The Continuously Variable Transmission: A Simulated Tuning Approach

A Major Qualifying Project Report:

Submitted to the Faculty of

WORCESTER POLYTECHNIC INSTITUTE

In Partial Fulfillment of the Requirements for the

Degree of Bachelor of Science

By:

Timothy R. DeGreenia. tddegrenia@gmail.com

Date: November 26, 2013

Approved by:

Professor John M. Sullivan
# Contents

Abstract ................................................................................................................................. 5

Chapter 1. Introduction ........................................................................................................ 6

Chapter 2. Background .......................................................................................................... 7

Chapter 3. Tuning Program .................................................................................................. 17
  3.1 Desired Vehicle Dynamics ............................................................................................ 18
  3.2 Engine Power Diagram ................................................................................................ 19
  3.3 Gear Ratio Calculations ............................................................................................... 21
    CVT ................................................................................................................................. 22
    Intermediate .................................................................................................................... 23
    Total .................................................................................................................................. 23
  3.4 Derivation of Torque Diagram Calculations ................................................................. 24
  3.5 Low Ratio: Engagement Phase ..................................................................................... 28
  3.6 One to One Ratio: Straight Shift Phase ....................................................................... 30
  3.7 High Ratio: Shift Out Phase/ Back Shifting ................................................................. 31

Chapter 4. Simulation Results .............................................................................................. 32
  4.1. 2012 Tune ................................................................................................................... 32
  4.2 2013 Tune .................................................................................................................... 41

Chapter 5. Conclusion ......................................................................................................... 46

References ............................................................................................................................. 47
Table of Figures

Figure 1: Conventional Automatic Transmission with Planetary Gear Sets .............................................. 7
Figure 2: Engine speed is depicted against vehicle speed. The dashed line represents the rise and fall of engine speed as each gear exchange is made. ............................................................................. 8
Figure 3: CVT system in which the two belt driven pulleys represent the CVT and primary gear reduction. The secondary chain reduction acts to increase the range of adjustability of the CVT gear ratio........... 9
Figure 4: CVT speed diagram in which the dashed line represents engine speed. The advantage of the CVT is that it allows the engine speed to remain constant thus producing steady power production through the majority of vehicle advancement. ................................................................. 10
Figure 5: Cutaway of CVT clutch components ....................................................................................... 11
Figure 6: CVT in idle state prepared for low ratio vehicle launch............................................................ 13
Figure 7: CVT begins low ratio operation as belt is gripped within primary pulley, system begins to rotate, and vehicle launch and acceleration occur................................................................. 13
Figure 8: Belt diameter is the same between the pulleys creating the section of consistent rate of acceleration through the majority of the range of vehicle speed. .................................................................... 14
Figure 9: High gear ratio of CVT in which the output axle rotates at nearly the same speed as the input crankshaft. Torque and resistance are low while speed slowly climbs to its limits. ................................. 15
Figure 10: CVT speed diagram. The area represented by A: idle range of the transmission. B: engine engagement speed. C: the belt is gripped. D: low ratio. F: straight shift acceleration. G: High ratio shift out. .................................................................................................................................. 15
Figure 11: Engine Power Diagram with horsepower production against engine speed. Each slope represents a different power band..................................................................................................................... 19
Figure 12: All relationships that affect gear ratio ..................................................................................... 21
Figure 13: 2012 WPI FSAE Power vs Engine Speed Diagram ................................................................. 32
Figure 14: 2012 Speed Diagram ............................................................................................................. 37
Figure 15: 2013 Dynamic Profile in Comparison to Previous Year. 2013 is shown in red. ....................... 44

Table of Tables

Table 1: Gear Ratio Parameters .............................................................................................................. 21
Table 2: Torque Diagram Parameters for Delineation ........................................................................... 24
Table 3: Component Parameters for Low Ratio CVT Actuation ........................................................... 28
Table 4: CVT Component Parameters for Straight Shift ......................................................................... 30
Table 5: 2012 Input Parameters ............................................................................................................ 33
Table 6: 2012 Gear Ratio Values from Calculation ................................................................................ 34
Table 7: 2012 Active Adjustable Components ....................................................................................... 38
Table 8: 2013 Input Parameters ............................................................................................................ 41
Table 9: 2013 Active Components ........................................................................................................ 45
### Table of Equations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation 1: Gear Ratio of CVT in Low Ratio</td>
<td>22</td>
</tr>
<tr>
<td>Equation 2: Gear Ratio of CVT in Straight Shift</td>
<td>22</td>
</tr>
<tr>
<td>Equation 3: Gear Ratio of CVT in High Ratio</td>
<td>22</td>
</tr>
<tr>
<td>Equation 4: Gear Ratio of Secondary Sprocket Chain Reduction</td>
<td>23</td>
</tr>
<tr>
<td>Equation 5: Total Vehicle Gear Ratio at Low CVT Engagement</td>
<td>23</td>
</tr>
<tr>
<td>Equation 6: Total Vehicle Gear Ratio at Straight Shift CVT Operation</td>
<td>23</td>
</tr>
<tr>
<td>Equation 7: Total Vehicle Gear Ratio at High Ratio CVT Operation</td>
<td>23</td>
</tr>
<tr>
<td>Equation 8: James Watt Horsepower to Rotational Torque Relationship</td>
<td>24</td>
</tr>
<tr>
<td>Equation 9: Conservation of Power from Input to Output</td>
<td>25</td>
</tr>
<tr>
<td>Equation 10: Unit Conversion Between Input Power and Input Torque Relationship</td>
<td>25</td>
</tr>
<tr>
<td>Equation 11: Simplified Conversion of Horsepower into Torque</td>
<td>25</td>
</tr>
<tr>
<td>Equation 12: Equivalent conversion From Input Power to Output Torque</td>
<td>25</td>
</tr>
<tr>
<td>Equation 13: Final Gear Ratio of System in Relation to Comparative Angular Velocities and Radii</td>
<td>25</td>
</tr>
<tr>
<td>Equation 14: Relationship of Output Torque to Input Torque Dependent upon Gear Ratio Caused by Varying Angular Velocities</td>
<td>26</td>
</tr>
<tr>
<td>Equation 15: Output Torque in terms of Known Variables, Input Power, Input Angular Velocity, Gear Ratio</td>
<td>26</td>
</tr>
<tr>
<td>Equation 16: Output Torque at Engine Engagement Speed and Phase</td>
<td>26</td>
</tr>
<tr>
<td>Equation 17: Output Torque at Optimal Engine Speed and Straight Shift Phase</td>
<td>26</td>
</tr>
<tr>
<td>Equation 18: Output Torque at Peak Engine Speed and High Ratio Phase</td>
<td>26</td>
</tr>
<tr>
<td>Equation 19: Output Angular Velocity in Terms of Input Angular Velocity and Active Gear Ratio</td>
<td>27</td>
</tr>
<tr>
<td>Equation 20: Output Angular Velocity at Engine Engagement Speed and Low Gear Ratio</td>
<td>27</td>
</tr>
<tr>
<td>Equation 21: Output Angular Velocity at Optimal Engine Speed and Straight Shift Gear Phase</td>
<td>27</td>
</tr>
<tr>
<td>Equation 22: Output Angular Velocity at Peak Engine Speed and High Gear Ratio</td>
<td>27</td>
</tr>
<tr>
<td>Equation 23: Vehicle Velocity in Terms of Output Angular Velocity and Tire Radius</td>
<td>27</td>
</tr>
<tr>
<td>Equation 24: Simplified Vehicle Velocity</td>
<td>27</td>
</tr>
<tr>
<td>Equation 25: Vehicle Velocity During Low Ratio Engagement</td>
<td>27</td>
</tr>
<tr>
<td>Equation 26: Vehicle Velocity During Straight Shift Engagement</td>
<td>28</td>
</tr>
<tr>
<td>Equation 27: Vehicle Velocity at During High Ratio Operation</td>
<td>28</td>
</tr>
<tr>
<td>Equation 28: Pressure Spring Force in Terms of Spring Constant and Compression Length</td>
<td>29</td>
</tr>
<tr>
<td>Equation 29: Newton's Second Law</td>
<td>29</td>
</tr>
<tr>
<td>Equation 30: Velocity in Terms of Flyout Radius and Angular Velocity of Input Shaft</td>
<td>29</td>
</tr>
<tr>
<td>Equation 31: Flyweight Force in Terms of Flyweight Mass and Rotational Velocity</td>
<td>29</td>
</tr>
<tr>
<td>Equation 32: Equivalence of Pressure Spring Force and Flyweight Force</td>
<td>29</td>
</tr>
<tr>
<td>Equation 33: Flyweight Mass in Terms of Pressure Spring Force and Flyweight Velocity with Conversion Factors Included</td>
<td>29</td>
</tr>
<tr>
<td>Equation 34: Belt Force as Flyweight Force overcomes Pressure Spring Force</td>
<td>30</td>
</tr>
<tr>
<td>Equation 35: Torque Spring Engagement as Torque Spring Force and Belt Force Approach Equivalence</td>
<td>30</td>
</tr>
<tr>
<td>Equation 36: Torque Spring Force in Terms of Flyweight Mass, Operating Engine Speed, and Pressure Spring Force</td>
<td>30</td>
</tr>
</tbody>
</table>
Abstract
This MQP defines an intuitive protocol for the tuning of the continuously variable transmission (CVT) for competition applications including the FSAE Design Competition. The tuning program explored in this report allows the reader to simulate transmission tuning affects on vehicle operation and make informed tuning decisions that save time, reduce cost, and provide more consistent tunes. This method was used to simulate alterations to the 2012 WPI FSAE vehicle tune and resulted in a vehicle prepared for racing conditions.
Chapter 1. Introduction
This report was developed as a result of the 2013 WPI formula vehicle redesign for the 2013 FSAE collegiate design competition. All aspects of the available 2012 vehicle were reconsidered and either refurbished or redesigned for optimal performance, including the engine, transmission, front and rear end suspension systems, and the body as described in the project report “Design and Optimization of a FSAE Vehicle”. As this process progressed, little was known about the tuning methodology of the CVT, how it functioned, or how it would impact the vehicle dynamics. With so much being done to the vehicle and such a short period of time to prepare it, only a limited amount of track testing was conducted thus transmission tuning became difficult to accomplish. This program aims to facilitate CVT tuning in situations that involve time and direct testing constraints for future tuners, and afford them the opportunity to play with the vehicle dynamics through simulation rather than operation. The use and success of this program will save time and resources commonly spent during the transmission tuning process. It will afford the tuner the opportunity to achieve an understanding of the dynamic affects that different tunes will have on the CVT system and then implement a successful tune in a timely manner for the rest of the team.
Chapter 2. Background
The purpose of any transmission is to translate engine performance into vehicle performance. A transmission accomplishes this task by providing a variety of gear ratios between the engine crankshaft and the output axle of the vehicle. Each gear ratio will result in a different vehicle profile of speed and torque while the engine operates at the same speed or range of speed. The goal of the transmission is to allow the engine to operate within an ideal state of power production, and to apply this power effectively to the track through the use of the appropriate gear ratio.

