial solution \(0=0\). This means that the system of equations has infinite solutions. Notice that the two lines are the same, so this is what we expect.
Appendix I: Advanced Algebra Lesson Plans

4. \[
\begin{align*}
y &= 3x - 2 \\
y &= 3x + 5
\end{align*}
\]

In this problem, you end up with a false solution \(-2 = 5\). This means that the system of equations has no solution. Notice that the lines are parallel, so this is what we expect.

**Elimination**

Another way to solve a system of equations is by the elimination method. In this method, you either add or subtract the two equations from each other and eliminate one variable. The result is one equation with one unknown, which can be solved. Once that variable is solved, you can substitute that value back into either original equation and solve for the other variable.

**Examples:**

1. \[
\begin{align*}
-2y &= x + 3 \\
2y &= x + 1
\end{align*}
\]
   \((-2, -0.5)\)

2. \[
\begin{align*}
3x + 2 &= 5y \\
3x - 2 &= 2y
\end{align*}
\]
   \((14/9, 4/3)\)

3. \[
\begin{align*}
y &= 2x + 3 \\
2y &= -x + 4
\end{align*}
\]
   \((-2/5, 11/5)\)

4. \[
\begin{align*}
x + y &= 3 \\
x - y &= 4
\end{align*}
\]
   \((7/2, -1/2)\)

**Class work:**

Solve the following algebraically. Use a method that makes solving them easier:

1. \[
\begin{align*}
y &= 3x + 2 \\
y &= 2x - 4 \\
-2y &= 3x + 2 \\
2y &= 2x + 4
\end{align*}
\]
   \((-6, -16)\) elimination or substitution

2. \[
\begin{align*}
y &= 3x + 2 \\
y - 3x &= 4 \\
y &= 3x + 2
\end{align*}
\]
   \((-6/5, 4/5)\) elimination

3. \[
\begin{align*}
3x - 3 &= y \\
x &= y - 3 \\
2y &= 4x - 3 \\
4y - 8x &= -6
\end{align*}
\]
   \((3,6)\) substitution

4. \[
\begin{align*}
40x - 20y &= 30 \\
20x - 30y &= 30 \\
-3x &= 5y \\
2x &= 3y - 2
\end{align*}
\]
   \((3/8, -3/4)\) elimination

5. \[
\begin{align*}
-3x &= 5y \\
2x &= 3y - 2 \\
x + y &= 2 \\
x - y &= 3
\end{align*}
\]
   \((-10/19, 6/19)\) elimination or substitution
Appendix I: Advanced Algebra Lesson Plans

9. \[
\begin{align*}
2x + y &= 3 \\
-2x - y &= 4
\end{align*}
\]
No solution

10. \[
\begin{align*}
x &= 3x - 2y + 3 \\
x &= -4x - 4y - 8
\end{align*}
\]
(-14/9, -1/18) elimination or substitution

**Homework:**

The remaining problems that have not been solved on the board will be for homework.
Appendix I: Advanced Algebra Lesson Plans

November 3-4

Massachusetts Curriculum Frameworks Addressed in this Lesson:

A-REI-6  Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

F-BF-1  Write a function that describes a relationship between two quantities.

Do Now:

11/3

Solve the following systems algebraically:

1. \[
\begin{align*}
    x - y &= 5 \\
    x + 3y &= 4
\end{align*}
\]

\[
(19/4, -1/4)
\]

2. \[
\begin{align*}
    y &= 3x - 2 \\
    y &= 2x + 4
\end{align*}
\]

\[
(6, 16)
\]

3. \[
\begin{align*}
    3x &= y + 2 \\
    -3x &= y - 2
\end{align*}
\]

\[
(2/3, 0)
\]

11/4

Can you construct a three dimensional object with exactly one surface using only a piece of paper?

Yes, you can. The result is a Mobius strip. (This do-now was just for fun!)

Steps for Solving a Word Problem Involving a System of Equations:

1. Determine the variables.
2. Assign letters to each variable.
3. Write equations for each statement.
4. Solve for each variable.
5. Answer the particular question.
6. Check your answer.

I have the students solve the following word problems using the above steps. I do a few of them with them before letting them try the rest on their own:

1. The admission fee at a small fair is $1.50 for children and $4.00 for adults. On a certain day, 2200 people enter the fair and $5050 is collected. How many children and how many adults attended?

   Answer: There were 1500 children and 700 adults.

   (Source: http://www.purplemath.com/modules/systprob.htm)
2. A landscaping company placed two orders with a nursery. The first order was for 13 bushes and 4 trees, and totaled $487. The second order was for 6 bushes and 2 trees, and totaled $232. The bills do not list the per-item price. What were the costs of one bush and of one tree?

Answer: Bushes cost $23 each; trees cost $47 each.

(Source: http://www.purplemath.com/modules/systprob2.htm)

3. Two small pitchers and one large pitcher can hold 8 cups of water. One large pitcher minus one small pitcher constitutes 2 cups of water. How many cups of water can each pitcher hold?

Answer: The small pitcher holds 2 cups of water. The large pitcher holds 4 cups of water.

(Source: http://regentsprep.org/Regents/math/ALGEBRA/AE3/PracWord.htm)

4. The Lakers scored a total of 80 points in a basketball game against the Bulls. The Lakers made a total of 37 two-point and three-point baskets. How many two-point shots did the Lakers make? How many three-point shots did the Lakers make?

Answer: 31 two-point shots and 6 three-point shots.

(Source: www.algebra-class.com/system-of-equations-word-problems.html)

5. Antonio loves to go to the movies. He goes both at night and during the day. The cost of a matinee is $6.00. The cost of an evening show is $8.00. If Antonio went to see a total of 12 movies and spent $86.00, how many night movies did he attend?

Answer: Antonio attended 7 night movies.

(Source: http://www.onemathematicalcat.org/Math/Algebra_II_obj/simple_word_probs.htm)

6. HomeMade Toys manufactures solid pine trucks and cars and usually sells four times as many trucks as cars. The net profit from each truck is $6 and from each car, $5. If the company wants a total profit of $29,000, how many trucks and cars should they sell?

Answer: 4000 trucks and 1000 cars

(Source: Advanced Mathematical Concepts, p. 58)

7. Mrs. Griffin wants to plant soybeans and corn on 100 acres of land. Soybeans require 6 hours of labor per acre, and corn requires 8 hours of labor per acre. If Mrs. Griffin has 660 hours available, how many acres of each crop should she plant?

Answer: 70 acres of soybean, 30 acres of corn

(Source: Advanced Mathematical Concepts, p. 61)

8. AMC Homes, Inc. is planning to build three- and four-bedroom homes in a housing development called Chestnut Hills. Public demand indicates a need for three times as many three-bedroom homes as four-bedroom homes. The net profit from each three-bedroom home is $20,000 and from each four-bedroom home, $25,000. If AMC Homes must net a total profit of $3,800,000 from this development, how many of each type of home should they build?

