Design and Simulation of a Space Vehicle Coulomb Tether Experiment

A Major Qualifying Project Report
Submitted to the Faculty
Of
Worcester Polytechnic Institute
In partial fulfillment of the requirements for the
Degree of Bachelor of Science
In Aerospace Engineering

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Date: Tuesday, March 16th 2010

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Keywords:
1. Space Vehicles
2. Coulomb Tether
3. Test Bed Construction

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Abstract

Coulomb tethering is an attractive option for controlling the relative motion of two or more spacecraft in space. As it is a developing technology, little research has been done into the feasibility of a control scheme involving a Coulomb Tether. The goal of this project was to construct several simulations and build an experimental setup for future research into controlling these forces. Theory was refined and analysis was performed to build the simulation. Results for one-dimensional, two and three-craft formulations were derived. Further, the experimental setup was designed and construction on it was begun.
**Executive Summary**

The overall purpose of this project was to develop a test bed for experiments involving Coulomb forces and methods of using those forces to control the separation distance of two craft. The test bed was modeled after an experiment designed by Carl R. Seubert (Seubert 2009) at the University of Colorado. This was accomplished by employing two approaches. One was to create several simulations of experiments and use control theory to design a controller that successfully maintains the separation distance under a certain set of assumptions, despite any errors or uncertainties that may arise. The second was to design and begin construction on a track which could provide a near forceless environment for later projects to use.

First, in Chapter 2, the theory that Seubert’s paper presented for their experiment was refined into more general linear expressions that could be used in designing a controller. The purpose of putting the expressions for Coulomb force into linear forms is so that the linear techniques of control theory can be applied, since most of the methods applied in the analysis of the simulations used system state vectors and matrices. The result of this linearization is an expression for the force $F_{ij}$ applied to the $j^{th}$ sphere by Coulomb interactions with the $i^{th}$ sphere, given by Equation 2.6:

$$F_{ij} = \frac{\rho^2}{k_c x_d^2} \left( \delta V_i V_{j,d} + V_{i,d} V_{j} + V_{i,d} \delta V_j \right) - 2 \frac{V_{i,d} V_{j,d} \rho^2}{k_c x_d^3} \delta x$$

Where $\rho$ is the sphere’s radius (both spheres where assumed to have equal radius), $k_c$ is the Coulomb constant, $x_d$ is the separation distance, $\delta x$ is the error in position ($\delta x = x - x_d$), $V_i$ and $V_j$ are the voltages on the $i^{th}$ and $j^{th}$ spheres, respectively, and $\delta V_i$ and $\delta V_j$ are
the errors in voltage on the $i^{th}$ and $j^{th}$ spheres, respectively ($\delta V_i = V_i - V_{i,d}$ and $\delta V_j = V_j - V_{j,d}$, where $V_{i,d}, V_{j,d}$ are the nominal voltages required to hold the sphere at $x_d$ with zero velocity). This was the equation used for the bulk of the analysis.

Two simulations were made in MATLAB software and documented in Chapter 3. The first simulation, Case 1, explored the case with two spheres, with only one that is allowed to move. The results of this first simulation showed that an external force was necessary to bring the system to equilibrium at a distance $x_d$ from the stationary sphere. It was determined that this force would be the force due to gravity acting on this cart due to a small tilt angle, orientated so that the mobile sphere would move towards the stationary one if the Coulomb force was not there to counteract gravity. It was shown that this case would move the cart asymptotically towards the specified equilibrium, and that the results of the simulation could be applied to the experiment under development. The second simulation, Case 2, investigated a three-craft experiment where two end spheres were kept stationary and a middle one was allowed to be mobile. The results of this simulation demonstrated that, although the system was controllable with a single input, two inputs were needed to assure the system moved asymptotically to its specified equilibrium. Furthermore, since the experiment only required one moving sphere, the final test rig could be adapted to perform this case with only minimal modifications to the experimental setup.

The main purpose of this project was to begin construction on the test bed. Several parts of this goal were accomplished and documented in Chapter 4. First, the air
bearing track was designed, manufactured, and tested. The test bed consists of a long block of Delrin plastic shaped into a track with 29 equally spaced sections which direct pressurized air in four directions around the track. A cart was then designed to wrap around the track and be a stable platform for one of the two spheres to be placed on it. The picture to the right shows the final track assembly, without the cart on top of it. The mass of the sphere-cart assembly needed to be carefully controlled so that the pressurized air could provide an adequate amount of lift on the cart to remove friction from the assembly. The results of testing the track were encouraging, showing that future projects employing the track have a high chance of being successful. Several other necessary components, such as the high voltage power supply and high pressure air compressor, were bought and tested. Components for an air valve control system, which would direct the air to the tubes directly below the cart, were also purchased. However, the valves did not arrive on time for the completion of this project, so they were left untested.

Finally, several recommendations were noted in Chapter 5. Since this project team could not accomplish all the tasks it had set out to accomplish with this project, future project teams will be necessary to complete the experimental setup and begin testing different methods of controlling the forces. In particular, safety equipment
needs to be developed, the test bed needs to be mounted along with the air control system, and a software-based controller for the voltages needs to be developed so that the power supply can be used for an experiment.
Acknowledgements

We would like to express thanks to Neil Whitehouse and Professor J. Blandino for their technical assistance on this project. Also, we would like to thank our advisor Professor Hussein for his guidance throughout the project.
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1 Introduction

It has been known for some time that satellites accumulate an electrostatic charge during the duration of their space-borne activities. Recently, there has been research into productive uses for these charges (Hussein, 137-157). This research, looking into the generation and application of Coulomb forces between satellites, provides a theoretical basis for controlling the motion of satellites in Earth orbit.

Coulomb Formation Flight, or CFF, is a concept based on the manipulation of the electrostatic charges that naturally accumulate on satellites in deep space. Using these charges, it is possible to tether and control a multi-spacecraft system without the use of physical connections between them. However, this concept is still in its relative infancy and requires more research before it can be practically applied.

Due to restrictions on the size of satellites (either due to payload limits or simple feasibility issues), certain limits are set on what can be done in space. Satellite telescopes can only see so far, and communication satellites can only produce certain signal strength. Yet through the use of a tether based system, groups of several satellites can be linked together in order to increase their strength in these areas many times over. For instance, a formation of multiple interferometric satellites can have an equivalent resolution to a lens aperture that would be several times larger than could be put on any single satellite currently. The CFF concept provides a method for tethering these craft together without adding any other structural mass and with minimal propellant usage, as well as utilizing the latent charges that will inevitably build up in outer space.
Throughout the last decade, there has been increasing research into the subjects pertaining to Coulomb forces and satellites. Early research looked into the use of inter-craft Coulomb forces as a method of providing an ability to fly multiple spacecraft in formation without using conventional propulsion methods (King 2002). Hussein and Schaub expand on this research area of research, looking into the dynamics of a three-craft Coulomb tether flight formation (Hussein, 137-157). Complimentary research by Seubert and Schaub looked into pairing physical spacecraft tethers with an electrostatic charge system to keep the tethers taunt (Seubert 2008).