A conventional transmission increases vehicle speed while maintaining the engine’s operating range through the use of gear sets. The transmission in Fig.1 can transmit power entering from the engine crankshaft into useable power at the output shaft through the use of a clutching mechanism that engages specific gears based upon engine and vehicle speed. As the gear ratio increases, the rotational speed or angular velocity of the output axel increases in relation to the angular velocity of the engine crankshaft.

Figure 1: Conventional Automatic Transmission with Planetary Gear Sets

As will be discussed further in Chapter 3, engine performance creates power bands in which certain ranges of engine speed produce a specific level and rate of horsepower output. A transmission can be tuned to take advantage of this effect. The tuner can choose a power band that results in lower power output and higher fuel efficiency for common driving or perhaps a power band within higher engine speeds that produces higher power output for aggressive driving. The conventional transmission commonly functions within a single engine power band and allows the engine speed to fluctuate within this power band as each gear exchange is made. In this way, the conventional transmission increases vehicle speed while the engine operates in the same power band for each gear ratio as shown in Fig.2 provided by Aaen Clutch Tuning Handbook.
Figure 2: Engine speed is depicted against vehicle speed. The dashed line represents the rise and fall of engine speed as each gear exchange is made.

The conventional four-speed transmission in Fig. 2 shows the rise and fall in engine speed between 6000 and 11000 rpm as each gear exchange occurs and the vehicle increases speed. For a conventional transmission, the fluctuation in engine speed at each gear exchange is used to facilitate fluid gear transfer. As vehicle speed increases and gears are exchanged, the slope of engine speed and vehicle speed at each gear is different. Each gear and respective slope represents a different ratio of gain in vehicle speed per gain in engine speed. It is this change in slope that makes each gear useful by producing a different range of vehicle speed and production of torque. Steeper slopes such as in first gear in Fig. 2 represent high torque situations in which the gear ratio is low and the output axle rotates more slowly than the engine crankshaft. The same engine power is transferred which results in torque for overcoming vehicle inertia and the vehicle accelerates quickly but only to a limited speed. As higher gears are achieved, it is more difficult for the same engine power to propel the vehicle, so the rate of vehicle acceleration and the application of torque decrease but the overall vehicle speed rises because the output axle rotates almost as quickly as the crankshaft at high gear ratios.

A continuously variable transmission (CVT) is a lightweight, gear reduction system that utilizes a few regulatory mechanisms to achieve the same end as a conventional automatic transmission. The comparative size and complexity of a common CVT system is shown in Fig. 3.
As can be seen in Fig. 3, the CVT is a compact system and as will be described, it does not require the use of bulky gear sets or as many components as in the conventional transmission. A CVT system is comprised of two conical pulleys and a belt. As the sheaves of each pulley move closer or farther away from one another, their conical shape causes the belt to rise and fall between the sheaves of each pulley. Depending upon the state of the belt within each pulley, the active gear ratio is changed. Instead of switching between bulky fixed gears which only supply a limited number of gear ratios, the CVT pulleys create a continuous exchange of gear ratios by constantly altering the state of the belt between them. This provides a range of gear ratios limited by the pulley diameters and every possible gear ratio that is provided within that range is available for use. The regulatory mechanisms that allow for control of the pulley diameters include engine speed, flyweights, two springs, and a torque feedback mechanism called a helical torque ramp. When these mechanisms work in concert, they act to increase vehicle speed fluidly while maintaining engine speed at a single value instead of fluctuating within a single power band. This feature of engine speed maintenance is possible due to the continuous and more inclusive variety of gear ratios that the CVT offers. Figure 4 provided by Aaen Clutch Tuning Handbook depicts the comparative effectiveness of vehicle advancement using a CVT.
Figure 4: CVT speed diagram in which the dashed line represents engine speed. The advantage of the CVT is that it allows the engine speed to remain constant thus producing steady power production through the majority of vehicle advancement.

A CVT system utilizes the same range of available gear ration as the conventional transmission. However, its design allows for any point within the range of adjustability to be used in order to maintain a single “optimal” operating engine speed instead of a range of engine speed as in the conventional transmission. In a sense, the variety of CVT gear ratios fills in the gaps of the conventional step gears. In this way, engine performance does not need to be interrupted to exchange gears because the transmission creates an automatic and a more fluid exchange. In other words, the transmission adjusts between all available points fluidly and allows engine performance to remain constant while the transmission does the work of providing seamless gear advancement. While the conventional transmission fluctuates between 6000 and 11000rpm as in Fig. 2, the CVT allows the speed of the engine to reach a value of power output determined to be effective for racing applications, 9000rpm in Fig.4, and then remain steady at this engine speed while the vehicle traverses the course. This quality is advantageous because the output power of the engine is predictable and consistent which minimizes losses in speed and power application as the transmission exchanges gears for overcoming course obstacles effectively.

Figure 5 is a diagram of the entire transmission and the regulatory components that make the changes in belt diameter and thus gear ratio possible. The arrows depict the direction of actuation of the pulleys and mechanisms involved.
Operation of the CVT is based upon the interaction of a number of regulatory mechanisms as shown in Fig. 5: engine speed, pressure spring force, flyweight mass, torque spring force, and the angle of the helical torque ramp. Together, the pulleys and the components which actuate them...
interact to create gear transfer not at points but within the following four phases: idle, low range acceleration, one to one straight shift progression, and high range or shift out.

The primary pulley which is driven by the engine contains the flyweights and pressure spring. Their interaction regulates the idle and low range phases of operation and causes the pulley to close and grip the belt during launch of the vehicle. As engine speed increases, the flyweights spin and gain centrifugal force. The flyweights push against the spider tower and pressure spring. Once engine speed reaches a determined engagement level, the force created by the flyweights is sufficient to depress the pressure spring. This action results in movement of the outside sheave of the primary pulley towards the inside sheave thereby gripping the belt and causing engagement and rotation of the transmission system in the low gear ratio range. As engine speed increases following vehicle launch, the sheaves continue to approach one another thus changing the gear ratio by raising the belt within the primary pulley.