Answer: 120 three-bedroom homes and 40 four-bedroom homes
Appendix I: Advanced Algebra Lesson Plans

(Source: Advanced Mathematical Concepts, p.61)

9. In one week, a music store sold 9 guitars for a total of $3611. Electric guitars sold for $479 each and acoustic guitars sold for $339 each. How many of each type of guitars were sold?
Answer: 4 electric guitars, and 5 acoustic guitars

(Source: Algebra 2, page 165)

10. One evening, 76 people gathered to play doubles and singles table tennis. There were 26 games in progress at one time. A double game requires 4 players and a singles game requires 2 players. How many games of each kind were in progress at one time if all 76 people were playing?
Answer: 12 double games and 14 single games

(Source: Algebra 2, page 166)

11. During one calendar year, a state trooper issued a total of 375 citations for warnings and speeding tickets. Of these, there were 37 more warnings than speeding tickets. How many warnings and how many speeding tickets were issued?
Answer: 206 warnings and 169 speeding tickets

(Source: Algebra 2, page 157)

12. Flying with the wind, a plane flew 1000 miles in 5 hours. Flying against the wind, the plane could fly only 500 miles in the same amount of time. Find the speed of the plane in calm air and the speed of the wind.
Answer: The wind speed is 50mph and the plane speed is 150mph

(Source: Algebra 2, page 166)
Massachusetts Curriculum Frameworks Addressed in this Lesson:

A-REI-12  Graph solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality).

A-CED-3  Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

Do Now:

11/7

Graph $y > -2x - 5$ and $y \leq x + 3$ on the same plane.

11/8

SAT question:

Suppose $a \Box b = a + 4b$ and $a \Delta b = a + 3b$

If $4 \Delta (5y) = (5y) \Box 4$, what is the value of $y$?

Lesson:

The do-now is an example of graphing a system of inequalities. To do this, all you have to do is graph two inequalities using the same rules as before, but this time they will be on the same graph. Remember that the shaded region is where the solution to the inequalities is. Also, use a dotted line for a strict inequality, and solid lines otherwise. In the do now, the solution to the system is where the shaded regions intersect. This is called a feasible region.

Students should already be familiar with graphing inequalities, so I do not hesitate to have them work out a few examples for themselves:
When there is about 15 minutes left, I introduce them to a few word problems. When solving word problems, it is a good idea to follow the following steps:

**Step 1: Identify variables**

**Step 2: Assign algebraic letters to each variable.**

**Step 3: Write a system of inequalities.**

When there is about 15 minutes left, I introduce them to a few word problems. When solving word problems, it is a good idea to follow the following steps:

Step 1: Identify variables

Step 2: Assign algebraic letters to each variable.

Step 3: Write a system of inequalities.
Appendix I: Advanced Algebra Lesson Plans

Step 4: Graph the system of inequalities. Remember where to shade and what type of lines to use.

Step 5: Answer the question.

Step 6: Check your work. Does the answer to the question make sense?

34. **SUMMER JOBS** You can work at most 20 hours next week. You need to earn at least $92 to cover your weekly expenses. Your dog-walking job pays $7.50 per hour and your job as a car wash attendant pays $6 per hour. Write a system of linear inequalities to model the situation.

---

**MULTIPLE REPRESENTATIONS** The Junior-Senior Prom Committee must consist of 5 to 8 representatives from the junior and senior classes. The committee must include at least 2 juniors and at least 2 seniors. Let $x$ be the number of juniors and $y$ be the number of seniors.

a. **Writing a System** Write a system of inequalities to describe the situation.

b. **Graphing a System** Graph the system you wrote in part (a).

c. **Finding Solutions** Give two possible solutions for the numbers of juniors and seniors on the prom committee.

A college wants to admit anywhere from 1000 to 1200 students next year. At least 400 students must have a high school GPA of at least 3.7 and no more than 600 students may have a high school GPA of 3.7 or lower. Write a system of inequalities that describes the situation and graph it.

**Homework:**

p.171, #17, 18, 20, 21, 23, 24
Appendix I: Advanced Algebra Lesson Plans

November 9

Massachusetts Curriculum Frameworks Addressed in this Lesson:

A-REI-6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Do now:

Use elimination to solve \[
\begin{cases}
3x - 4y = 5 \\
-3x + 2y = 1
\end{cases}
\] (-7/3, -3)

Use substitution to solve \[
\begin{cases}
y = 2x + 3 \\
2x - 5y = -2
\end{cases}
\] (-13/8, -1/4)

Class work:

Use elimination to solve the following systems of equations:

9. \[
\begin{align*}
3x + y + z &= 14 \\
-x + 2y - 3z &= -9 \\
5x - y + 5z &= 30
\end{align*}
\]

10. \[
\begin{align*}
2x - y + 2z &= -7 \\
-x + 2y - 4z &= 5 \\
x + 4y - 6z &= -1
\end{align*}
\]

12. \[
\begin{align*}
4x - y + 2z &= -18 \\
-x + 2y + z &= 11 \\
3x + 3y - 4z &= 44
\end{align*}
\]

13. \[
\begin{align*}
5x + y - z &= 6 \\
x + y + z &= 2 \\
x + 3y &= 4
\end{align*}
\]

Use substitution to solve the following systems of equations:

15. \[
\begin{align*}
x + y - z &= 4 \\
3x + 2y + 4z &= 17 \\
-x + 5y + z &= 8
\end{align*}
\]

16. \[
\begin{align*}
2x - y - z &= 15 \\
4x + 5y + 2z &= 10 \\
-x - 4y + 3z &= -20
\end{align*}
\]

18. \[
\begin{align*}
x + 3y - z &= 12 \\
2x + 4y - 2z &= 6 \\
-x - 2y + z &= -6
\end{align*}
\]

19. \[
\begin{align*}
2x - y + z &= -2 \\
6x + 3y - 4z &= 8 \\
-3x + 2y + 3z &= -6
\end{align*}
\]

9. (1, 5, 6) 13. Infinite solutions 18. No solution

10. (-3, -1, -1) 15. (3, 2, 1) 19. (0, 0, -2)

12. (0, 8, -5) 16. (5, 0, -5)

Homework: p.182, 11, 14, 17, 20
Appendix I: Advanced Algebra Lesson Plans

November 10

Massachusetts Curriculum Frameworks Addressed in this Lesson:

A-REI-6    Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

F-BF-1    Write a function that describes a relationship between two quantities. (Actually, we are dealing with three variables now, but this standard is close enough.)

Do Now:

Solve \( \begin{cases} x + y + z = 6 \\ x - 2y + 2z = 3 \\ x - y - z = -4 \end{cases} \)

Solve the following word problems. Use the same steps that you used to solve word problems with just two variables:

7. Determine the variables.
8. Assign letters to each variable.
9. Write equations for each statement.
10. Solve for each variable.
11. Answer the particular question.
12. Check your answer.

1. Find the equation of the parabola that passes through the points (-1, 9), (1, 5), and (2, 12).
2. Janet is spending her clothes allowance on school clothes. If she buys 3 blouses, 2 skirts, and 4 pairs of jeans, she will spend $292. If she buys 4 blouses, 1 skirt, and 3 pairs of jeans, she will spend $252. If jeans cost $4 more than skirts, what is the price of each item?
3. At Morgan’s Fine Cuisine, meals are served a la carte. That is, each item on the menu is priced separately. Jackie and Ted Parris went to Morgan’s to celebrate their anniversary. Jackie ordered prime rib, 2 side dishes, and a roll. Ted ordered prime rib, 3 side dishes, and 2 rolls. Jackie’s meal came to $36 and Ted’s meal came to $44. If the prime rib is three times as expensive as a side dish, what is the cost of each item?
4. In a superhero video game, you earn points by defeating enemies, by completing missions unscathed, and by performing stealth attacks. Ben defeated 20 enemies, completed 2 missions unscathed, preformed 4 stealth attacks, and earned a total of 220 points. Amanda earned 210 points by defeated 30 enemies, completed 1 mission unscathed, and performing 5 stealth attacks. Alex defeated 50 enemies and performed 2 stealth attacks, but he always got hurt. Alex earned 140 points. How many points are defeating enemies, completing missions unharmed and performing stealth attacks worth?
5. In a casino game, you can win big if you land on a green, blue or red segment on a spinning wheel. A person must pay $50 to play the game. Hannah won $12,000 by landing on green twice, on blue three times, and on red once. Susan won $8,000 by landing on blue, green, and
red once. Chris won $20,000 by landing on green once, blue twice, and red three times. How much money do you win by landing on each of the three colors?