Other recent research focused on the exploration of the one-dimensional Coulomb forces problem. This research included studies that proved the stability of such a system (Wang 2007), as well as the construction of a one-dimensional Coulomb force test bed (Seubert 2009).

Continued study into Coulomb Formation Flight can yield many important benefits to the use of satellites. The very low use of propellant is a major benefit. An electron-beam device or other method of charging can be used to create an efficient technique for altering or maintaining the position of the satellite by producing an attractive or repulsive force between spacecraft. The ability to create electro-statically tethered spacecraft formations gives the benefits of a physically tethered system without the need to create the physical tether structures, which add mass, additional physical constraints, and undesirable forces.

There are two main goals of this project. The first is to design and build a test rig that can be used in the future to explore the application and control of electrostatic
forces. The second is to design several control simulations in MATLAB that can demonstrate the controllability and stability of the nonlinear systems. This will allow for future testing of individual control schemes in simplified computer simulations.

The experimental test rig first looks to verify the use of electrostatic charges to create a change in position of the free craft. To set up a situation where this can occur, an air bearing track system is used to remove any frictional forces on the craft. When properly calibrated, the only active force should be the electrostatic force between the two sphere-craft. When the power system is active, sensors should be able to measure the motion of the free sphere to verify the force.

These sensor readings can then be cross-checked with the simulated controller. This control simulation can be used to verify the controllability and stability of the one-dimensional system. With control parameters and proper track conditions, the simulation can also be used to control the movement of the spheres through voltage modification.

Through these two methods of experimentation, this project will prove that the use of Coulomb forces to move spacecraft is feasible and show how these forces can be applied to various one-dimension formations of spacecraft in the field.
2  Modeling and Controlling Coulomb Forces

2.1  Coulomb Charge Equations

The underlying equation of motion for the force on a charged object in space due to another charged object can be derived from $F_{tot,j} = \sum F_{ij}$. The equation has a form identified from research (Seubert 2009, 4):

$$\vec{F}_{ij} = k_c \frac{q_i(t)q_j(t)}{r_{ij}^2} e\left(\frac{-r_{ij}}{\lambda_d}\right)\left[1 + \frac{r_{ij}}{\lambda_d^2}\right]\hat{r}_{ij}$$

Equation 2.1

Where: $q_i(t)$, $q_j(t)$ are the charges on the $i^{th}$ and $j^{th}$ spheres respectively, in [C]

$k_c$ is the Coulomb constant, in [Nm$^2$/C$^2$]

$\lambda_d$ is the Debye length, in [m]

$r_{ij}$ is the separation from the $i^{th}$ to the $j^{th}$ spheres, in [m]

$\hat{r}_{ij}$ is a unit vector pointing from the $i^{th}$ to the $j^{th}$ spheres (dimensionless)

For the experiment, the movement is confined only in the x-direction. This will simplify the expression for the force into a scalar equation in the x-direction. Moreover, Equation 2.1 can be rewritten using a linear relation for the charge $q$ on a sphere of radius $\rho$. This equation has the form (Seubert 2009, 4):

$$q(t) = \frac{\rho V(t)}{k_c}$$

Equation 2.2

Where: $V(t)$ is the applied voltage across the sphere, in [V]

$\rho$ is the sphere’s radius, in [m]
According to Seubert (2009), “In the atmosphere of the laboratory it is assumed that the shielding or force decay is negligible as the relative separation distances are very small.” Therefore, the Debye Length $\lambda_d$ can be proven to be effectively infinite. Assuming this, the equation for the force will be simplified immensely. By combining Equation 2.1 and Equation 2.2 with all of these assumptions, the magnitude of total force on the $j^{th}$ sphere is found to be:

$$F_{ij} = F_{ij} (x, V_i(t), V_j(t), t) = m \ddot{x} = \frac{V_i(t)V_j(t) \rho^2}{\chi^2} \frac{1}{k_c}$$

Equation 2.3

This equation is a $2^{nd}$ order, nonlinear ODE in $x$. Therefore, linearization techniques must be applied in order to put this equation in a form that can be used for the derivation of the controller. The Coulomb constant in the denominator of the fraction makes this force very small. In fact, without the large values of voltage being used for this experiment ($\pm 30$ kV), the effect of this force would be too small to have any significant effect. Therefore, all other external forces must be minimized to be as small as physically possible in order to make certain that the electrostatic force is the dominant force in the experiment.

One of the primary aims of this project is to apply control theory to build a controller for the experiment that will hold two charged spheres a specified distance away from each other despite the existence of sensor errors and disturbances. However, in order to build the controller, the equation for $F_{ij}$ must be linearized so that
appropriate linear-based control theory techniques can be applied. Using the Taylor

Series expansion:

\[ F_{ij} = F_{ij} \left( x_d, V_{i,d}(t), V_{j,d}(t) \right) + \frac{\partial F_{ij}}{\partial x} \bigg|_d \delta x + \frac{\partial F_{ij}}{\partial V_i} \bigg|_d \delta V_i + \frac{\partial F_{ij}}{\partial V_j} \bigg|_d \delta V_j \]

Equation 2.4

Where: \( \delta x = x - x_d \), \( \delta V_i = V_i(t) - V_{i,d} \), and \( \delta V_j = V_j(t) - V_{j,d} \)

The following expression for \( F_{ij} \) is obtained:

\[ F_{ij} = \frac{V_{i,d} V_{j,d} \rho^2}{k_c x_d^2} - 2 \frac{V_{i,d} V_{j,d} \rho^2}{k_c x_d^3} \delta x + \frac{V_{j,d} \rho^2}{k_c x_d^2} \delta V_i + \frac{V_{i,d} \rho^2}{k_c x_d^2} \delta V_j \]

Equation 2.5

This simplifies to:

\[ F_{ij} = \frac{\rho^2}{k_c x_d^2} \left( \delta V_i V_{j,d} + V_{i,d} V_{j,d} + V_{i,d} \delta V_j \right) - 2 \frac{V_{i,d} V_{j,d} \rho^2}{k_c x_d^3} \delta x \]

Equation 2.6

Equation 2.6 will be the primary equation used for the analysis of the experiment, as well as the analysis of the simulations for systems with two or more spheres.