Driven by the primary pulley, the secondary pulley contains the torque spring and helical torque ramp which regulate the splitting of the secondary pulley and occurrence of the straight shift and high range phases of operation. As the belt rises in the primary pulley and increases gear ratio, a tension force is created in the rotating secondary pulley. This force pushes against the torque spring as the belt tries to decrease its diameter in the secondary pulley. The force becomes sufficient to depress the torque spring once engine speed reaches its “optimal” value, 9000rpm in Fig. 4. At this time, the belt lowers in the secondary pulley and reaches the same diameter in both pulleys creating a one to one ratio called the straight shift. It is at this time that the CVT is most sensitive to gear adjustment in order to maintain a constant engine speed through vehicle advancement. As the system reaches the extent of its performance range and vehicle velocity approaches its limits, the belt sits highest in the primary pulley as the sheaves are almost touching and the belt sits lowest in the secondary pulley as the sheaves are far apart creating the high ratio range.

As engine speed climbs, the interaction of these mechanisms forms the respective diameters of each pulley and thus the appropriate gear ratio for useful application of power to the track during vehicle advancement. The state of the vehicle and regulatory components of the transmission during each phase is as follows:

1.) Idle
   a. Vehicle is at rest.
   b. Engine speed is below CVT engagement speed, 5000rpm in Fig. 4, and does not create enough centrifugal force in the flyweights for mechanism interaction.
   c. Flyweight force is not sufficient to actuate the pressure spring and close the primary pulley for belt engagement.
   d. Belt seats low in primary pulley and high in secondary pulley prepared for high torque, low ratio engagement shown in Figure 6.
2.) Engagement (Low Range)
   
a. Vehicle accelerates in low gear range so as to overcome standing inertia of vehicle. The output axle rotates more slowly than the input crankshaft while transferring the same power to the track creating a high torque situation for vehicle launch.

b. Engine speed is sufficient to cause flyweight and pressure spring interaction and proceeds to climb toward optimal operating speed and power output.

c. Interaction of flyweight and pressure spring forces causes primary pulley sheaves to clamp belt thus engaging the CVT system and causing vehicle acceleration.

d. Belt begins to rise in the primary pulley as engine speed increases, though the belt remains high in the secondary pulley until optimal engine speed is attained. Belt diameter is smaller in the primary pulley than in the secondary pulley through low range acceleration as shown in Fig. 7.

e. Engine speeds from engagement to optimal power output, 5000rpm and 9000rpm respectively in Fig. 4, define the period of low range gear ratio of the CVT.

---

*Figure 6: CVT in idle state prepared for low ratio vehicle launch*

*Figure 7: CVT begins low ratio operation as belt is gripped within primary pulley, system begins to rotate, and vehicle launch and acceleration occur.*
3.) Straight Shift (1:1 Acceleration)
   a. Vehicle accelerates consistently through its range of speed while engine speed remains constant and the transmission is stable in a one to one ratio.
   b. Engine speed is at optimal output and is sustainable at this level of power production.
   c. The primary pulley is mostly engaged/ closed which creates sufficient belt force to engage/ split the secondary pulley at this optimal level of engine operation.
   d. Belt diameter begins to drop in the secondary pulley. Since both pulley springs are in operation, belt diameter between the pulleys equalizes and creates the 1:1 straight shift ratio shown in Fig. 8 allowing the engine speed to remain at a single level of performance, 9000rpm in Fig. 4.
   e. The vehicle accelerates and the engagement of both pulleys allows for sensitive torque feedback. This means that as the vehicle encounters various track conditions, the transmission pulleys will automatically produce minor gear ration adjustments across their continuous range in order to maintain optimal engine speed.

![Figure 8](image-url): Belt diameter is the same between the pulleys creating the section of consistent rate of acceleration through the majority of the range of vehicle speed.

4.) Shift Out (High Range)
   a. Vehicle approaches top speed.
   b. Engine output begins to exceed the optimal value and degrades in sustainability but increases slightly in power production as will be discussed in Chapter 3.
   c. Both pulleys become fully engaged causing the belt to sit at its highest in the primary pulley and lowest in the secondary pulley thus creating the high range gear ratio shown in Fig. 9.
d. The high range gear ratio lasts from optimal engine speed to engine peak at which point the engine cannot operate any faster and the transmission is at its limit of actuation and gear exchange.

Figure 9: High gear ratio of CVT in which the output axle rotates at nearly the same speed as the input crankshaft. Torque and resistance are low while speed slowly climbs to its limits.

These same effects during transmission system operation can be visualized by a speed diagram as before. The speed diagram in Fig. 10 provides a visual example of the phase advancement described above as engine speed and vehicle speed increase. (Aaen)


From the diagram displayed in Fig. 10, the transmission phases can be extrapolated. Within range A, the idle phase is active and the vehicle is stationary. Once the engine speed at point B is attained, the flyweights depress the pressure spring and the primary pulley closes on the belt
thus engaging the system and beginning launch through range C. The range of D shows the low ratio gain of vehicle speed as the engine speed climbs towards optimal and the secondary pulley remains unengaged. Point E marks the engine speed at which the secondary pulley is engaged and belt diameters begin to equalize within the pulleys. The majority of the range of vehicle acceleration occurs through F during straight shift and again marks the most efficient period of output from the vehicle. This steady but high output period is why the CVT transmission can be more useful than conventional transmissions. Finally, G marks the high ratio range at which point both pulleys are fully engaged and as can be seen, the engine speed exceeds its optimal levels. A speed diagram as above can be helpful in visualizing the steps through which a CVT progresses, but it is only through the combination of this information with the specific engine statistics and desired vehicle dynamics that makes this system effective. The engine and transmission must act in concert and be tuned so as to utilize the performance of the engine effectively. Chapter 3 will further describe the desired relationship between these two systems so that proper tuning can be achieved and areas of concern or further refinement can be addressed appropriately to attain desired vehicle performance without compromising the rest of the tune.

Common tuning methods involve trial and error testing with track observations as the primary reasoning for tuning alterations and decisions. This tuning method can be time consuming and inconsistent due to the need to test, alter the transmission, and then retest. Furthermore, the risk of damage to the engine or vehicle may increase as engine speeds or mechanism forces exceed what is expected due to improper tunes. In racing situations, time is valuable and a proper tune is of great importance for a healthy operating vehicle and a good race. An understanding of what is to be expected in the way of engine and vehicle reaction to tunes is important.

The tuning program that follows reduces the time needed for track testing of the vehicle and avoids the risk of damage to the vehicle by providing a simulated tuning approach that describes the reasoning and mentality behind tuning adjustments. By utilizing this approach, entire tunes can be implemented off of the track and slight alterations can be accounted for prior to risking vehicle damage during operation. In addition, tuning scenarios and options can be simulated and then determined to be feasible or realistic given the resources available and outcome expected.
Chapter 3. Tuning Program
The delineation that follows will define the process required to determine what kind of dynamic vehicle profile is desired, as well as how to achieve that profile through a tune that allows the engine and transmission to co operate effectively. The goal is to gain an understanding of the relationship between engine performance and the use of that performance through gear reduction to the end of affecting vehicle dynamics. This program will not only provide the calculations needed to install an appropriate tune, but it will also supply a basis of reasoning behind the tuning goals. Chapter 4 will illustrate examples of tuning decisions and alterations made in the field based upon the dynamic and operation goals explored here. The systems and stages to be explored in this program include the engine, the transmission gearing including the secondary chain reduction, and the components that actuate gear exchange for each respective stage of operation. Once the desired engine performance and the range of available gear ratios are established, these components will be combined in formulae in order to create a relationship of vehicle performance to that of engine operation. Finally, the regulatory components of the CVT that make gear change and dynamics possible will be studied and the calculations for adjustment presented.