**MARKETING** The marketing department of a company has a budget of $30,000 for advertising. A television ad costs $1000, a radio ad costs $200, and a newspaper ad costs $500. The department wants to run 60 ads per month and have as many radio ads as television and newspaper ads combined. How many of each type of ad should the department run each month?

Homework: p.184, 42, 43, 46
Appendix I: Advanced Algebra Lesson Plans

Extra Credit

(I decided not to give this assignment, because the students that really don’t need extra credit will do it, and the students that need extra credit won’t do it. I regret my decision.

Example:

- Find the partial fraction decomposition of the following:

  \[
  \frac{5x + 7}{x^3 + 2x^2 - x - 2}
  \]

  The denominator factors as \((x + 2)(x + 1)(x - 1)\), so these will be the denominators in the decomposition. That is, I’m looking for \(A\), \(B\), and \(C\) in the following:

  \[
  \frac{A}{x + 2} + \frac{B}{x + 1} + \frac{C}{x - 1}
  \]

  Multiplying both sides by the common denominator, I get:

  \[
  5x + 7 = A(x + 1)(x - 1) + B(x + 2)(x - 1) + C(x + 2)(x + 1)
  \]

  \[
  = A(x^2 - 1) + B(x^2 + x - 2) + C(x^2 + 3x + 2)
  \]

  \[
  = (A + B + C)x^2 + (B + 3C)x + (-A - 2B + 2C)1
  \]

  Comparing coefficients (by thinking of "\(5x + 7\)" as "\(0x^2 + 5x + 7\"), I get:

  \[
  A + B + C = 0
  \]

  \[
  B + 3C = 5
  \]

  \[
  -A - 2B + 2C = 7
  \]

  Solving this system, I get \(A = -1\), \(B = -1\), and \(C = 2\). Then the partial fraction decomposition is:

  \[
  \frac{-1}{x + 2} + \frac{-1}{x + 1} + \frac{2}{x - 1}
  \]

Problem:

Find the partial fraction decomposition of \(\frac{4x^2 + 7x + 6}{x^3 + x^2 - 4x - 4}\)
Appendix I: Advanced Algebra Lesson Plans

Solutions:

1. \[ a(-1)^2 + b(-1) + c = 9 \]
\[ a(1)^2 + b(1) + c = 5 \]
\[ a(2)^2 + b(2) + c = 12 \]

Simplifying the three equations, I get:

\[ 1a - b + c = 9 \]
\[ 1a + b + c = 5 \]
\[ 4a + 2b + c = 12 \]

I won't display the solving of this problem, but the result is that \( a = 3, b = -2, \) and \( c = 4, \) so the equation is:

\[ y = 3x^2 - 2x + 4 \]

2. $28 for blouses, $32 for skirts, $36 for jeans
3. Prime ribs cost $21, side dishes cost $7, and rolls cost $1 each.
4. 2 points for defeating each enemy, 50 points for completing missions unscathed, and 20 points for stealth attacks.
5. $1000 for green, $2000 for blue, $5000 for red.
6. 18 TV ads, 30 radio ads, 12 newspaper ads each month.

Extra Credit:

\[ \frac{2}{x + 2} + \frac{3}{x - 2} - \frac{1}{x + 1} \]
Appendix I: Advanced Algebra Lesson Plans

November 14 (Review Day)

Do Now:

Complete the super strip in 5 minutes (or try to). The super strip is 100 fill-ins where students have to choose two numbers that add up to \(x\) and multiply to \(y\). For example, one fill-in would ask what pair of numbers adds up to 7 and multiply to 10, and students would write 5 and 2. This will prepare students for factoring.

Review:

1. Graph \(4x + y = 8\) and \(2x - 3y = 18\) and find the solution to the system of equations if one exists. (Section 3.1)

2. Graph \(2x + y = 4\) and \(2x + y = 1\) and find the solution to the system of equations if one exists. (Section 3.1)

Solve algebraically. (Section 3.2)

Graph the solutions to these systems of inequalities. (Section 3.3)

Homework:

Study for Wednesday’s test
Appendix I: Advanced Algebra Lesson Plans

Solutions:

1. ![Graph with equations 2x - 3y = 18 and 4x + y = 8]

2. ![Graph with equations 2x + y = 4 and 2x + y = 1]

Remainder of solutions are found in the teacher's textbook.
Appendix I: Advanced Algebra Lesson Plans

Advanced Algebra
November 17th, 2011

Standard Form of a Parabola

Objectives:

- Know that the standard form of a parabola is \( y = ax^2 + bx + c \)
- Know that if \( a \) is negative, the parabola is a “frown” and if \( a \) is positive, the parabola is a “smile”
- Know that if \(|a| > 1\), then the parabola is narrower than \( y = x^2 \) and if \(|a| < 1\), then the parabola is wider than \( y = x^2 \)
- Know that the vertex of a parabola occurs when \( x = -\frac{b}{2a} \), and that the vertical line of symmetry is on \( x = -\frac{b}{2a} \)
- Know that the y-intercept of the parabola is \((0, c)\)

Massachusetts Curriculum Frameworks Addressed in this Lesson:

F-IF-7a  Graph linear and quadratic functions and show intercepts, maxima, and minima.

Do Now:

Graph \( y = 2x^2 \) and \( y = -2x^2 \). What is the difference between the two parabolas?

Lesson:

I will introduce standard form of the parabola, then ask the students what the values of \( a \), \( b \), and \( c \) are for the parabolas from the do-now. Students may be able to recognize that when \( a \) is negative, the parabola is a frown, and when \( a \) is positive the parabola is a smile.

I will hand out graphing calculators to the students so that they can generate graphs quicker. I have them compare the following graphs:

\[
\begin{align*}
  y &= x^2 \\
  y &= 3x^2 \\
  y &= -3x^2 \\
  y &= \frac{1}{3}x^2 \\
  y &= -\frac{1}{3}x^2 
\end{align*}
\]
Appendix I: Advanced Algebra Lesson Plans

They should recognize that the graphs with negative coefficient are frowns, and the ones with positive coefficients are smiles. I will ask them if they can determine when a graph is narrower or wider than $y = x^2$. If they cannot, I encourage them to produce more graphs.

Then, I ask them to graph:

$$y = x^2 + 1$$
$$y = x^2 + 2$$
$$y = x^2 + 3$$

I will ask them what the $y$-intercept for them are, which are $(0, c)$. They know that the $y$-intercept occurs when $x=0$, so the only term that remains is the $c$ value.

Finally, I will tell them that the vertex (maximum or minimum) point on a parabola is when $x = -\frac{b}{2a}$. I also tell them that the vertical line of symmetry occurs on the same $x$ value.