2.2 Control Theory

Knowing that two spacecraft with electrostatic charges on them will generate force is only the first step into the CFF research. Once it is determined that electrostatics can generate enough force to create a tether between two or more spacecraft, something has to be done to control how these forces act. By harnessing the charges
and manipulating them to certain optimal values, it will be possible to generate a true tether. This will be accomplished through the use of various linear control analysis and techniques. In order to analyze different cases with a fixed number \( N \) of spheres holding a distinct charge \( q \), it was necessary to develop a general algorithm to confirm stability and controllability of the system. The specific cases and results will be covered in the next chapter.

### 2.2.1 Setup of State Vectors of the Linearized System

In order to linearize the system, a state vector is needed to describe the variables that will govern the dynamics of the system. In a general Coulomb system of \( N - 1 \) Coulomb tethers consisting of \( N \) spheres (\( P \) of which are allowed to move freely), the state vector \( X \) is composed of the positions and velocities of the \( P \) mobile spheres. The remainder \( N - P \) spheres that are restricted to being stationary need not be considered in the dynamics of the system because they are not allowed to move. In matrix form, the 1st order differential equation for the error in state \( (\delta \dot{x} = x - x_d \text{ and } \delta \dot{x} = \dot{x}) \) is represented as a matrix equation: \( \delta \dot{X} = A\delta X + BV \). Fully expanded, the equation is given in general by:

\[
\begin{pmatrix}
\delta \dot{x}_1 \\
\delta \dot{x}_2 \\
\vdots \\
\delta \dot{x}_P
\end{pmatrix} = \begin{pmatrix}
\delta x_1 \\
\delta x_2 \\
\vdots \\
\delta x_P
\end{pmatrix} = \begin{bmatrix}
A_{1,1} & \cdots & A_{1,2P} \\
\vdots & \ddots & \vdots \\
A_{2P,1} & \cdots & A_{2P,2P}
\end{bmatrix}
\begin{pmatrix}
\delta X_1 \\
\delta X_2 \\
\vdots \\
\delta X_{2P}
\end{pmatrix} + \begin{bmatrix}
B_{1,1} & \cdots & B_{1,N} \\
\vdots & \ddots & \vdots \\
B_{2P,1} & \cdots & B_{2P,N}
\end{bmatrix}
\begin{pmatrix}
V_1 \\
V_2 \\
\vdots \\
V_N
\end{pmatrix}
\]

Equation 2.7
Where: The dimension of \( X \) is \( 2P \times 1 \), the dimensions of \([A]\) is \( 2P \times 2P \), the dimensions of \([B]\) is \( 2P \times N \), and the dimensions of \([V]\) is \( N \times 1 \).

Using this matrix equation, the result from Equation 2.6 can be applied to any case being analyzed and the matrices \( A \) and \( B \) can be found to satisfy Equation 2.7 for the system. Specific expressions for these Matrices \( A \) and \( B \) will be given in Chapter 3.

### 2.2.2 Stability and Controllability Analysis

After the nonlinear equations have been put into a matrix form, the \( A \) and \( B \) matrices that are found can be used to perform several analyses for the stability and controllability of the system.

To find the stability of the system, the eigenvalues of the \( A \) matrix must be found. This can be done by setting the $\det[A - \lambda I] = 0$. With these eigenvalues, it can be determined if the system is stable by finding out if, or under what conditions, the eigenvalues have negative real parts. If there are positive real parts for a specific case, then it is known that the nonlinear system is not stable and a way must be found to stabilize the system. However, if the real parts are zero (i.e. the eigenvalues are purely imaginary), then further nonlinear analysis must be performed to determine stability. Negative real parts show that the system is stable, which is the optimal result.

To determine controllability of the system, both the \( A \) and \( B \) matrices are used. The algorithm used to determine controllability involves the composition of a matrix \( R \), which is defined as $R = [B \ AB \ A^2B \ \cdots \ A^{2P-1}B]$. A controllable system will have a full row rank \( R \) matrix (i.e. \( \text{Rank}(R) = 2P \)) which means \( R \) will have at least \( 2P \) columns.
which are linearly independent of each other. This implies that using Coulomb forces to control relative motion between spacecraft will be successful under an appropriately designed control system.

### 2.2.3 Linear Controller Design

In order to design a linear controller, gain coefficients $K_{kl}$ are needed ($k$ for all $N$ voltages that are controlled and $l$ for all $P$ spheres that are mobile). These gain coefficients will linearly translate the errors in position and velocity into voltage corrections (either positive or negative) to attempt to bring the system to rest at the specified equilibrium point (Ogata 2009). In matrix notation, the vector $\{V\}$ can be rewritten as a matrix containing all the gain coefficients $[K]$ right multiplied by the state error vector $\{\delta X\}$ (e.g. $V = K\delta X$), which gives the following as the linear equation of motion:

$$
\begin{pmatrix}
\delta X_1 \\
\delta X_2 \\
\vdots \\
\delta X_{2P}
\end{pmatrix}
= 
\begin{pmatrix}
A_{1,1} & \cdots & A_{1,2P} \\
\vdots & \ddots & \vdots \\
A_{2P,1} & \cdots & A_{2P,2P}
\end{pmatrix}
\begin{pmatrix}
\delta X_1 \\
\delta X_2 \\
\vdots \\
\delta X_{2P}
\end{pmatrix}
+ 
\begin{pmatrix}
B_{1,1} & \cdots & B_{1,N} \\
\vdots & \ddots & \vdots \\
B_{2P,1} & \cdots & B_{2P,N}
\end{pmatrix}
\begin{pmatrix}
K_{1,1} & \cdots & K_{1,2P} \\
\vdots & \ddots & \vdots \\
K_{N,1} & \cdots & K_{N,2P}
\end{pmatrix}
\begin{pmatrix}
\delta X_1 \\
\delta X_2 \\
\vdots \\
\delta X_{2P}
\end{pmatrix}
$$

**Equation 2.8**

From this equation, we can write the matrix $\tilde{A} = A + BK$, defined such that $\delta \ddot{X} = \tilde{A}\delta X$. The closed loop system is stable when the eigenvalues of the $\tilde{A}$ matrix have negative real parts. This ensures that the values chosen for $K_{kl}$ will move the system towards a point of equilibrium (given that the errors being corrected are small). By finding out under what conditions the values of $K_{kl}$ will fail to satisfy this constraint on the eigenvalues, the appropriate coefficients can be chosen.
3 Simulation Setup and Results

3.1 Simulating Coulomb Forces

Two simulations were created in order to fully explore different configurations for the experiment. Both simulations were designed to have movement constrained to only one direction, and both take into account the ±30kV limit on the voltage that the power sources could provide. The first simulation, Case 1, was designed to emulate the full experiment, with one sphere kept fixed and the other mobile. This would allow the prediction of the results for the experiment. Case 2 is a derivative of the three-craft problem and has the potential to progress into a physical experiment if so desired by future MQP groups. In Case 2, the two spheres at both ends are kept stationary, while the middle one is allowed to move. Both cases will be designed and analyzed using the techniques described in Chapter 2.