Evaluate Engine Performance Goals
Describe mentality for utilizing engine performance

Describe Need and Calculation of Gear Ratios

Produce Formulae for Vehicle Dynamics based upon Engine Speed and Active Gear Ratio

Describe Effect of Component Adjustment
Provide calculations for adjustment
3.1 Desired Vehicle Dynamics
In the field of competitive racing, the desired vehicle dynamics will be much different than a common daily commute vehicle. The engine will need to operate at a higher level of power production in order to achieve more rapid vehicle launch off of the line, speed recovery in corners, as well as to achieve higher vehicle speeds in the straights. Although increased power output is a large part of allowing for more aggressive driving, this power is useless if it cannot be transferred to the track effectively. The transmission must allow for quick and appropriate gear transitions so that vehicle inertia can be controlled. At launch, a vehicle must overcome its standing inertia in order to gain speed and obtain a beneficial start. If the power output is too high, the vehicle may lose traction but if it is too low, it may be just as ineffective at leaving the line. When exiting corners, vehicle momentum must be increased or re established so low gear ratios must be engaged to supply the torque necessary for increasing speed. When exiting corners, a high gear ratio may not supply the torque needed to overcome the standing inertia of the vehicle and the system may bog down or experience a period of pause as it tries to establish grip and acceleration. Likewise, as vehicle speed increases, low gear ratios limit the range of vehicle speed and must transition in order to extend this range. Engine speed and active gear ratio play the largest roles in achieving desired vehicle dynamics, but the regulatory components of the transmission make it all possible and all adjustable.
3.2 Engine Power Diagram

Proper evaluation of the power diagram of the engine in use in the vehicle is the first step to achieving the proper tune in the vehicle. Each phase of the transmission is triggered by mechanism interaction based upon engine speed and power output. If engine speeds for a desired profile are incorrectly determined, a properly tuned transmission wouldn’t matter because the engine itself may operate at a level that creates a loss of necessary power. Evaluation of the power diagram can make or break a tune. The stock power diagram for a Yamaha Phazer engine is provided in Figure 11 as an example. For practical tuning, a dynamometer test of the exact engine in use is more reliable for providing the engine power and speed data required to create a one off tune.

![Power Output vs. Engine Speed](image)

**Figure 11:** Engine Power Diagram with horsepower production against engine speed. Each slope represents a different power band.

In general, as engine speed increases, so too does the horsepower output of the engine. However, the rate of horsepower gain per increase in engine speed may vary depending upon the operating range of engine speed. The various slopes presented in Figure 11 depict this effect of varying horsepower gains which are called power bands in engine performance. Depending upon the power band in operation, the engine will produce a different level of power output as well as gain or lose that power at a different rate. For a conventional step transmission, the range of the power band remains nearly constant as the transmission advances through the gears. This effect is shown in Fig. 2 as the engine speed rises and falls between 6000rpm and 11000rpm during each gear exchange. In a daily commute car, this range would be between 1000-5000rpm in
which power production is lowered due to the reduced desire for aggressive driving as in Figure 11. This low range would allow for high fuel mileage and a steadily paced acceleration of the vehicle. In sport or racing applications where fuel efficiency is traded for higher output of power, the common range may be between 7500-9000rpm or even 9000-11000rpm as in Fig. 11. In general, a transmission that allows the engine to function at these levels of performance will experience increased torque output at lower gear ratios and increased vehicle speeds at higher gear ratios. For racing scenarios, fluctuating engine speed and engagement of appropriate gear ratios based upon speed and course obstacles can make maintaining performance cumbersome. This is where a CVT is beneficial because it simply increases engine speed no matter the power band and then remains constant during vehicle advancement. Its ability to automatically backshift to the appropriate gear ratio during corners and hills makes it much easier to manage for the driver.

For a racing application with the engine performance shown in Fig. 11 and a CVT to control that performance, the power bands become quite important in achieving the desired vehicle characteristics during the different transmission phases. Whereas the conventional transmission utilizes a single power band, the CVT can utilize the benefits of a few. For example, assuming a high performance situation, a tuner may consider an engine speed of 5000rpm for engagement and the engine speeds between 9000 and 10000rpm in Fig. 11 for the straight shift transmission phase. For engagement, 5000rpm is a good place to start because the power band is not so aggressive that the car loses traction during launch. As you may notice, once the vehicle advances through the low range, the power band chosen for the “all important” straight shift phase produces a high amount of power but also maintains a fairly level or less steep slope. This means that fluctuation in power production within this range is subtle and fairly consistent making it desirable for the straight shift stage of transmission operation. This range is also desirable because it is not the highest power band that the engine produces which means that increased power will still be available if the high range or shift out phase of the transmission is reached, and performance will be less likely to degrade as shown at about 11500rpm.
3.3 Gear Ratio Calculations
As discussed, gear ratio is just as important as engine production during establishment of desired vehicle dynamics. Once operating engine speeds are determined, it is the active gear ratio that translates the engine speed into vehicle speed and the engine power into vehicle performance. All changes in diameter between the input crankshaft and output axel or tire affect the gear ratio and thus the relationship of engine speed to vehicle speed including within the CVT, the secondary chain reduction, and between the chain reduction and output axle. Figure 3 is provided again so as to provide a visual representation of each component directly influencing active gear ratio.

This section presents the formulae necessary to determine the gear ratio of the CVT including the secondary chain reduction, or intermediate gear set, and output shaft so as to obtain a direct comparison of engine speed to vehicle speed and engine power to output performance. Appropriate calculation of gear ratio is the next step to creating a vehicle profile and effectively applying engine power to the track.

Listed in Table 1 are the parameter descriptions and notation involved in defining the gear ratios of the CVT phases and intermediate gear set. The subtext “l”, “s”, “h” refer to the active transmission phases low ratio, straight shift, and high ratio, while “P” and “S” refer to the primary and secondary pulleys respectively, and “c” and “int” refer to CVT and intermediate gearsets respectively.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Parameter</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{Pl}$</td>
<td>Radius Primary in Low Ratio/engagement</td>
<td>Inches</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>-------</td>
</tr>
<tr>
<td>(r_{ps})</td>
<td>Radius Primary in Straight Shift</td>
<td>Inches</td>
</tr>
<tr>
<td>(r_{ph})</td>
<td>Radius Primary in High Ratio</td>
<td>Inches</td>
</tr>
<tr>
<td>(r_{sl})</td>
<td>Radius Secondary in Low Ratio</td>
<td>Inches</td>
</tr>
<tr>
<td>(r_{ss})</td>
<td>Radius Secondary in Straight Shift</td>
<td>Inches</td>
</tr>
<tr>
<td>(r_{sh})</td>
<td>Radius Secondary in High Ratio</td>
<td>Inches</td>
</tr>
<tr>
<td>(n_{pint})</td>
<td>Number of teeth on primary sprocket/intermediate gear set</td>
<td>Constant in Teeth</td>
</tr>
<tr>
<td>(n_{sint})</td>
<td>Number of teeth on secondary sprocket</td>
<td>Constant in Teeth</td>
</tr>
<tr>
<td>(r_o)</td>
<td>Radius of output shaft/tire</td>
<td>Inches</td>
</tr>
<tr>
<td>(G_{cl}, G_{cs}, G_{ch})</td>
<td>Gear ratio of CVT in low, straight, and high phases</td>
<td>Unitless</td>
</tr>
<tr>
<td>(G_{int})</td>
<td>Gear ratio of intermediate gear set</td>
<td>Unitless</td>
</tr>
<tr>
<td>(G_{FL}, G_{FS}, G_{FR})</td>
<td>Final gear ratio of entire system from crankshaft to tire dependent upon phase</td>
<td>Unitless</td>
</tr>
</tbody>
</table>

**CVT**

The CVT gear ratios are determined solely by pulley diameter dependent upon the active phase. The ratio is the relationship of the secondary pulley radius to the primary pulley radius in inches creating a unitless constant. Below are the gear formulae of the CVT in each phase of transition.

The low ratio measurements are taken when the belt sits lowest in the primary pulley and highest in the secondary pulley. The measurement is from the center of the pulley to the median thickness of the belt.

\[
1. \quad G_{cl} = \frac{r_{sl}}{r_{pl}}
\]

*Equation 1: Gear Ratio of CVT in Low Ratio*

Straight shift gear ratios are always 1 at which point the belt diameter is the same in each pulley.

\[
2. \quad G_{cs} = \frac{r_{ss}}{r_{ps}}
\]

*Equation 2: Gear Ratio of CVT in Straight Shift*

High ratio measurements are taken similarly to the low ratio measurements; however, the belt is high in the primary pulley and low in the secondary pulley.

\[
3. \quad G_{ch} = \frac{r_{sh}}{r_{ph}}
\]

*Equation 3: Gear Ratio of CVT in High Ratio*
**Intermediate**

While the CVT gear ratios have a direct influence upon engine operation, the secondary chain reduction, or intermediate gear set, does not. Instead, this gear set is used in order to extend the active gear range on vehicle performance. While the CVT adjusts gear ratio, the size and diameters are usually constrained which may prevent the vehicle from behaving as desired. Alone, the CVT ratio may be too low resulting in low output speed or too high resulting in high output speeds that break traction. The constant intermediate gear set helps to adjust the CVT range to an effective level. During fine tuning, if the CVT and engine are operating in sync but the vehicle still seems slow, the intermediate gear set would be the necessary area of adjustment.