Class work & Homework:

Determine the following:

a. The vertex of the parabola  
b. The vertical line of symmetry  
c. The $y$-intercept of the parabola  
d. If the parabola is wider or narrower than $y = x^2$  
e. If the parabola is a frown or a smile

1. $y = 3x^2 - 2x + 5$  
2. $y = x^2 + x + 1$  
3. $y = -9x^2 + 2x - 10$  
4. $y = x^2 + 3x - 4$  
5. $y = -2x^2 - x + 2$  
6. $y = 5x^2 - x$  
7. $y = (x - 2)(-x + 4)$  
8. $y = \pi x^2 + \sqrt{2}x + 3$
Appendix I: Advanced Algebra Lesson Plans

Advanced Algebra

November 18th, 2011

Graphing Parabolas

Objectives:

- Students will learn how to graph parabolas by finding the vertex, the vertical line of symmetry, the y-intercept, and by determining if the shape is a frown or a smile.

Massachusetts Curriculum Frameworks Addressed in this Lesson:

F-IF-7a               Graph linear and **quadratic** functions and show intercepts, maxima, and minima.

Do Now:

For $y = 2x^2 - 2x + 4$, determine the vertex, the vertical line of symmetry, the y-intercept, and whether the parabola is a frown or a smile.

Answer: vertex at (1/2, 7/2), line of symmetry is $x = \frac{1}{2}$, y-intercept is (0, 4), smile

Lesson:

How to graph a parabola:

1. Graph the vertex.
2. Graph the vertical line of symmetry.
3. Graph the y-intercept.
4. Reflect the y-intercept over the line of symmetry.
5. Pick one easy value for $x$, such as 0 or 1, and find the corresponding $y$ value. Then plot that point.
6. Reflect that point along the line of symmetry.
7. Draw a curve between the points.

So to graph $y = 2x^2 - 2x + 4$, we have:
Appendix I: Advanced Algebra Lesson Plans

I ask students to write solutions down to homework problems 4 thru 8. I ask students if there are any questions on that assignment, then I ask them to graph the parabolas from their homework.

1. Vertex at (-1.5, -6.25). y-intercept at (0, -4)
2. Vertex at (-1/4, 2.125). y-intercept at (0, 2)
3. Vertex at (1/10, -0.05). y-intercept at (0, 0)
4. \( y=-x^2+6x-8 \)  Vertex at (3, 1), y-intercept at (0, -8)
5. Vertex at (-0.2251, 4.5734), y-intercept at (0, 3)

I will have students show their graphs to the class, and I may compare their graphs with the graphs produced by a graphing calculator.

Some students may finish graphing those problems before the end, so I will tell them to graph more quadratics:

\[
\begin{align*}
y &= 2x^2 - x + 5 \\
y &= -3x^2 + 2x - 3 \\
y &= x^2 - 10x - 2 \\
y &= (x - 3)(x + 5) \\
y &= (2x + 2)(x - 1)
\end{align*}
\]

**Homework:**

p. 240 22-32 even
Appendix I: Advanced Algebra Lesson Plans

Advanced Algebra

November 21st, 2011

Vertex Form and Intercept Form

Objectives:

• Students will be able to graph a parabola using the vertex form of a quadratic equation.
• Students will be able to graph a parabola using the intercept form of a quadratic equation.

Massachusetts Curriculum Frameworks Addressed in this Lesson:

F-IF-7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

Do Now:

Complete the super strip.

Lesson:

A quadratic equation in the form \( y = a(x - h)^2 + k \) is in vertex form. Vertex form has the following properties:

• The vertex is \((h, k)\)
• The axis of symmetry is \(x = h\)
• The graph is a smile if \(a > 0\), and a frown if \(a < 0\)

Example:

Graph \( y = 2(x - 3)^2 + 1 \)

Step 1: Identify the constants \(a, h,\) and \(k\): \(a = 2, h = 3, k = 1\)

Step 2: Identify and plot the vertex: The vertex is \((3, 1)\)

Step 3: Identity and draw the axis of symmetry: The axis of symmetry is \(x = 3\)

Step 4: Evaluate the function for two values of \(x\): \((1, 9)\) and \((2, 3)\)

Step 5: Reflect the two points in the axis of symmetry: \((5, 9)\) and \((4, 3)\)
Class work:

Graph the following functions:

1. \( y = (x - 2) + 2 \)
2. \( y = -(x + 2) - 1 \)
3. \( y = 2(x - 3) + 3 \)
4. \( y = -2(x + 2) \)
5. \( y = -(x - 1) + 3 \)

Lesson (after lunch):

\( y = a(x - p)(x - q) \) is the intercept form of a quadratic equation. Intercept form has the following properties:

- The x-intercepts are \( p \) and \( q \)
- The axis of symmetry is \( x = \frac{p + q}{2} \)
- The vertex has the x coordinate \( x = \frac{p + q}{2} \)
- The graph is a smile if \( a > 0 \), and a frown if \( a < 0 \).

Example:

Graph \( y = 2(x - 3)(x - 1) \)

\( A = 2, \ p = 3, \ q = 1 \). The graph has x-intercepts when \( x = 3 \) and when \( x = 1 \), so two points on the graph are \( (0, 3) \) and \( (0, 1) \). The axis of symmetry is \( x = (3+1)/2 = 4/2 = 2 \). The vertex is also has the x-coordinate \( x = 2 \), and its y-coordinate is \( 2(2-3)(2-1) = -2 \). So the vertex is \( (2, -2) \). Since \( a > 0 \), it is a smile.
Class work:

Graph the following functions:

1. \( y = (x - 1)(x - 4) \)
2. \( y = 2(x + 1)(x + 2) \)
3. \( y = -2(x - 3)^2 \)
4. \( y = -(x - 2)(x + 2) \)
5. \( y = -2(x - 1)(x + 2) \)

Homework:

Finish the ten problems above.
Appendix I: Advanced Algebra Lesson Plans

November 22, Vertex Bingo

<p>| | | | | |</p>
<table>
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<td>(0, 0)</td>
<td>(5, -1)</td>
<td>(2, -1)</td>
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<td>(1, 0.5)</td>
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<td>(2, -9)</td>
<td>(0, 1)</td>
<td>(-1.25, 7.125)</td>
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<td>12.</td>
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<td>14.</td>
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<td>16.</td>
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<td>y = x^2 - 3x + 4</td>
<td>y = 3x^2 - 2x</td>
<td>y = (x - 3)(x + 2)</td>
<td>y = (x + 1)(x - 2)</td>
<td>y = -2(x - 1)^2 - 1</td>
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<td>17.</td>
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<td>19.</td>
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<td>y = (x - 3)^2 + 4</td>
<td>y = 2x^2 + 4x - 5</td>
<td>y = 3x^2 - 10x - 3</td>
<td>y = (x - 2)^2</td>
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<td>y = -3x^2 + 3x</td>
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<td>y = (x - 4)(x + 2)</td>
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<td>y = 2x^2 - x + 12</td>
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<td>y = -(x - 2)^2 + 5</td>
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<td>y = (x - 2)(x - 4)</td>
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<td>y = (x - 2)(x + 5)</td>
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Appendix I: Advanced Algebra Lesson Plans

Advanced Algebra

November 28th, 2011

Factoring $x^2+bx+c$

Massachusetts Curriculum Frameworks Addressed in this Lesson

A-SSE-2 Use the structure of an expression to identify ways to write it.

A-REI-4b Solve quadratic equations by... factoring.

Do Now:

Complete the super strip.