3.1.1 The Coulomb Tether Experiment Simulation (Case 1)

For the two-craft experiment, Equation 2.6, derived in the previous chapter, can be simplified even further. First, the charge on the $i^{th}$ sphere, $q_i(t)$, is held to a constant value ($q_i(t) = q_S$) by applying a constant voltage $V_S$, so $V_i(t) = V_S$. Also, the voltage on the $j^{th}$ sphere, $V_j(t)$, will be renamed to $V_c(t)$. $V_{c,d}$ is defined to be the desired voltage on the moving sphere attached to the cart, which is the voltage that keeps the cart at a specified distance $x_d$ away from the stationary sphere when the velocity of the cart is zero. The voltages $V_{c,d}$ and $V_S$ are both of the same sign, so their product is positive. Using these conventions, the linearization simplifies to:
\[ m\ddot{\delta x} = \frac{V_c V_c(t) \rho^2}{k_c x_d^2} - 2 \frac{V_s V_c d \rho^2}{k_c x_d^3} \dot{\delta x} \]

Equation 3.1

From this result, Equation 3.1 can be written in matrix form. To accomplish this, the 2\textsuperscript{nd} order differential equation in \(x\) must be reduced to two 1\textsuperscript{st} order differential equations, which is consistent with control theory analysis notation. This matrix equation for the state error vector \(\{\delta x\}\) is:

\[
\begin{bmatrix}
\delta \dot{x} \\
\end{bmatrix} = A \delta x + Bu = \begin{bmatrix}
0 & 1 \\
-2 \frac{V_s V_c d \rho^2}{mk_c x_d^3} & 0
\end{bmatrix} \begin{bmatrix}
\delta x \\
\end{bmatrix} + \begin{bmatrix}
0 \\
\frac{V_s \rho^2}{k_c x_d^2}
\end{bmatrix} \{V\}
\]

Equation 3.2

Where: The vector \(\{\delta x\}\) is composed of \(\delta x_1 = \delta x, \delta x_2 = \dot{\delta x}\)

The vector \(\{V\}\) is composed of \(V = V_c(t)\)

Using stability analysis on the matrix A, the real parts of the eigenvalues are found to be zero. Due to this result, further nonlinear analysis must be performed in order to know if the system is stable or not in this configuration. To circumvent this problem, an external force due to gravity on the cart is added. This ensures that an equilibrium point exists (since without external forces, the two sphere case was found to have no equilibrium) and allows the system to move asymptotically towards that point, if designed correctly. In the real experiment, this force due to gravity would be present anyways since it is not feasible to make the track perfectly level. The angle of tilt that will minimize the effect of gravity has to be chosen to be small enough so that it does not overpower the Coulomb force. This angle was experimentally found with the
simulation, and is about five hundredths of a degree. The direction of tilt was also found to satisfy equilibrium only if the track was inclined away from the stationary sphere, showing that in the absence of Coulomb forces, the cart would slide toward the stationary sphere. Figure 3.1 shows the configuration for Case 1.

Figure 3.1: Diagram of Case 1

The results of equation 3.2 also allow the controllability of the system to be confirmed. As mentioned in Section 2.2.2, this system is controllable if the matrix \( R = [B | AB] \) has rank 2 (where 2 is the dimension of the A matrix). Using this test, it can be shown that system is controllable in this tilted configuration and that no further control inputs are necessary other than the voltage on the cart.

### 3.1.2 3-Craft Case with One Mobile Sphere (Case 2)

The second simulation that was developed was Case 2. Case 2 consists of three spheres: two of which are stationary (on the left and right) and another sphere that is allowed to move on a cart in the center. Since there is only one moving sphere, the vector \( \delta x \) has only two components: the error in position and velocity of the center
mobile sphere. Also, as was shown with the simulation, only voltages $V_1$ and $V_3$ need to be controlled for the system to move asymptotically towards equilibrium. Therefore, using the results of Equation 2.6 applied to the general matrix equation $\delta \dot{X} = A \delta X + BV$, the $A$ and $B$ matrices are found to be:

$$A = \begin{bmatrix}
2V_{2,d} \rho^2 \frac{0}{mk_c} & \frac{V_{3,d}}{(L-x_d)^3} & \frac{V_{1,d}}{x_d^3} & 1
\end{bmatrix}
$$

Equation 3.3

$$B = \begin{bmatrix}
0 & V_{2,d} \rho^2 \\
\frac{0}{mk_c x_d^2} & \frac{V_{2,d} \rho^2}{mk_c(L-x_d)^2}
\end{bmatrix}
$$

Equation 3.4

These matrices $A$ and $B$ allow stability and controllability tests to be performed on the system. Using the linear techniques presented in Section 2.2.2, the real parts of the eigenvalues (of the $A$ matrix) are zero, showing that further nonlinear analysis is necessary. Using the simulation, it can be shown that the system for Case 2 will always be stable for small velocities ($< 0.1$ m/s) as long as all three optimal voltages ($V_{1,d}$, $V_{2,d}$, and $V_{3,d}$) have the same sign. The matrix $R$ composed of the matrices $A$ and $B$ (such that $R = [B \mid AB]$ ) has more than full column rank, proving that it is actually controllable with only one input. This case uses two control inputs based on results from the simulation. Figure 3.2 shows the configuration for Case 2.
3.2 MATLAB Simulation Results

Once both cases were analyzed using the techniques from Chapter 3, a MATLAB program was written to model how various linear controllers would react to each nonlinear system. MATLAB’s ode45 differential equation solver was utilized for this purpose. While each voltage would be controlled by the derived linear controller (the $K_{kl}$ Gain Coefficients), it is important to note that the full non-linear expression for $F_{ij}$ (the accelerations taken from dividing the expressions for $F_{ij}$ by the mass of the cart, $m$) would be used instead of the derived matrix expression obtained in the previous section. This would verify that the assumptions taken hold true in numerical simulation.