Similar to the calculation of the CVT gear ratios, the radius of the sprockets in the intermediate gear set can be used for calculation. However, a more accurate and somewhat simpler calculation involves the relationship between the number of teeth on the secondary sprocket to that of the driven sprocket on the output shaft.

\[
4. \quad G_{int} = \frac{n_{sint}}{n_{pint}}
\]

*Equation 4: Gear Ratio of Secondary Sprocket Chain Reduction*

**Total**

The final and totally encompassing gear ratio of the system is the combination of the two gear ratios above as well as the ratio between the intermediate set and the output shaft. Below is the calculation of the total gear ratio of the system in each phase of CVT engagement.

\[
5. \quad G_{Fl} = G_{cl} * G_{int} * r_o
\]

*Equation 5: Total Vehicle Gear Ratio at Low CVT Engagement*

\[
6. \quad G_{Fs} = G_{cs} * G_{int} * r_o
\]

*Equation 6: Total Vehicle Gear Ratio at Straight Shift CVT Operation*

\[
7. \quad G_{Fh} = G_{ch} * G_{int} * r_o
\]

*Equation 7: Total Vehicle Gear Ratio at High Ratio CVT Operation*

Now that engine performance from Chapter 3.2 and gear ratio information are established, a proper dynamic speed profile can be created that illustrates the conversion of engine speed into vehicle performance by way of gear reduction.
3.4 Derivation of Torque Diagram Calculations

Once the engine power diagram has been reviewed and the gear ratio measured, most of the information is known for determining the correct components for installation as well as the vehicle dynamics characteristic of the system. As has been previously described, the torque diagram simply provides a visual representation of the vehicle profile to be achieved by the integration of the engine and CVT. The purpose here is to provide the governing equations to create an accurate profile delineating the relationship between climbing engine speed and accelerating vehicle speed. It will be shown that the value of the active gear ratio and the engine speed as well as their rate of change will influence each phase in the torque diagram.

The parameter descriptions and nomenclature utilized to create the torque diagram at all phases are listed below. The subtext “in” and “out” used in the formulae refers to the input and output shafts of the system while the letters “l”, “s”, and “h” refer to low ratio, straight shift, and high ratio respectively.

Table 2: Torque Diagram Parameters for Delineation

<table>
<thead>
<tr>
<th>Notation</th>
<th>Parameter</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{int}}, P_{\text{ins}}, P_{\text{inh}}$</td>
<td>Rated horsepower of the engine</td>
<td>Horsepower (hp)</td>
</tr>
<tr>
<td>$P_{\text{outl}}, P_{\text{outs}}, P_{\text{outh}}$</td>
<td>Output power at the tires</td>
<td>Horsepower (hp)</td>
</tr>
<tr>
<td>$\tau_{\text{in}}$</td>
<td>Input torque from crankshaft</td>
<td>Foot pounds (ft-lb)</td>
</tr>
<tr>
<td>$\tau_{\text{out}}$</td>
<td>Output torque at the tires</td>
<td>Foot pounds (ft-lb)</td>
</tr>
<tr>
<td>$\omega_{\text{in}}$</td>
<td>Angular velocity of input crankshaft</td>
<td>Rotations per minute (rpm)</td>
</tr>
<tr>
<td>$\omega_{\text{out}}$</td>
<td>Angular velocity of the tires</td>
<td>Rotations per minute (rpm)</td>
</tr>
<tr>
<td>$G_F$</td>
<td>Final gear ratio of system including fixed gear reduction</td>
<td>Unitless</td>
</tr>
<tr>
<td>$v_F$</td>
<td>Velocity of the vehicle</td>
<td>Miles per Hour (mph)</td>
</tr>
</tbody>
</table>

The formulas that follow are delineated from power and torque equations and will be used to define each phase of the torque diagram necessary for depicting vehicle dynamics. The goal is to utilize the known inputs, engine power and engine speed, or angular velocity, from the power diagram as well as the gear ratio of the system, in order to define the output torque and vehicle speed at each phase.

The definition of power used here will be in the form of horsepower in which the following is true from James Watt in Machine Design:

$$8. \ 1 \text{hp} = 33,000 \frac{ft-lbf}{\text{min}}$$

Equation 8: James Watt Horsepower to Rotational Torque Relationship
In general, the input power from the engine will be equivalent to the output power at the tires. Losses due to friction and belt slip in the pulleys do occur; however, for the purposes of this study, a perfectly operating system will be assumed and these losses will be negated for these calculations.

9. \( P_{in} = P_{out} \)

Equation 9: Conservation of Power from Input to Output

As power transfers through the CVT system from engine to output shaft by means of rotation, the relationship of angular velocity in RPM between the two shafts allows the horsepower generated from the input shaft to be converted into useful torque in ft-lb at the output shaft.

10. \[ P_{in} = 33,000 \frac{ft-lbf}{min} = \tau_{in} \cdot \omega_{in} = \tau_{in} ft-lbf \cdot \omega_{in} \frac{rev}{min} \cdot \frac{2\pi \text{ rad}}{rev} \]

Equation 10: Unit Conversion Between Input Power and Input Torque Relationship

After simplification of the conversion factors, the formula is as follows:

11. \[ P_{in} = \frac{\tau_{in} \cdot \omega_{in} ft-lbf}{5252 \ min} \]

Equation 11: Simplified Conversion of Horsepower into Torque

Per equation 9, the following is also true.

12. \[ P_{in} = \frac{\tau_{in} \cdot \omega_{in} ft-lbf}{5252 \ min} = \frac{\tau_{out} \cdot \omega_{out} ft-lbf}{5252 \ min} \]

Equation 12: Equivalent conversion From Input Power to Output Torque

This power and torque relationship is caused by the change in rotational speed or angular velocity between the input and output shafts. This change, when the same amount of power is produced and transferred, causes a variation in the torque that is applied to the track. If the output axle rotates more slowly such as in low ratio engagement, more power is applied to the track per revolution of the output shaft thus creating a high torque application of power. When the output shaft rotates almost as quickly as the input shaft, there are more revolutions of the output shaft per horsepower meaning that each revolution is less powerful and produces less torque but the vehicle travels more distance. Equation 13 shows another way to rationalize gear ratio in terms of input angular velocity to output angular velocity caused by varying radii.

13. \[ G_F = \frac{\omega_{in}}{\omega_{out}} = \frac{\tau_{out}}{\tau_{in}} \]

Equation 13: Final Gear Ratio of System in Relation to Comparative Angular Velocities and Radii
When the ratio of angular velocities in equation 13 is applied to equation 12, output torque can be simplified in terms of the input torque and active gear ratio dependent upon engine speed and transmission phase.

14. $\tau_{out} = \tau_{in} \times \frac{\omega_{in}}{\omega_{out}} = \tau_{in} \times G_F$

Equation 14: Relationship of Output Torque to Input Torque Dependent upon Gear Ratio Caused by Varying Angular Velocities

By reorganizing equations 14 and 12, we obtain the output torque in terms of the known calculable input variables: input power, input angular velocity, and gear ratio of the system.

15. $\tau_{out\ ft-lb} = \frac{P_{in}hp \times G_F \times 5252 rev_{rad}}{\omega_{in_{min}}}$

Equation 15: Output Torque in terms of Known Variables, Input Power, Input Angular Velocity, Gear Ratio

Now the output torque will be solved at each phase of engagement of the CVT and thus at the various engine speeds, engine powers, and active gear ratio at vehicle engagement, optimal performance and peak performance.