Lesson:

Factoring is a process of grouping terms in a polynomial into a compact form so that solving for variables is easier.

When you want to factor a trinomial (a trinomial is a polynomial with three terms that are added together) in the form $x^2+bx+c$:

1. Identify $b$ and $c$
2. First find all the possible pairs of numbers that multiply together to get $c$.
3. Then, add the two numbers from each pair, and choose the numbers that add up to $b$.
4. Call these two numbers $a$ and $b$. Then the factorization of $x^2+bx+c$ is $(x+a)(x+b)$.

Examples:

1. Factor $x^2+5x+6$.
   - $B = 5$ and $c = 6$.
   - $1x6, -1x-6, 2x3$ and $-2x-3$ are all the possible combinations to get $6=c$.
   - $1+6=7, -1+(-6)=-7, 2+3=5, -2+(-3)=-5$. Of these combinations, only $2+3=b=5$.
   - Choose $a=2$ and $b=3$ or $a=3$ and $b=2$. It doesn't matter what you choose.
   - The factored form is $(x+2)(x+3)$

2. Factor $x^2-4x-5$
   - $B=4$ and $c=-5$
   - $1x-5$ and $-1x5$ are the only possible combinations to get $-5=c$.
   - $1+(-5)=-4$ and $-1+5=4$. Of these combinations, only $-1+5=b=4$.
   - Choose $a=-1$ and $b=5$ or $a=5$ and $b=-1$. It doesn't matter which you choose.
   - The factorization is $(x-1)(x+5)$

3. Factor $x^2-3x+2$
Appendix I: Advanced Algebra Lesson Plans

- B=-3 and c=2
- 1x2 and -1x-2 are the only possible combinations to get 2=c.
- 1+2=3 and -1+(-2)=-3. Of these combinations, only -1+(-2)=b=-3.
- Choose a=-1 and b=-2 or a=-2 and b=-1. It doesn’t matter which you choose.
- The factorization is (x-1)(x-2)

Class work:

Factor the following trinomials. (Some students will be extremely quick doing these, because of the super strip exercises).

1. \(x^2+7x+12\) \(= (x+3)(x+4)\)
2. \(x^2-x-6\) \(= (x+2)(x-3)\)
3. \(x^3+3x-28\) \(= (x+7)(x-4)\)
4. \(x^2+x-12\) \(= (x+4)(x-3)\)
5. \(x^2-7x+12\) \(= (x-4)(x-3)\)
6. \(x^2-9x+18\) \(= (x-3)(x-6)\)
7. \(x^2+7x-18\) \(= (x-2)(x+9)\)
8. \(x^2-3x-70\) \(= (x-10)(x+7)\)
9. \(x^2-2x-48\) \(= (x-8)(x+6)\)
10. \(x^2+8x-48\) \(= (x-4)(x+12)\)
11. \(x^2+16x+55\) \(= (x+5)(x+11)\)
12. \(x^2-13x+36\) \(= (x-4)(x-9)\)
13. \(x^2-3x-88\) \(= (x+8)(x-11)\)
14. \(x^2-17x+60\) \(= (x-5)(x-12)\)
15. \(x^2+3x-54\) \(= (x-6)(x+9)\)

Special Cases:

1. The difference of two squares: \(a^2-b^2=(a-b)(a+b)\)
   a. \(x^2-4=(x-2)(x+2)\)
   b. \(x^2-9=(x-3)(x+3)\)
2. Perfect square trinomials: \(a^2+2ab+b^2=(a+b)^2\) and \(a^2-2ab+b^2=(a-b)^2\)
   a. \(x^2+4x+4=(x+2)^2\)
   b. \(x^2-8x+16=(x-4)^2\)

Factoring can help you solve quadratic equations.

Examples:

1. Solve \(x^2+14x+49=0\).
   \(x^2+14x+49=(x+7)^2=0\). Squaring both sides yields \((x+7)=0\). So, \(x=-7\).

2. Solve \(x^2+2x-24=0\).
Appendix I: Advanced Algebra Lesson Plans

\(x^2+2x-24=(x+6)(x-4)=0\). To make the left-hand side equal to zero, either the first term may be zero or the second term may be zero, since multiplication by zero is zero. \((x+6)=0\) or \((x-4)=0\). So \(x=-6\) and \(x=4\).

3. Solve \(x^2+x-6\)
   \(x^2+x-6=(x-2)(x+3)=0\). So either \((x-2)=0\) or \((x+3)=0\). So \(x=2\) and \(x=-3\).
   When the same solution pops up twice, we say that that solution has \textit{multiplicity of two}.

Class work:

Solve the following quadratic equations:

1. \(x^2+4x-21=0\) \hspace{1cm} x=3 \hspace{1cm} x=-7
2. \(x^2-8x+16=0\) \hspace{1cm} x=4, multiplicity two
3. \(x^2-81=0\) \hspace{1cm} x=\pm 9
4. \(x^2+8x=14\) \hspace{1cm} x=2 \hspace{1cm} x=-7
5. \(x^2=25\) \hspace{1cm} x=\pm 5
6. \(x^2+49x+49\) \hspace{1cm} x=\pm 7
7. \(x^2+22x+120\) \hspace{1cm} x=-10 \hspace{1cm} x=-12
8. \(x^2=12x-32\) \hspace{1cm} x=4 \hspace{1cm} x=8
9. \(x^2=10x-25\) \hspace{1cm} x=5 multiplicity two
10. \(x^2-3x=40\) \hspace{1cm} x=-5 \hspace{1cm} x=8

Homework:

Page 256, #24-38 even
Appendix I: Advanced Algebra Lesson Plans

Advanced Algebra

November 29th 2011

Factoring ax^2+bx+c

Massachusetts Curriculum Frameworks Addressed in this Lesson

A-SSE-2 Use the structure of an expression to identify ways to write it.

A-REI-4b Solve quadratic equations by... factoring.

Do Now:

Solve the following:

1. \( x^2 + 13x = 30 \)  \( x = 2, -15 \)
2. \( x^2 - x - 12 = 0 \)  \( x = -3, 4 \)
3. \( x^2 = -25 - 10x \)  \( x = -5, \text{ multiplicity 2} \)
4. \( x^2 + 14x + 48 = 0 \)  \( x = -8, -6 \)

Lesson

I decided to abandon this lesson plan in favor of the method described on page 195 in the Appendix. The method below was in the book, and I thought that there must be an easier way to factor...

How to factor the trinomial \( ax^2 + bx + c \):

1. List all pairs of positive numbers that multiply to \( a \). Call these numbers \( k \) and \( l \).
2. List all pairs of numbers that multiply to \( c \). Call these numbers \( m \) and \( n \). If \( c \) is positive, \( m \) and \( n \) must have the same sign, but if \( c \) is negative, \( m \) and \( n \) must have opposite signs.
3. Compute \((kx+m)(lx+n)\) for each pair of numbers from step 1 and step 2. Choose the correct factorization.

Note that \( ax^2 + bx + c = (kx + m)(lx + n) = kln^2 + (kn + lm)x + mn \)
Appendix I: Advanced Algebra Lesson Plans

Example:

Factor $5x^2 - 17x + 6$.

**Solution**

You want $5x^2 - 17x + 6 = (kx + m)(lx + n)$ where $k$ and $l$ are factors of 5 and $m$ and $n$ are factors of 6. You can assume that $k$ and $l$ are positive and $k \geq l$. Because $mn > 0$, $m$ and $n$ have the same sign. So, $m$ and $n$ must both be negative because the coefficient of $x$, $-17$, is negative.