3.2.1 Case 1 Results

As mentioned previously, the difficulties encountered in making the two-craft Case 1 have equilibria were solved through the addition of an angle of tilt to the track. This provided an opposing force that would attract the mobile sphere towards the stationary sphere. When coupled with the repulsive force of the Coulomb tether
between the two spheres, there exists an exact equilibrium at the point where the cart had zero velocity and the Coulomb force equals the constant force due to gravity on the cart. Since the Coulomb force was so small, the force due to gravity \( F_g = mg \sin \theta \) had to be small as well. To achieve this, the angle needed to be extremely small since \( m \) and \( g \) are fixed. The simulation was designed so an angle could be chosen (between 0 and .1 degrees) and from that, the optimal voltage \( V_{c,d} \) would be chosen to guarantee equilibrium at the point \( x_d \). A value for the angle of tilt of .05 degrees was chosen for the following simulations.

Using the simulation, it was found that the system could be stabilized to its equilibrium for most values of initial distance from the first sphere (well within the range the full experiment would operate in). It was also found that the system was extremely sensitive to initial velocity. A value on the order of 1 m/s would guarantee that the system would fail to converge. The limit for initial velocity was numerically found to be around 10 to 20 cm/s. Figure 3.3 and Figure 3.4 below show results from the simulation. Figure 3.3 shows both ideal and real simulations (without and with tilt and voltage saturation on the system) for the initial condition that \( x_i = 1.5 \text{ m} \) and the cart’s initial velocity is zero. Figure 3.4 shows both ideal and real simulations for \( x_i = 0.25 \text{ m} \) and zero initial velocity. Both cases had an \( x_d = 0.5 \text{ m} \), as indicated by the horizontal black line on all graphs.
Figure 3.3: Case 1 when $x_i < x_d$

Figure 3.4: Case 1 when $x_i > x_d$
3.2.2 Case 2 Results

For Case 2, the results of several preliminary simulations helped refine the results of the stability and controllability analysis given in the previous section. The case where the middle sphere was operated with the opposite sign as the two end spheres was explored first. Observations revealed that this configuration was unstable since the middle sphere would become attracted to one of the end spheres and steadily drift towards it until the spheres would crash. Since the system was found to be indeterminately stable for the case where they were all of positive sign, observations from the simulation were necessary to determine if the system could converge to the specified equilibrium. The resulting observations revealed that the system was stable, but the center sphere would oscillate back and forth in between the left and right spheres indefinitely if it was perturbed from its equilibrium position. This led to configuring the setup to control the voltages on the two end spheres instead of the central sphere. After further trial and error, it was found that the gain coefficients $K_{kl}$ needed to control the system would depend entirely on the error in velocity (i.e. any non-zero velocity) of the cart. These values would ensure the system converged asymptotically to its equilibrium and moved towards $x_d$ with the limit as $t \to \infty$. Figure 3.5 and Figure 3.6 both illustrate how the system reacts for two separate values of $x_d$. Figure 3.5 shows the system when $x_i = 0.5$ m and Figure 3.6 shows the system when $x_i = 2.5$ m.
Figure 3.5: Case 2 when $x_i < x_d$

System When $x_d = 2\, m$

System When $x_d = 1\, m$

Figure 3.6: Case 2 when $x_i > x_d$
4   Experimental Setup and Results

4.1   Forceless Environment

   The primary operating specification for the 1-D test track was that it could simulate a “deep space environment” to the satellite. For this to happen there must be near zero net force on the satellite (from the environment) at all times. Two major design concepts were considered. The first design was a magnetic levitation system, while the second used air to counteract gravity.

4.1.1   Magnetic Levitation

   The magnetic levitation system was one of the very first options considered. By using magnets to suspend the experiment it would be possible to easily simulate the deep space environment that was necessary. One advantage of using a magnetic system, as opposed to other concepts, was that a magnetic system could be run in a vacuum which would prohibit any arcing of charges that could occur in a normal earth environment. Though the arcing could pose a small problem, it can be avoided, while the major problem with the magnetic levitation system could not.

   When using a system that employs rare earth magnets, it is known that magnets have a tendency to seek a position of minimum energy. This would be counter-intuitive to an experiment seeking to minimize outside forces since additional forces would be necessary to move the magnet out of those positions. Therefore, the magnetic levitation system was proven inadequate for the needs of the experiment, and other options were considered.
4.1.2 Air Bearing Levitation

An air bearing system, while complex, was more practical for this test bed. By passing air through holes in a track (similar to an air hockey table) it is possible to counter the force of gravity, and levitate the cart slightly. While levitated, the cart would encounter very little friction. Though there were a handful of problems that arose with this configuration, none were large enough to make its use unjustifiable.

The first problem is that in the medium of air, high voltage electrostatic sources will arc if they come close enough to each other. A simple way to avoid this was to restrain the cart so that its proximity to any other source of arcing would be limited to a specified distance. The second issue was undesirable torques due to the movement of air around the cart. While the air coming from the track underneath the cart provided the force to levitate the cart and prevent friction, there was also air moving around the outer edges of the cart that would create torques. The forces from this flow of air could create large enough torques on the cart to be able to keep the cart in contact with the track, creating undesirable friction and preventing the cart from moving properly. In order to restrict the air flow so that the air could be concentrated underneath the cart, an air control system had to be implemented which limited the airflow to only those pipes that were directly underneath the cart.

4.2 Air Bearing Track

The air bearing track is an integral part of the experimental setup that is part of this project’s goals. The design of the track must take into account many factors. The track must be an effective test bed for one-dimensional motion, which is reliant on the
air levitation system keeping the cart level and free from frictional forces. The air distribution system must also be able to effectively channel the air into the track and the air flow must be evenly distributed through the structure. The various incarnations of this track each compensated for these design requirements in different ways.

The final track, which best conforms to the design standards, has the following specifications:

<table>
<thead>
<tr>
<th>Final Track Specifications</th>
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<tbody>
<tr>
<td>Height</td>
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<tr>
<td>Width</td>
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<td>Length</td>
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<td>Composition</td>
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<tr>
<td>Half-Pipe Specifications</td>
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<td>Depth</td>
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<tr>
<td>Air Channel Specifications</td>
</tr>
<tr>
<td>Top</td>
</tr>
<tr>
<td>Side</td>
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<tr>
<td>Bottom</td>
</tr>
</tbody>
</table>

Table 4.1: Final Track Specifications

4.2.1 Track One

Track One was the original design concept for the air bearing track. This early design did not include an air channel design. The overall design featured an accentuated half-pipe shape, which maximized the size of the air pocket. However, this design was ultimately discarded due to a number of factors.

Figure 4.1: Track One Solidworks Model
Prime among these was that this early design did not take into account the use of the cart, so the free sphere would have been inherently unstable. The air pocket would have also been more difficult to create and maintain because of its larger size. Figure 4.1 shows the Solidworks model that was created for Track One.

4.2.2 Track Two

Track Two was the first design to include a design for the air channel system as well as the first to include relative dimensioning. The track’s air system was designed with one entrance at the bottom of each series and four outlet ports each. The top of the track was designed to provide a stable air cushion for the cart.