16. $\tau_{eng\ ft-lb} = \frac{P_{eng}hp \times G_F \times 5252 rev}{\omega_{eng_{min}}}$

Equation 16: Output Torque at Engine Engagement Speed and Phase

17. $\tau_{opt\ ft-lb} = \frac{P_{opt\ hp} \times G_F \times 5252 rev}{\omega_{opt_{min}}}$

Equation 17: Output Torque at Optimal Engine Speed and Straight Shift Phase

18. $\tau_{peak\ ft-lb} = \frac{P_{peak\ hp} \times G_F \times 5252 rev}{\omega_{peak_{min}}}$

Equation 18: Output Torque at Peak Engine Speed and High Ratio Phase

We obtain the angular velocity of the output shaft by reorganizing equation 15 dependent upon the gear phase that the transmission is operating in. This equation obtains the values important in defining the speed diagram, namely the vehicle speed. The output angular velocity will define the vehicle speed and thus vehicle dynamics in relation to the engine speed provided in the power diagram. Each output angular velocity will be dependent upon the engine speed as well as the active phase or gear ratio of the transmission allowing vehicle profile progression to be depicted.
19. $\omega_{out\ min}^{rev} = \frac{P_{in}*G_F*5252\ rev}{\tau_{out\ ft-lb}} = \frac{\omega_{in\ min}^{rev}}{G_F}$

Equation 19: Output Angular Velocity in Terms of Input Angular Velocity and Active Gear Ratio

20. $\omega_{out\ min}^{rev} = \frac{\omega_{eng\ min}^{rev}}{G_{Fl}}$

Equation 20: Output Angular Velocity at Engine Engagement Speed and Low Gear Ratio

21. $\omega_{out\ opt\ min}^{rev} = \frac{\omega_{opt\ min}^{rev}}{G_{Fs}}$

Equation 21: Output Angular Velocity at Optimal Engine Speed and Straight Shift Gear Phase

22. $\omega_{out\ peak\ min}^{rev} = \frac{\omega_{peak\ min}^{rev}}{G_{Fh}}$

Equation 22: Output Angular Velocity at Peak Engine Speed and High Gear Ratio

The radius and angular velocity of the output shaft during each transmission phase define the torque diagram in relatable terms of engine speed to expected vehicle speed for the tuner. The general formula and the calculation of vehicle speed at each ratio phase is as follows.

23. $v_F mph = r_{out\ in} * \frac{1\ ft}{12\ in} * \frac{1\ mile}{5280\ ft} * \omega_{out\ min}^{rad} * \frac{60\ s}{min} * \frac{rev}{2\pi\ rad} * \frac{60\ min}{1\ hr}$

Equation 23: Vehicle Velocity in Terms of Output Angular Velocity and Tire Radius

Simplified, equation 23 looks as follows in general and for each phase of transmission operation:

24. $v_F mph = \frac{r_{out\ in}}{63,360\ in}\ miles * \frac{573.25\ omega_{out\ rev}}{hour}$

Equation 24: Simplified Vehicle Velocity

25. $v_F mph = \frac{r_{out\ in}^{eng}}{63,360\ in}\ miles * \frac{573.25\ omega_{out\ eng\ rev}}{hour}$

Equation 25: Vehicle Velocity During Low Ratio Engagement
26. \[ v_{FS} \text{mph} = \frac{r_{out} \text{in}}{63,360 \text{in}} \text{miles} * \frac{573.25 \omega_{out, opt} \text{rev}}{\text{hour}} \]

Equation 26: Vehicle Velocity During Straight Shift Engagement

27. \[ v_{FH} \text{mph} = \frac{r_{out} \text{in}}{63,360 \text{in}} \text{miles} * \frac{573.25 \omega_{out, peak} \text{rev}}{\text{hour}} \]

Equation 27: Vehicle Velocity at During High Ratio Operation

By utilizing the engine power diagram and equation 24, a speed diagram depicting the desired vehicle profile can be produced at every input engine speed and active gear ratio. In addition, equations 12 and 15 can be used to determine output power and torque at each engine speed and gear ratio. Now the transmission components will be solved for that will allow the vehicle to fit the profile created through the calculations above.

### 3.5 Low Ratio: Engagement Phase

Listed below are the parameters involved in defining the components active in the engagement phase of the transmission.

Table 3: Component Parameters for Low Ratio CVT Actuation

<table>
<thead>
<tr>
<th>Notation</th>
<th>Parameter</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_{PS} )</td>
<td>Force Pressure Spring</td>
<td>Pounds (lb)</td>
</tr>
<tr>
<td>( k_{PS} )</td>
<td>Stiffness of Spring</td>
<td>Pounds per Inch ( \frac{lb}{\text{in}} )</td>
</tr>
<tr>
<td>( x_{PS} )</td>
<td>Compression Length of Spring</td>
<td>Inches</td>
</tr>
<tr>
<td>( F_{fw} )</td>
<td>Force Flyweight</td>
<td>Pounds (lb)</td>
</tr>
<tr>
<td>( m_{fw} )</td>
<td>Mass Flyweight</td>
<td>Grams (g)</td>
</tr>
<tr>
<td>( v_{fw} )</td>
<td>Velocity of Flyweight</td>
<td>Feet per Second ( \frac{ft}{s} )</td>
</tr>
<tr>
<td>( r_{fw} )</td>
<td>Flyweight Radius</td>
<td>Inches</td>
</tr>
<tr>
<td>( \omega_{in} )</td>
<td>Angular Velocity of Input Shaft (Crankshaft)</td>
<td>Revolutions per Minute ( \text{rpm} )</td>
</tr>
<tr>
<td>( \omega_{eng} )</td>
<td>Engagement Speed of Engine</td>
<td>Revs per Minute ( \text{rpm} )</td>
</tr>
</tbody>
</table>

When tuning a CVT, the first component to be chosen is the pressure spring. The force and rate of the pressure spring set the engagement speed of the transmission and its rate of progression toward the next phase. By matching the flyweight force with the force of the pressure spring, one can set the engine speed at which engagement occurs. Utilizing this relationship, a flyweight mass can be determined based upon the desired engine engagement speed and the pressure spring force to overcome.

The pressure spring force is determined by Hooke’s law as in Machine Design and is commonly designated by a color code depending on the manufacturer. When it is determined that flyweight
masses are not realistic for installation, the pressure spring can be swapped out for a different force and the flyweights recalculated according to Aaen.

\[
28. \quad F_{PS}lb = k_{PS} \frac{lb}{in} \times x_{PS}in
\]

Equation 28: Pressure Spring Force in Terms of Spring Constant and Compression Length

The flyweight force acting against the pressure spring force prior to engagement is dependent upon the mass of the flyweights, the fly out radius of the weights, the engine speed which causes the flyweights to gain energy, and the number of flyweights which in this case is 3.

\[
29. \quad F = m * a = \frac{1}{2} m * v^2
\]

Equation 29: Newton’s Second Law

\[
30. \quad \frac{vf}{s} = rin * \omega \frac{rev}{min} * \frac{1min}{60s} * \frac{2\pi rev}{rev} * \frac{1ft}{12in}
\]

Equation 30: Velocity in Terms of Flyout Radius and Angular Velocity of Input Shaft

The equation that follows is delineated from Newton’s second law in equation 25 and the relationship of rotational to translational velocity in equation 26.

\[
31. \quad F_{fw}lb = \frac{1}{2} m_{fw}g * \frac{lb}{453g} \left( \frac{vf}{s} \right)^2 * n = \frac{3}{2*453} * m_{fw} * \left( \frac{\frac{2\pi f_{fw}\omega_{in}}{720} ft}{s} \right)^2
\]

Equation 31: Flyweight Force in Terms of Flyweight Mass and Rotational Velocity

The CVT becomes engaged when the flyweight force at the given engine speed overcomes the pressure spring force.

\[
32. \quad F_{PS}lb = F_{fw}lb
\]

Equation 32: Equivalence of Pressure Spring Force and Flyweight Force

At this moment when the pressure spring force and flyweight force are equivalent, engagement occurs. The pressure spring force and engine speed are known, so we solve for the appropriate flyweight mass to install with the following formula combining equations 27 and 28.

\[
33. \quad m_{fw}g = \frac{2*453*F_{PS}}{3*\left( \frac{2\pi f_{fw}\omega_{eng} ft}{720} \right)^2}
\]

Equation 33: Flyweight Mass in Terms of Pressure Spring Force and Flyweight Velocity with Conversion Factors Included

Now that the regulatory components of the engagement phase have been chosen, the profile of the speed diagram for this phase can be depicted. The engine engagement speed, power at that...
speed, and the gear ratio during this engagement phase will be used to determine the vehicle speed through the entirety of this low ratio engagement phase for use of creating the torque profile.

### 3.6 One to One Ratio: Straight Shift Phase

Once the engagement phase of the transmission has been set and a flyweight mass has been chosen, the torque spring force must be calculated for the chosen flyweight mass and optimal engine speed. This torque spring force will mark the beginning of the straight shift phase at which the transmission moves from low gear to its optimal one to one acceleration phase.