<table>
<thead>
<tr>
<th>$k, l$</th>
<th>5, 1</th>
<th>5, 1</th>
<th>5, 1</th>
<th>5, 1</th>
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</thead>
<tbody>
<tr>
<td>$m, n$</td>
<td>-6, -1</td>
<td>-1, -6</td>
<td>-3, -2</td>
<td>-2, -3</td>
</tr>
<tr>
<td>$(kx + m)(lx + n)$</td>
<td>$(5x - 6)(x - 1)$</td>
<td>$(5x - 1)(x - 6)$</td>
<td>$(5x - 3)(x - 2)$</td>
<td>$(5x - 2)(x - 3)$</td>
</tr>
<tr>
<td>$ax^2 + bx + c$</td>
<td>$5x^2 - 11x + 6$</td>
<td>$5x^2 - 31x + 6$</td>
<td>$5x^2 - 13x + 6$</td>
<td>$5x^2 - 17x + 6$</td>
</tr>
</tbody>
</table>

The correct factorization is $5x^2 - 17x + 6 = (5x - 2)(x - 3)$.

Class work:

Solve the following equations:

1. $3x^2 + 5x + 2 = 0$  
   \[-\frac{2}{3}, -1\]
2. $2x^2 + 6x + 4 = 0$  
   \[-1, -2\]
3. $2x^2 + 7x - 4 = 0$  
   \[\frac{7}{2}, -4\]
4. $3x^2 + 12x - 15 = 0$  
   \[1, -5\]
5. $2x^2 - 12x - 14 = 0$  
   \[-1, 7\]
6. $5x^2 - 33x + 40 = 0$  
   \[\frac{8}{5}, 5\]
7. $2x^2 + 11x + 15 = 0$  
   \[-\frac{5}{2}, -3\]
8. $3x^2 - 9x - 12 = 0$  
   \[-1, 4\]
9. $2x^2 + 7x - 49 = 0$  
   \[\frac{7}{2}, -7\]
10. $2x^2 - 7x - 15 = 0$  
    \[-\frac{3}{2}, 5\]
11. $2x^2 - 9x + 9 = 0$  
    \[3/2, 3\]
12. $3x^2 + 7x - 20 = 0$  
    \[5/3, -4\]
13. $7x^2 + 19x - 6 = 0$  
    \[2/7, -3\]
14. $6x^2 - x - 35 = 0$  
    \[5/2, -7/3\]
15. $9x^2 + 9x + 2 = 0$  
    \[-\frac{2}{3}, 1/3\]

Special Cases:

The difference of two squares: $a^2 - b^2 = (a + b)(a - b)$
Appendix I: Advanced Algebra Lesson Plans

- $9x^2 - 64 = (3x)^2 - 8^2 = (3x + 8)(3x - 8)$
- $4x^2 - 9 = (2x)^2 - 3^2 = (2x - 3)(2x + 3)$

Perfect square trinomials: $a^2 + 2ab + b^2 = (a + b)^2$ and $a^2 - 2ab + b^2 = (a - b)^2$

- $4x^2 + 20x + 25 = (2x)^2 + 2(2x)(5) + 5^2 = (2x + 5)^2$
- $36x^2 - 12x + 1 = (6x)^2 - 2(6x)(1) + 1^2 = (6x - 1)^2$

Factoring out monomials first:

- $5x^2 - 80 = 5(x^2 - 16) = 5(x - 4)(x + 4)$
- $-2x^2 - 4x + 30 = -2(x^2 + 2x - 15) = -2(x - 3)(x + 5)$
- $3x^2 - 27x = 3x(x - 9)$

Class Work:

Factor the expression. If the expression cannot be factored, say so.

1. $7x^2 - 20x - 3$
2. $5z^2 + 16z + 3$
3. $2w^2 + w + 3$
4. $3x^2 + 5x - 12$
5. $4u^2 + 12u + 5$
6. $4x^2 - 9x + 2$

Factor the expression.

7. $16x^2 - 1$
8. $9y^2 + 12y + 4$
9. $4r^2 - 28r + 49$
10. $25s^2 - 80s + 64$
11. $49z^2 + 42z + 9$
12. $36n^2 - 9$

Factor the expression.

13. $3s^2 - 24$
14. $8t^2 + 38t - 10$
15. $6x^2 + 24x + 15$
16. $12x^2 - 28x - 24$
17. $-16n^2 + 12n$
18. $6z^2 + 33z + 36$

Homework:

Page 263, #4-30 even. To be collected!
Appendix I: Advanced Algebra Lesson Plans

Advanced Algebra

November 30th 2011 to December 1st 2011

Properties of Square Roots and Rationalizing Denominators

Massachusetts Curriculum Frameworks Addressed in this Lesson

A-SSE-2 Use the structure of an expression to identify ways to write it.

I could not find any standards on simplifying square roots and rationalizing denominators, but they are essential skills to have to solve quadratic equations, which will be introduced in a few days.

Do Now:

Solve:

1. \(3x^2 + 22x - 16 = 0\)  \[-8 \quad 2/3\]
2. \(2x^2 - 11x - 21 = 0\)  \[7 \quad -3/2\]
3. \(4x^2 - 18x + 20 = 0\)  \[5/4 \quad 2\]

I did not have time to discuss special cases when \(a\) does not equal 1, so I will focus on that before lunch.

Special Cases:

The difference of two squares: \(a^2 - b^2 = (a + b)(a - b)\)

- \(9x^2 - 64 = (3x)^2 - 8^2 = (3x + 8)(3x - 8)\)
- \(4x^2 - 9 = (2x)^2 - 3^2 = (2x - 3)(2x + 3)\)

Perfect square trinomials: \(a^2 + 2ab + b^2 = (a + b)^2\) and \(a^2 - 2ab + b^2 = (a - b)^2\)

- \(4x^2 + 20x + 25 = (2x)^2 + 2(2x)(5) + 5^2 = (2x + 5)^2\)
- \(36x^2 - 12x + 1 = (6x)^2 - 2(6x)(1) + 1^2 = (6x - 1)^2\)

Factoring out monomials first:

- \(5x^2 - 80 = 5(x^2 - 16) = 5(x - 4)(x + 4)\)
- \(-2x^2 - 4x + 30 = -2(x^2 + 2x - 15) = -2(x - 3)(x + 5)\)
- \(3x^2 - 27x = 3x(x - 9)\)

Class Work:

Factor the expression. If the expression cannot be factored, say so.

1. \(7x^2 - 20x - 3\)
2. \(5z^2 + 16z + 3\)
3. \(2w^2 + w + 3\)
4. \(3x^2 + 5x - 12\)
5. \(4u^2 + 12u + 5\)
6. \(4x^2 - 9x + 2\)
Appendix I: Advanced Algebra Lesson Plans

Lesson (after lunch):

Properties of square roots:

- If \( x^2 = c \), then \( x = \pm \sqrt{c} \)
- \( \sqrt{x} \) is only defined for real numbers when \( x \geq 0 \)
- \( \sqrt{x} = x^{1/2} \)
- \( \sqrt{ab} = \sqrt{a} \sqrt{b} \)
- \( \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \)

Examples:

- \( \sqrt{\frac{1}{2} \cdot 2 \cdot 2} = \sqrt{2^2 \cdot 2} = 2\sqrt{2} \)
- \( \sqrt{6\sqrt{21}} = \sqrt{126} = \sqrt{9\sqrt{14}} = 3\sqrt{14} \)
- \( 16^{\frac{1}{2}} + 89^{\frac{1}{2}} = \sqrt{16} + \sqrt{89} = 4 + 9 = 13 \)
- \( \sqrt{\frac{144}{4}} = \sqrt{144} \div \sqrt{4} = 12 \div 2 = 6 \)

I tried to prevent disaster by presenting students this. It only proved to be semi-effective.