There were, however, a number of drawbacks to this track. The track was thicker than preferred and the air channel system setup was not optimal for the purposes of the experiment. Also, on the top surface of the track, the straight angles on the rise towards the centerline would have been difficult to machine. These issues were addressed in the next version of the track. Figure 4.2, Figure 4.3, and Figure 4.3 show Solidworks models of this design.
4.2.3 Track Three

There were a number of improvements made to the third track. To improve machinability, the rise of the top surface took the form of a smooth curve rather than an angle. This resulted in two separate air pockets taking form beneath the cart. The top surface air channels were also moved closer to the centerline to increase the effect of the air levitation system. In addition to these changes, the track itself was thinned.
down to better conform to the experimental concept of a final track design. Figure 4.5 and Figure 4.6 illustrate Solidworks wireframe models of Track Three.

4.2.4 Track Four

The design of Track Four improves upon the type of track design used for Track Three. Since the Delrin block that the track was to be machined from was two inches in height, the height of the designed track was changed to be two inches at its peak. Also, for machinability reasons, the height of the larger holes in the bottom of the track was increased. The depth of the curved air pockets was also decreased to allow the air
channels to lead further up the side of the track to make the air bearing system more effective. Figure 4.7 shows a wireframe model of Track Four, made in Solidworks.

Figure 4.7: Track Four Wireframe Model

4.2.5 Track Five

The changes made to the track design in Track Four were improved upon in Track Five to make air bearing system more effective. The top and side air channels were moved closer the centerline and top edge, respectively. To help accomplish this, the surface curve was diminished even further than in Track Four. Figure 4.8 and Figure 4.9 highlight these changes.

Figure 4.8: Track Five Solid Model
4.2.6 Track Six

Track Six was the final track design and the only one to be machined. This design was characterized by the implementation of a wide, half-pipe track with small ridges, using what was learned from earlier versions of the track. This design increased the stability of the cart, as earlier designs would have been more susceptible to the cart tipping. The air channel system was also simplified through the use of two separate sets of intake holes on the bottom of the track, which were also tapped to allow the connectors for the hoses to attach. The separate intake holes allowed the air to be evenly distributed while also permitting vertical and horizontal output holes to be added to improve machinability concerns. Figure 4.10, Figure 4.11, and Figure 4.12 show the final track configuration.
Figure 4.10: Track Six Solid Model

Figure 4.11: Track Six Wireframe Model

Figure 4.12: Track Six Air Channel Model
4.3 Airflow System

In order to restrict airflow to the area underneath the cart, a control system is needed to shut-off the valves when the cart was not directly over them. The control system consisted of a row of 29 infrared sensors (one that related to each row of holes in the track), and 29 solenoid valves. Each of the sensors was connected through an electrical circuit shown adopted from Seubert in Figure 4.13.

![Figure 4.13: Air Flow Control Circuit (Seubert 2009, 11)](image)

This circuit receives a signal from the infrared sensor and relays it to the corresponding solenoid valve. The sensor is simply a position sensor, so it can only sense one of two things. It can “see” the cart, or it can “see” nothing. For the case when it senses the cart, a signal will be sent through the circuit, amplified, and then massed through a transistor which will close a second part of the circuit. When this becomes closed, power is provided to the corresponding solenoid valve. The valves are “normally-closed” meaning that unless they are activated no air will pass through them. When the power reaches a valve, it will open and allow air to freely flow until it receives a signal that the cart is no longer in front of the corresponding sensor. Implementing this
control system into the air bearing track would have removed a very large possibility for error from the experiment from external torques.

The components for the circuit were purchased. One branch of the circuit was constructed and tested to ensure the components that were bought would function properly with the circuit. Unfortunately, the solenoid valves that were needed are not easily attainable in larger quantities. A minimum lead time of 10 weeks for the necessary 29 valves was a major issue. Because of time constraints and ordering issues, there was no way to procure the valves on time for the completion of this first MQP. This would mean that the air control system could not be implemented into this version of the project, and would have to be explored in future projects. In order to be able to test the air system without the solenoid valves, an interim manifold needed to be designed. The manifold was created by using a 34 inch length of PVC pipe with a radius of 1.5 inches. 29 holes were tapped into the length of the pipe, equally spaced at 1 inch apart. A barbed nipple was then screwed into each of the holes and connected to the track via ¼ in. flexible tubing. Caps were placed onto each end of the pipe to make it airtight.

Once all of the tubes were connected to the underside of the track by barbed nipples similar to the ones in the temporary manifold, it became necessary to raise the track off of the ground so it did not lie on the tubing. In order to do this, a “table” was created for just this purpose. The table measures 30in. x 6in. on the outside and on the inside it is hollow to allow the tubes to pass freely to the manifold. Due to the unique need for the complete absence of conductive materials, the table had to be constructed...
entirely out of wood. Initially reinforced with only wood glue, holds were later drilled at each joint. These holes were then filled with a wooden dowel and sealed off to ensure complete stability of the table. Figure 4.14 and Figure 4.15 show the final track that was constructed. Figure 4.14 shows the track in its final configuration with the manifold attached and Figure 4.15 shows the underside of the track with the air tubes attached.
The air for the air-bearing system will be provided to the track using an air compressor. Initially, a small, low cost compressor was used to provide air to the track. However, it was found that the small compressor was inadequate for the needs of the test-bed. The small compressor that was initially chosen was a 3 gallon unit with a maximum PSI of 100 and a 1/3HP motor to compress the air. The 1/3 HP engine could not keep the tank full of air when in use, and the compressor tank would be empty in under 8 seconds. In order to amend this, as well as simulate the operation of the solenoid valve system that would be used on the track, all of the tubes except the 12 under the track were closed by using pinch valves. This left a length of 12 inches on the track for the 10.5 inch cart to sit on. The small compressor was tried again under this new configuration. Once again the engine could not keep up with the exiting airflow from the tank, and the tank was empty in 15 seconds. In addition to the poor capability of the motor, and volume of the tank, the pressure behind it was also unacceptable. On every test, the 100 PSI of the compressor failed to lift the cart noticeably for any significant period of time. The lack of power and capacity was too large of a problem for the smaller compressor to be used.

This smaller compressor was returned, and replaced with another Craftsman compressor with a larger tank size of 26 gallons. The compressor is rated for 160 PSI maximum, and the engine had a peak horsepower of 1.5. Figure 4.16 shows the larger air compressor that was purchased for providing air pressure for the project. By drilling a hole into the end of the
temporary PVC manifold and threading a standard quick connect valve, it was possible to create a way to easily attach and remove the compressor from the manifold.