#### Table 4: CVT Component Parameters for Straight Shift

<table>
<thead>
<tr>
<th>Notation</th>
<th>Parameter</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F_{TS})</td>
<td>Force Torque Spring</td>
<td>Pounds (lb)</td>
</tr>
<tr>
<td>(F_{fw})</td>
<td>Force Flyweight</td>
<td>Pounds (lb)</td>
</tr>
<tr>
<td>(F_{b})</td>
<td>Belt Force</td>
<td>Pounds (lb)</td>
</tr>
<tr>
<td>(m_{fw})</td>
<td>Mass Flyweight</td>
<td>Grams (g)</td>
</tr>
<tr>
<td>(r_{fw})</td>
<td>Flyweight Radius</td>
<td>Inches</td>
</tr>
<tr>
<td>(\omega_{opt})</td>
<td>Optimal Speed of Engine</td>
<td>Revs per Minute (rpm)</td>
</tr>
</tbody>
</table>

The force interacting with the torque spring is the belt force defined by the flyweight force after engagement at rising engine speed.

\[
34. \quad F_{b} lb = F_{fw} lb - F_{PS} lb
\]

*Equation 34: Belt Force as Flyweight Force overcomes Pressure Spring Force*

When the belt force and torsion spring force are equivalent, the straight shift phase will begin at which point the torsion spring compresses and the secondary pulley splits reducing belt diameter and increasing gear ratio.

\[
35. \quad F_{TS} lb = F_{b} lb
\]

*Equation 35: Torque Spring Engagement as Torque Spring Force and Belt Force Approach Equivalence*

The force of the torque spring is defined by the flyweight force produced by the chosen flyweights at the optimal engine speed. Equations 27, 30, and 31 are used to synthesize the following equation.

\[
36. \quad F_{TS} lb = \frac{3}{2} \times \frac{m_{fw}}{453} \times \left(\frac{F_{fw} \times \omega_{opt} \times ft}{720 s}\right)^2 - F_{PS} lb
\]

*Equation 36: Torque Spring Force in Terms of Flyweight Mass, Operating Engine Speed, and Pressure Spring Force*
After defining the torque spring that will actuate the transmission at the optimal engine speed, the peak power and speed of the vehicle can be reached and maintained from low ratio engagement through high ratio shift out.

### 3.7 High Ratio: Shift Out Phase/ Back Shifting

There are no calculations necessary for the high ratio shift out phase so information will be provided concerning reducing vehicle speed once it has been obtained. As the calculations for the speed diagram and individual components have all been explored for advancement of the vehicle, the next step would be to establish control over the deceleration or back shifting of the vehicle. The calculations will exceed the scope of this study however the operations that occur can be described.

During deceleration, also termed “back shifting”, engine speed is largely not lost. Instead, the engine remains at the optimal engine speed and the transmission adjusts (back shifts) to a lower gear ratio appropriate for re acceleration when it is desired again. In this way, engine performance remains available and ready to overcome the track conditions and vehicle inertia and continue racing. What makes this action possible is the helical torque ramp located in the secondary pulley of the transmission. The angle of this torque ramp regulates force feedback from track conditions and reduced vehicle speeds through the secondary of the transmission. The secondary of the transmission is sensitive to this torque feedback and will close slightly thereby lowering gear ratio and reducing output shaft speed although input speed will remain mostly unchanged.

A properly tuned CVT for vehicle advancement through top speed along with the control over vehicle speed during deceleration can result in an aggressive and predictable vehicle for competition. The added efficiency of the system in maintaining engine performance makes it a lethal combination when intricate track conditions are expected. Luckily, the ease with which it can be tuned when completely understood makes for a fun tuning project with enough depth and variability to keep one working for a better tune.
Chapter 4. Simulation Results
This chapter will explore the iterative process that was used to tune the 2013 WPI FSAE vehicle and provide the mentality and reasoning behind the alterations to the previous 2012 tune.

4.1. 2012 Tune
The 2013 WPI FSAE vehicle was a tuning project that provides further insight into the consequences of transmission tuning decisions. Based upon a limited number of field observations as well as simulation, dynamic characteristics of the vehicle were assumed and the calculations and delineations provided in Chapter 3 were used in order to produce a tune with new dynamic goals. The observations and results of this process are provided so as to give an understanding of the use of the delineations provided in Chapter 3.

The first step in establishing a vehicle profile and creating a program for tuning adjustment is to determine the unchanging variables of the system as well as the desired dynamic performance. To this end, engine performance, radii measurements for gear ratio calculations and active regulatory components will be needed.

A 600cc Yamaha Phazer engine as described in the 2013 project report by Alspaugh et. al. was used for the WPI formula vehicle. The engine power diagram supplied by the manufacturer is shown in Fig. 37.

![Power Output vs. Engine Speed](image_url)
The 2012 team that originally tuned the vehicle aimed to obtain an aggressive vehicle launch by utilizing an engagement speed of 8500rpm. Their optimal engine speed was chosen to be 9000rpm and a peak speed of 11000rpm was chosen. Though the intent was laudable, the tune that was installed did not reflect the desired output performance as will be described by the field observations made in 2013. Table 5 lists the active input parameters that were used in the 2012 tune.

Table 5: 2012 Input Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine Speed at Engagement</td>
<td>$\omega_{\text{eng}}$</td>
<td>8500</td>
<td>Rpm</td>
</tr>
<tr>
<td>Engine Speed at Straight Shift</td>
<td>$\omega_{\text{opt}}$</td>
<td>9000</td>
<td>Rpm</td>
</tr>
<tr>
<td>Engine Speed at Peak</td>
<td>$\omega_{\text{peak}}$</td>
<td>11000</td>
<td>Rpm</td>
</tr>
<tr>
<td>Engine Power at Engagement</td>
<td>$P_{\text{eng}}$</td>
<td>62</td>
<td>Hp</td>
</tr>
<tr>
<td>Engine Power at Optimal Straight Shift</td>
<td>$P_{\text{opt}}$</td>
<td>68</td>
<td>Hp</td>
</tr>
<tr>
<td>Engine Power at Peak</td>
<td>$P_{\text{peak}}$</td>
<td>90</td>
<td>Hp</td>
</tr>
<tr>
<td>Low Ratio Primary Radius</td>
<td>$r_{P_l}$</td>
<td>1</td>
<td>Inches</td>
</tr>
<tr>
<td>1:1 Ratio Primary Radius</td>
<td>$r_{P_S}$</td>
<td>3</td>
<td>Inches</td>
</tr>
<tr>
<td>High Ratio Primary Radius</td>
<td>$r_{P_h}$</td>
<td>5</td>
<td>Inches</td>
</tr>
<tr>
<td>Low Ratio Secondary Radius</td>
<td>$r_{S_l}$</td>
<td>7</td>
<td>Inches</td>
</tr>
<tr>
<td>1:1 Ratio Secondary Radius</td>
<td>$r_{S_S}$</td>
<td>3</td>
<td>Inches</td>
</tr>
<tr>
<td>High Ratio Secondary Radius</td>
<td>$r_{S_h}$</td>
<td>1</td>
<td>Inches</td>
</tr>
<tr>
<td>Chain Reduction Teeth Primary</td>
<td>$n_{p\text{int}}$</td>
<td>16</td>
<td>Constant</td>
</tr>
<tr>
<td>Chain Reduction Teeth Secondary</td>
<td>$n_{s\text{int}}$</td>
<td>67</td>
<td>Constant</td>
</tr>
<tr>
<td>Output Shaft/Tire Radius</td>
<td>$r_o$</td>
<td>10</td>
<td>Inches</td>
</tr>
<tr>
<td>Pressure Spring Force</td>
<td>$F_{PS}$</td>
<td>123</td>
<td>Pounds (lb)</td>
</tr>
<tr>
<td>Flyweight Mass</td>
<td>$m_{f\text{w}}$</td>
<td>70</td>
<td>Grams (g)</td>
</tr>
<tr>
<td>Torque Spring Force</td>
<td>$F_{TS}$</td>
<td>140</td>
<td>Pounds (lb)</td>
</tr>
</tbody>
</table>
Calculations for active gear ratio, output torque, and vehicle speed will be calculated and introduced to a speed diagram so as to evaluate the effectiveness of the transmission tune through a visual representation of the dynamic characteristics.

Equations 1-7 from Chapter 3 represent the calculations for the active gear ratio in use in the vehicle. In order to obtain these values, substitute the parameters listed in Table 5.