Note that \( \sqrt{a} + \sqrt{b} \neq \sqrt{a + b} \) and that \( \sqrt{a} - \sqrt{b} \neq \sqrt{a - b} \)

**Proof:** Let \( a = 9 \) and \( b = 4 \). Then \( \sqrt{9} + \sqrt{4} = \sqrt{13} \), but \( \sqrt{9} + \sqrt{4} = 3 + 2 = 5 \)
Appendix I: Advanced Algebra Lesson Plans

Class work:

**Simplify the expression.**

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<thead>
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<tbody>
<tr>
<td>1.</td>
<td>$\sqrt{27}$</td>
<td>2.</td>
<td>$\sqrt{98}$</td>
</tr>
<tr>
<td>3.</td>
<td>$\sqrt{10} \cdot \sqrt{15}$</td>
<td>4.</td>
<td>$\sqrt{8} \cdot \sqrt{28}$</td>
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<tr>
<td>5.</td>
<td>$\sqrt{\frac{9}{64}}$</td>
<td>6.</td>
<td>$\sqrt{\frac{15}{4}}$</td>
</tr>
<tr>
<td>7.</td>
<td>$\sqrt{\frac{11}{25}}$</td>
<td>8.</td>
<td>$\sqrt{\frac{36}{49}}$</td>
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**Rationalizing Denominators:**

It is conventional to avoid square roots on the denominator. To rationalize an expression in the form $\frac{a}{\sqrt{b}}$, you can multiply by $\frac{\sqrt{b}}{\sqrt{b}}$ without changing the value of the fraction because $\frac{\sqrt{b}}{\sqrt{b}} = 1$.

Then:

$$\frac{a}{\sqrt{b}} = \frac{a \cdot \sqrt{b}}{\sqrt{b} \cdot \sqrt{b}} = \frac{a\sqrt{b}}{b}$$

To rationalize an expression in the form $\frac{a}{b + \sqrt{c}}$, you can multiply by $\frac{b - \sqrt{c}}{b - \sqrt{c}}$ without changing the value of the fraction since $\frac{b - \sqrt{c}}{b - \sqrt{c}} = 1$. The expression $b - \sqrt{c}$ is called the conjugate of $b + \sqrt{c}$, and $b - \sqrt{c}$ and $b + \sqrt{c}$ are called conjugate pairs. They are called conjugate pairs because $(b + \sqrt{c})(b - \sqrt{c}) = b^2 - c$. Notice that this eliminates the square root in the denominator:

$$\frac{a}{b + \sqrt{c}} = \frac{a}{b + \sqrt{c}} \cdot \frac{b - \sqrt{c}}{b - \sqrt{c}} = \frac{a(b - \sqrt{c})}{b^2 - c}$$

**Examples:**

- $\sqrt{2} = \frac{\sqrt{2} \cdot \sqrt{2}}{2} = \frac{\sqrt{4}}{2}$
- $\sqrt{10} = \frac{\sqrt{10} \cdot \sqrt{10}}{\sqrt{10} \cdot \sqrt{10}} = \frac{\sqrt{100}}{10} = \frac{10}{10} = 1$
- $\sqrt{2 + \sqrt{2}} = \frac{\sqrt{2} + \sqrt{2}}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$
- $\sqrt{\frac{3}{10 + \sqrt{2}}} = \frac{\sqrt{3} \cdot \sqrt{10 - 2\sqrt{2}}}{10 - 2\sqrt{2}}$
- $\sqrt{\frac{5}{2 + \sqrt{22}}} = \frac{\sqrt{5} \cdot \sqrt{22 - 2}}{22 - 2} = \frac{5\sqrt{22 - 2}}{18}$

Class work:
Appendix I: Advanced Algebra Lesson Plans

Simplify the expression.

9. $\sqrt{\frac{6}{5}}$  
10. $\sqrt{\frac{9}{8}}$  
11. $\sqrt{\frac{17}{12}}$  
12. $\sqrt{\frac{19}{21}}$
13. $\frac{-6}{7 - \sqrt{5}}$  
14. $\frac{2}{4 + \sqrt{11}}$  
15. $\frac{-1}{9 + \sqrt{7}}$  
16. $\frac{4}{8 - \sqrt{3}}$

Homework:

Page 269 4-18 even

Remark:

The class only got up to the second example of simplifying radicals. December 1st was spent on the remaining material.

December 1st Do Now:

Simplifying radicals: $\sqrt{100}, \sqrt{800}, \sqrt{20}, \sqrt{40}$
Appendix I: Advanced Algebra Lesson Plans

Advanced Algebra

December 2\textsuperscript{nd}, 2011

Solving Quadratic Functions with Square Roots

Massachusetts Curriculum Frameworks Addressed in this Lesson:

A-REI-4b  Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.

Do Now:

Simplify the following radicals:

1. \(\sqrt{140}\)  
2. \(\sqrt{45}\)  
3. \(\sqrt{800}\)  
4. \(\frac{\sqrt{150}}{100}\)  
5. \(\frac{\sqrt{12}}{4+\sqrt{30}}\)  
6. \(\frac{\sqrt{8}}{2-\sqrt{80}}\)

The do now was a debacle. I suspect that many students did not do the homework. Students spent a lot of time computing huge numbers instead of simplifying things first. Many students also made algebraic mistakes rationalizing the denominators. It took nearly the whole class to go over the do now, and so today’s lesson was delayed until the next day. So much for a five minute warm up. At least students learned how to simply things more efficiently.

Solving Quadratic Equations with Square Roots:

Solve \(3x^2 + 5 = 41\).

\[
3x^2 + 5 = 41 \\
3x^2 = 36 \\
x^2 = 12 \\
\Rightarrow x = \pm \sqrt{12} \\
x = \pm \sqrt{4 \cdot \sqrt{3}} \\
x = \pm 2\sqrt{3}
\]

\(\Rightarrow\) The solutions are \(2\sqrt{3}\) and \(-2\sqrt{3}\).

How would you check your answer?

Word Problem:
Appendix I: Advanced Algebra Lesson Plans

The position of a falling object can be modeled by the equation \( h = -16t^2 + h_0 \) where \( h \) is the height in feet, \( h_0 \) is the altitude the object fell from, and \( t \) time in seconds. If an object fell from 100 feet, how long did it take to reach the ground?

**SOLVING QUADRATIC EQUATIONS** Solve the equation.

22. \( s^2 = 169 \)  
23. \( a^2 = 50 \)  
24. \( x^2 = 84 \)  
25. \( 6x^2 = 150 \)  
26. \( 4p^2 = 448 \)  
27. \( -3w^2 = -213 \)  
28. \( 7r^2 - 10 = 25 \)  
29. \( \frac{x^2}{25} - 6 = -2 \)  
30. \( \frac{t^2}{20} + 8 = 15 \)  
31. \( 4(x - 1)^2 = 8 \)  
32. \( 7(x - 4)^2 - 18 = 10 \)  
33. \( 2(x + 2)^2 - 5 = 8 \)

**Homework:**

Study for Wednesday’s test 4.1 thru 4.5
Appendix I: Advanced Algebra Lesson Plans

Advanced Algebra

December 5th 2011

Solving Quadratics with Square Roots

Massachusetts Curriculum Frameworks Addressed in this Lesson:

A-REI-4b Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.