4.4 Spacecraft

To simulate the spacecraft, two 10 inch diameter Styrofoam spheres are used. Each is wrapped in foil to enable their conductivity and ensure that they can be charged electro-statically to the necessary 10 - 30kV. One of the orbs will be mounted on a pole and have a constant charge on it. The second sphere will be mounted on a cart and have a variable voltage on it. It is through varying the voltage on this cart that the control system will be tested, and the feasibility of the CFF concept will be determined.

While the fixed orb will not require any modification, the orb that is on the cart must be changed. The orb and the cart (made from Ultra High Molecular Weight Polyethylene or UHMW) must weigh no more than 500g. This ensures that the air levitation system will be able support it along the length of the track. In order for the weight requirement to be met, the orb must be hollowed out so that only the shell remains. Once completed, it will be wrapped in foil and placed on the cart. The cart is 10.5 inches long, 6 inches wide, and .3 inches thick. This volume ensures the cart receives maximum lift, while keeping weight to a minimum. Figure 4.17 and Figure 4.18 show the sphere in the hands of one of the project members and placed on the cart.
4.5 Electrostatic Power Supply

The experiment will utilize a Spellman Dual Polarity Auto Reversing Electrostatic Power Supply. This supply will generate the necessary voltage (30 kV) while using very low amperage (300 μA) which results in a very reasonable power output (9W). Figure 4.19 shows the power supply and its connections. Due to its configuration, the power supply must be controlled from the front by a 25-pin analog connection. A piece was made that separated the 25-pin connector into individual wires. Five wires would ultimately become part of the output of the power supply, and 18 would be used to control and monitor the power supply. Two wires of the 25 had no use in the power supply. Wires were grouped in sets of five, with the color scheme in each group was designed to increase from 1 -5 according to ROGBV (Red, Orange, Green, Blue, Violet).
The power supply was connected to the computer system via a National Instruments USB-6008 device. This is a low-cost multifunction DAQ, which allows data to be transferred both to and from the power supply. It is compatible with LABView Signal Express, and connects to any computer via a USB cable.

4.6 Safety

Safety was a considerable concern. The experiment will be working with extremely high voltages (around 30,000V), but the amperage involved will be small enough that there are no safety hazards when working with the power supply. The output from the power supplies will be 9 Watts at its maximum. To prevent arcing off the spheres, everything in a 75 centimeter radius from the test bed will need to be non-conducting. Since there are such high voltages operating electro-statically, the entire project will essentially act as a very large capacitor and could discharge at any given time. For this reason, a large plastic room that surrounds the project needs to be constructed. The plastic itself should be grounded out, and this will alleviate any potential problems.

4.7 Results

After testing with the 26 gallon compressor, some problems were identified. While there was enough pressure to lift the cart, it was not possible to tell how well the system would work when the orb is attached or the valve system is being employed. With the clamps removed in order to observe the motion of the cart along the full length of the track, the air compressor could not produce the needed pressure of 100
psi in the manifold and tubes. This meant it could not suspend the cart over the track. However, since the manifold on the valve system will be much smaller in volume than our interim manifold, it is not prudent to say whether the air bearing system will be more or less effective than that current manifold. When the solenoid valves do become integrated into the design, there will be a maximum of 11 valves open at any time, as was accounted for from the pinching clips that were put on the tubes in our tests. In contrast, adding the sphere to the cart will add more weight to the cart, requiring more pressure to be applied to lift the cart. It is impossible to tell if the benefit from operating only 11 valves at a time would balance the extra weight due the orb.
5 Recommendations

Overall, this project was successful in completing the goals that were set. As was shown in Chapter 3, simulation results successfully showed that both the two and three-craft systems could be controlled by varying the voltage on one or more spheres, given the assumptions that were taken. Also, Chapter 4 demonstrated that significant progress was made towards designing a test bed for further control experiments. However, recommendations were developed in this Chapter so that future groups working on this project will have a baseline to develop their work.

To start with, the air control system must be constructed and integrated with the valves that were purchased but did not arrive in time for this project. The air control system is essential to the project because it would minimize the external torques that are present on the cart. It will also aide in amplifying the pressure in the tubes which will direct the air to levitate the cart. The control system circuit must be constructed from duplicates of the diagram shown in Figure 4.13. Each duplicate will connect eight infrared sensors to their corresponding solenoid valves. Once fully integrated into the system, the airflow from the compressor will become much more efficient, as only 11 of the solenoids will be open at a time. Once integrated, the circuit, valves, and sensors need to be mounted properly on a moving structure that will house the track as well. This will allow the experiment to be moved to a location where it will have the desired 75cm clearance from all conductive materials. Finally, the proper adapters need to be fitted to the valves so that the air compressor can be used to provide the necessary air efficiently and without leakage.
After the track has been setup and the air system has been integrated, work must be done on the infrastructure that will power and control the charges on the two spheres. This is important since the power supply that was bought for this project relies entirely upon an external source, such as a computer, to control the output of the power supply. Also, the power supply needs to be connected to an external power supply in order to operate, something that must be accounted for in the final test structure. Additionally, a method for the creation of surface charges on the aluminum-clad spheres must be researched and developed. Since the experiment will be using extremely high voltages, the methods for providing charges on the spheres must be rigorously tested to make sure no current leakage occurs out of the system. If current leakage occurs, the experiment will fail to perform correctly.

Next, a computer controller must be developed that can employ the results of the simulations in Chapter 3 and control the charges on the spheres. This is important to satisfy the overall goal of this project and develop a controller that will carefully control the distances between the two spheres. The design of this computer controller should be such that it is easily integrated with the test system, possibly employing programs such as LABview or MATLAB. Also, the computer controller should allow for changes to the control scheme to be made quickly and accurately, so that multiple schemas can be tested in a short amount of time.