Do Now:

I will spend some time showing students how to avoid calculating huge numbers. For the factoring do-
now, students just need to recognize that they are perfect square trinomials. For #1, simplify the fraction first, and for #2, simplify each square root first.

Factor:

1. $400x^2 + 480x + 144$
2. $64x^2 + 144x + 81$

Simplify:

1. $\sqrt{\frac{70}{20}}$
2. $\sqrt{\frac{200}{500 - \sqrt{600}}}$

When you factor, you want to check these three things (in order)

1. Is there a greatest common factor that you can factor out first?
2. Is it a difference of two squares?
3. Is it a perfect square trinomial?

Only then should you try to factor the “long” way.

Lesson: Solving Quadratic Equations with Square Roots:

Solve $3x^2 + 5 = 41$.

1. $3x^2 + 5 = 41$ Write original equation.
2. $3x^2 = 36$ Subtract 5 from each side.
3. $x^2 = 12$ Divide each side by 3.

...... $x = \pm \sqrt{12}$ Take square roots of each side.

$x = \pm \sqrt{4 \cdot 3}$ Product property

$x = \pm 2\sqrt{3}$ Simplify.

The solutions are $2\sqrt{3}$ and $-2\sqrt{3}$.
Appendix I: Advanced Algebra Lesson Plans

How would you check your answer?

**Word Problem:**

The position of a falling object can be modeled by the equation \( h = -16t^2 + h_0 \) where \( h \) is the height in feet, \( h_0 \) is the altitude the object fell from, and \( t \) time in seconds. If an object fell from 100 feet, how long did it take to reach the ground?

**SOLVING QUADRATIC EQUATIONS** Solve the equation.

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**Homework:**

Study for Wednesday’s test 4.1 thru 4.5
Appendix I: Advanced Algebra Lesson Plans

Advanced Algebra

December 6th 2011

Graphical Solutions to Quadratic Equations

Massachusetts Curriculum Frameworks Addressed in this Lesson:

A-SSE-2 Use the structure of an expression to identify ways to write it.
A-REI-4b Solve quadratic equations by... factoring.
F-IF-8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of context.
A-SSE-3a Factor a quadratic expression to reveal the zeros of the function it defines.

Do Now:

Solve the following equations:

1. \(5(x - 3)^2 = 10\)
2. \(12(x + 9)^2 - 144 = 0\)
3. \(20(2x - 7)^2 - 35 = 5\)

Many students might try to expand these expressions instead of eliminating the constants first.

Lesson:

I ask students to graph the following quadratic equations. Then, factor the right hand side:

1. \(y = x^2 - 3x - 10\) \((x-5)(x+2)\)
2. \(y = x^2 + 5x - 36\) \((x-4)(x+9)\)
3. \(y = 4x^2 - 7x - 2\) \((4x+1)(x-2)\)
4. \(y = 2x^2 + 7x - 4\) \((2x-1)(x+4)\)
If you wanted to solve the above equations where $y=0$, where would the solutions be on the graph?

$y=0$ is the $x$-axis, so the solution to the quadratic equations will be where the parabola intersects with the $x$-axis. These special points are called the zeros of a quadratic function. Notice that the zeros are equidistant from the vertical line of symmetry and the vertex.

Note that a quadratic equation can have at most two real solutions. In all four cases above, there are two real solutions. It is also possible to have exactly one real solution and no real solutions at all.
Appendix I: Advanced Algebra Lesson Plans

Example:

\[ y = x^2 - 10x + 25 = (x - 5)(x - 5) = (x - 5)^2 \]

When \( y=0 \), \( x \) can only equal 5. So 5 is the only real solution. Actually, the solution has **multiplicity of two**, because the same solution is duplicated twice.

![Graph of a parabola](image)

Example:

\[ y = x^2 + 4 \]

When \( y=0 \):

\[
\begin{align*}
0 &= x^2 + 4 \\
-4 &= x^2 \\
\pm\sqrt{-4} &= x
\end{align*}
\]

The square root of -4 is not a real number, so the quadratic equation has **no real solution**. If you look at the graph, you will notice that the parabola does not intersect with the x-axis:
The above example cannot be factored. However, just because a quadratic equation cannot be factored, that does not mean that there are no real solutions!

Example:

\[ y = x^2 - 2.8x - \sqrt{7} \]

This cannot be factored, yet the graph of the function clearly crosses the x-axis twice. Therefore \(0 = x^2 - 2.8\) has two real solutions.
Appendix I: Advanced Algebra Lesson Plans

Later on in this course, you will learn the quadratic formula to solve any quadratic equation.

Class work / Homework:

Graph the following functions. If possible, factor the right hand side and find all real solutions when $y=0$.

1. $y = x^2 + 4x - 5$
2. $y = x^2 + 6x + 8$
3. $y = 2x^2 - 5x + 3$
4. $y = 4x^2 + 2x - 2$
5. $y = x^2 + 6x + 9$
6. $y = -x^2 - 2$
Appendix I: Advanced Algebra Lesson Plans

Advanced Algebra

December 8th 2011

Finding Roots of Quadratic Equations

Massachusetts Curriculum Frameworks Addressed in this Lesson:

A-SSE-2 Use the structure of an expression to identify ways to write it.
A-REI-4b Solve quadratic equations by... factoring.
F-IF-8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of context.
A-SSE-3a Factor a quadratic expression to reveal the zeros of the function it defines.

Do Now:

Find the roots of the following quadratic equations:

1. \(y = 6x^2 - 31x + 18\)  
   \((3x-2)(2x-9)\)
2. \(y = 2x^2 - 19x + 24\)  
   \((2x-3)(x-8)\)
3. \(y = 4x^2 - 16x + 16\)  
   \((2x-4)^2\)

Lesson:

I will review and check last night’s homework. Then I will go over the do-now problems. I will then show students how to find the roots of the above quadratic equations using the graphing calculator. I will also show them how to find the y-intercept and the vertex using the graphing calculator.

1. Use the window \(-2\leq x\leq 7\) and \(-25\leq y\leq 100\)
2. Use the window \(-2\leq x\leq 12\) and \(-25\leq y\leq 100\)
3. Use the same window as 1.

Class work:

Using the graphing calculator, approximate the solutions to the following:

1. \(0 = 3x^2 - \sqrt{2}x - 3.5\)
2. \(0 = 5.49x^2 - \pi x - 2.33\)
3. \(9.73 = x(-8.21x - 3.2)\)
4. \(\frac{3x-2x^2}{\pi} = \sqrt{22}\)
Appendix I: Advanced Algebra Lesson Plans

Homework:

Find the roots of the following quadratic equations. Some may not have any roots.

1.  \( y = 8x^2 - 8x - 16 \)
2.  \( y = 5x^2 + 5x - 30 \)
3.  \( y = 9x^2 - 12x + 4 \)
4.  \( y = 4(x - 2)^2 - 4 \)
5.  \( y = 60x^2 - 90x - 270 \)
6.  \( y = 5 + 3(x + 1)^2 \)