Finally, before testing can begin, the proper safety equipment and housing needs to be developed. This is necessary so that any mishaps with the system itself can be contained and prevent any damage to human life or the equipment employed in the
experiment. Construction of the plastic shield, as well as testing and checking the experimental setup for 75cm clearance of conductive materials, are important assignments that must be completed before testing can begin.
Works Cited


%% MQP Experiment, Case 1 (MQP_Experiment_Case_1.m) %%

clear all; close all; clc;

% Parameters %

global P xd K1 K2 g Vs Vco theta

m = 0.5; % mass of cart [kg]
Kc = 8.987551787 * 10^9; % coulomb constant [N*m^2/C^2]
rho = 0.1016; % radius of spheres [m]
P = (rho^2)/(m*Kc); % Simplifying Parameter [N*m^4/kg*C^2]
xd = 0.5; % desired distance [m]
Vo = 10000; % Operating Voltage [V]
Vs = -2*Vo; % voltage on stationary sphere [V]
g = 9.81; % Gravity [m/s^2]
tiltAngle = .01; % Track Tilt Angle [degrees]
theta = tiltAngle*(pi/180); % Track Tilt Angle [radians]
Vco = (g*(xd^2)*sin(theta))/(P*Vs); % Correction Factor for tilt [V]

% Constant Coefficents %
Q = 1;
R = 5;
K1 = Q*(((xd^2)/(P*Vo))+(Vo/xd));
K2 = R*(((xd^2)/(P*Vo)));

% Time Span %

T = (0:0.05:300); % time vector [s]

% Initial Conditions %

xi1(1) = .25; % Initial Position [m]
xi1(2) = 2; % Initial Velocity [m/s]
xi2(1) = 3; % Initial Position [m]
xi2(2) = 2; % Initial Velocity [m/s]

% Differential Equation Solver %

[t,x1] = ode45('xsystem', t, xi1);
[t,x2] = ode45('xsystem_tilt', t, xi1);
[t,x3] = ode45('xsystem', t, xi2);
[t,x4] = ode45('xsystem_tilt', t, xi2);

% Plot Solution %

figure;

subplot(2,1,1);
plot(t, x1(:,1), 'r');
hold on;
plot(t, xd, 'k');
title('Ideal System [Both Graphs For Case xi < xd]');
xlabel('Time [s]');
ylabel('Distance from Stationary Sphere [m]');

subplot(2,1,2);
plot(t, x2(:,1), 'r');
hold on;
plot(t,xd,'k');
title('Real System with 1/100 deg Tilt');
xlabel('Time [s]');
ylabel('Distance from Stationary Sphere [m]');

figure;

subplot(2,1,1);
plot(t, x3(:,1), 'r');
hold on;
plot(t,xd,'k');
title('Ideal System [Both Graphs For Case xi > xd]');
xlabel('Time [s]');
ylabel('Distance from Stationary Sphere [m]');

subplot(2,1,2);
plot(t, x4(:,1), 'r');
hold on;
plot(t,xd,'k');
title('Real System with 1/100 deg Tilt');
xlabel('Time [s]');
ylabel('Distance from Stationary Sphere [m]');
function xdot = xsystem(t,x)
global P K1 K2 xd Vs

Vc = K1*(x(1)-xd) + K2*(x(2)); % voltage on cart sphere [V]

function xdot = xsystem_tilt(t,x)
global P K1 K2 xd Vs Vco g theta

Vc = Vco + K1*(x(1)-xd) + K2*(x(2)); % voltage on cart sphere [V]

if Vc > 30000
    Vc = 30000;
elseif Vc < -30000
    Vc = -30000;
end

xdot(1) = x(2);
xdot(2) = Vs*Vc*P/((x(1))^2) - g*sin(theta);

xdot = xdot';
Appendix B: MATLAB Code for Case 2

%% MQP Experiment, Case 2 (MQP_Experiment_Case_2.m) %
% This program models the dynamics of a linearly-controlled, 3 craft %
% system. The system is composed of two immobile spheres on either end %
% of a levitating track suspending a third, mobile sphere. The mobile %
% sphere has a constant voltage applied to it, while the other two %
% spheres have a voltage on them that is controlled by the designed %
% controller. The distance $x$ is measured from the first sphere to the %
% middle one, the first sphere is at $x=0$, and the two end spheres are %
% placed a distance $L$ away from each other.

clear all; close all; clc;

% Parameters 

```matlab
m = 0.5; % mass of cart [kg]
Kc = 8.987551787 * 10^9; % coulomb constant [N*m^2/C^2]
rho = 0.1016; % radius of spheres [m]
xd = 1.0; % desired distance of mobile sphere (0.3 < xd < 2.7) [m]
L = 3; % distance between stationary spheres [m]
Vo = 3000; % Operating Voltage [V]
V1o = (xd^2) * Vo; % Operating Voltage on Sphere 1
V2o = Vo; % Operating Voltage on Sphere 2
V3o = ((L - xd)^2) * Vo; % Operating Voltage on Sphere 3
Q = (rho^2)/(Kc*m); % Simplifying Coefficient of Constants 

% Constant Coefficents 

K1 = -1000000; % Velocity Error Scaling Coefficient on Sphere 1
K2 = (30000 - Vo)*10; % Velocity Error Scaling Coefficient on Sphere 2
K3 = 1000000; % Velocity Error Scaling Coefficient on Sphere 3
```

% Time Span 

```matlab
experimentTime = 15; % Experiement Run-Time [min]
tf = experimentTime*60; % Experiment Run-Time in seconds [s]
dt = tf/10000; % Scaled Time Step
t = (0:dt:tf); % time vector [s]
```

% Initial Conditions 

```matlab
x1i(1) = 0.5; % Initial Position (Between 0.3 m and 2.7 m) [m]
x1i(2) = 0; % Initial Velocity (Cannot Excede +/- 0.1 m/s) [m/s]
```

```matlab
x2i(1) = 2.5; % Initial Position (Between 0.3 m and 2.7 m) [m]
x2i(2) = 0; % Initial Velocity (Cannot Excede +/- 0.1 m/s) [m/s]
```

% Differential Equation Solver 

```matlab
[t,x1] = ode45('xsystem_case2', t, x1i);
[t,x2] = ode45('xsystem_case2', t, x2i);
```
%% Plot Solution %%

figure;

subplot(2,1,1);
plot(t, x1(:,1), 'r');
hold on;
plot(t,xd,'k');
title('System for xi < xd');
xlabel('Time [s]');
ylabel('Distance from First Sphere [m]');

subplot(2,1,2);
plot(t, x2(:,1), 'b');
hold on;
plot(t,xd,'k');
title('System for xi > xd');
xlabel('Time [s]');
ylabel('Distance from First Sphere [m]');

%% Function (xsystem_case2.m) %%
function xdot = xsystem_case2(t,x)

global Q L V1o V2o V3o K1 K2 K3

V1 = V1o + K1*(x(2)); % voltage on sphere 1 [V]
V2 = V2o + K2*(x(2)); % voltage on sphere 2 [V]
V3 = V3o + K3*(x(2)); % voltage on sphere 3 [V]

if V1 > 30000
    V1 = 30000;
elseif V1 < -30000
    V1 = -30000;
end

if V3 > 30000
    V3 = 30000;
elseif V3 < -30000
    V3 = -30000;
end

%% Differential Equations %%
xdot(1) = x(2);
xdot(2) = V2*Q*((V1./(x(1)^2)) - (V3./(L -x(1))^2));

xdot = xdot';