1. \[ G_{cl} = \frac{r_{sl}}{r_{pl}} = \frac{7\text{in}}{1\text{in}} = 7 \]

   Equation 37: Gear Ratio of CVT in Low Ratio

2. \[ G_{cs} = \frac{r_{ss}}{r_{ps}} = \frac{3\text{in}}{3\text{in}} = 1 \]

   Equation 38: Gear Ratio of CVT in Straight Shift

3. \[ G_{ch} = \frac{r_{sh}}{r_{ph}} = \frac{1\text{in}}{5\text{in}} = .2 \]

   Equation 39: Gear Ratio of CVT in High Ratio

4. \[ G_{int} = \frac{n_{sint}}{n_{pint}} = \frac{67}{16} = 4.2 \]

   Equation 40: Gear Ratio of Secondary Sprocket Chain Reduction

5. \[ G_{FL} = G_{cl} \times G_{int} \times r_{o} = 7 \times 4.2 \times 10 = 294 \]

   Equation 41: Total Vehicle Gear Ratio at Low CVT Engagement

6. \[ G_{FS} = G_{cs} \times G_{int} \times r_{o} = 1 \times 4.2 \times 10 = 42 \]

   Equation 42: Total Vehicle Gear Ratio at Straight Shift CVT Operation

7. \[ G_{Fh} = G_{ch} \times G_{int} \times r_{o} = .2 \times 4.2 \times 10 = 8.4 \]

Table 6 lists the calculated gear ratios.

**Table 6: 2012 Gear Ratio Values from Calculation**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low CVT Ratio</td>
<td>(G_{cl})</td>
<td>7</td>
<td>Unitless</td>
</tr>
<tr>
<td>Straight Shift CVT Ratio</td>
<td>(G_{cs})</td>
<td>1</td>
<td>Unitless</td>
</tr>
<tr>
<td>High CVT Ratio</td>
<td>(G_{ch})</td>
<td>.2</td>
<td>Unitless</td>
</tr>
<tr>
<td>Intermediate Gear Ratio</td>
<td>(G_{int})</td>
<td>4.2</td>
<td>Unitless</td>
</tr>
<tr>
<td>Final Gear Ratio Low</td>
<td>( G_{FL} )</td>
<td>294</td>
<td>Unitless</td>
</tr>
<tr>
<td>------------------------</td>
<td>--------------</td>
<td>-----</td>
<td>---------</td>
</tr>
<tr>
<td>Final Gear Ratio</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Straight Shift</td>
<td>( G_{FS} )</td>
<td>42</td>
<td>Unitless</td>
</tr>
<tr>
<td>Final Gear Ratio High</td>
<td>( G_{FH} )</td>
<td>8.4</td>
<td>Unitless</td>
</tr>
</tbody>
</table>

Equations 16-18 from chapter 3 will be used to determine the output torque in terms of the known input power, engine speed, and gear ratio at each phase of transmission operation. The values of substitution are provided in Tables 5 and 7.

8. \[ \tau_{\text{eng}} \text{ ft}-\text{lb} = P_{\text{eng}} \frac{hp \times G_{FL} \times 5252 \text{rev}}{\omega_{\text{eng}} \frac{\text{rev}}{\text{min}}} = \frac{62hp \times 2.94 \times 5252 \text{rev}}{8500 \frac{\text{rev}}{\text{min}}} = 112.6 \text{ftlb} \]

Equation 43: Output Torque at Engine Engagement Speed and Phase

9. \[ \tau_{\text{opt}} \text{ ft}-\text{lb} = P_{\text{opt}} \frac{hp \times G_{FS} \times 5252 \text{rev}}{\omega_{\text{opt}} \frac{\text{rev}}{\text{min}}} = \frac{68hp \times 42 \times 5252 \text{rev}}{9000 \frac{\text{rev}}{\text{min}}} = 16.7 \text{ftlb} \]

Equation 44: Output Torque at Optimal Engine Speed and Straight Shift Phase

10. \[ \tau_{\text{peak}} \text{ ft}-\text{lb} = P_{\text{peak}} \frac{hp \times G_{FH} \times 5252 \text{rev}}{\omega_{\text{peak}} \frac{\text{rev}}{\text{min}}} = \frac{90hp \times 0.8 \times 5252 \text{rev}}{11000 \frac{\text{rev}}{\text{min}}} = 3.4 \text{ftlb} \]

Equation 45: Output Torque at Peak Engine Speed and High Gear Ratio

In order to obtain vehicle velocity in mph for use in a speed diagram, equations 20-22 and 25-27 from Chapter 3 must be used. The values to substitute are provided in Tables 5 and 7.

11. \[ \omega_{\text{outeng}} \frac{\text{rev}}{\text{min}} = \frac{\omega_{\text{eng}} \frac{\text{rev}}{\text{min}}}{G_{FL}} = \frac{8500 \frac{\text{rev}}{\text{min}}}{294} = 28.91 \frac{\text{rev}}{\text{min}} \]

Equation 46: Output Angular Velocity at Engine Engagement Speed and Low Gear Ratio

12. \[ \omega_{\text{outopt}} \frac{\text{rev}}{\text{min}} = \frac{\omega_{\text{opt}} \frac{\text{rev}}{\text{min}}}{G_{FS}} = \frac{9000 \frac{\text{rev}}{\text{min}}}{42} = 214.28 \frac{\text{rev}}{\text{min}} \]

Equation 47: Output Angular Velocity at Optimal Engine Speed and Straight Shift Gear Phase

13. \[ \omega_{\text{outpeak}} \frac{\text{rev}}{\text{min}} = \frac{\omega_{\text{peak}} \frac{\text{rev}}{\text{min}}}{G_{FH}} = \frac{11000 \frac{\text{rev}}{\text{min}}}{8.4} = 1,309.5 \frac{\text{rev}}{\text{min}} \]

Equation 48: Output Angular Velocity at Peak Engine Speed and High Gear Ratio
14. \[ v_{FL} \text{ mph} = \frac{r_{outin}}{63,360 in} \text{ miles} \times \frac{573.25 \omega_{outeng \text{ rev}}}{\text{hour}} = \frac{10 \times 573.25 \times 28.91}{63360} = 2.61 \text{ mph} \]  

Equation 49: Vehicle Velocity During Low Ratio Engagement

15. \[ v_{FS} \text{ mph} = \frac{r_{outin}}{63,360 in} \text{ miles} \times \frac{573.25 \omega_{outopt \text{ rev}}}{\text{hour}} = \frac{573.25 \times 214}{63360} = 19.36 \text{ mph} \]  

Equation 50: Vehicle Velocity During Straight Shift Engagement

16. \[ v_{FH} \text{ mph} = \frac{r_{outin}}{63,360 in} \text{ miles} \times \frac{573.25 \omega_{outpeak \text{ rev}}}{\text{hour}} = \frac{573.25 \times 1309.5}{63360} = 118.47 \text{ mph} \]  

Equation 51: Vehicle Velocity at During High Ratio Operation

From the calculations above, the following speed diagram represents the dynamic vehicle profile that the 2012 team was aiming for. It also depicts the profile that they actually achieved through their tune. Engine speed is plotted against vehicle speed and the values discovered in equations 14 to 16 above mark the critical points or vehicle speeds at which transmission phase changes were expected to occur.
The speed diagram in Fig. 14 shows the aggressive approach desired by the 2012 team. The tune that was implemented however did not reflect this desire and resulted in a wholly different speed diagram and a vehicle tune that was not appropriate for competition applications as is described later in the field observations.

From the figure, the 2012 goal constituted an aggressive tune defined by high launch speeds and quick attainment of optimal engine speed. Unfortunately, this approach does not produce what is desired. The high engagement speed results in loss of tire traction during vehicle launch, and the range between engagement speed and optimal engine speed does not lend enough time for the vehicle to gain velocity and momentum before the gear ratio adjusts to a low torque setting. Such a rapid transition to a low torque gear ratio causes useful power to not be effectively applied to the track for vehicle acceleration.

The actual tune that the 2012 team achieved was a result of the following calculations from Equations 31-36 from Chapter 3. Given, the pressure spring characteristics, these equations define the necessary flyweight mass and torque spring appropriate for the operating engine speeds during engagement and straight shift actuation.

\[
17. \quad F_{fw lb} = \frac{3}{2*453} \times m_{fw} \times \left(\frac{2\pi f_{fw} \times \omega_{in} \times f_t}{720}\right)^2
\]

*Equation 52: Flyweight Force in Terms of Flyweight Mass and Rotational Velocity*
18. \[ F_{PS} lb = F_{fw} lb \]

**Equation 53: Equivalence of Pressure Spring Force and Flyweight Force**

The input engine speed and desired pressure spring force to attain are known from Table 5. Reorganization of Equations 31 and 32 above define the desired flyweight mass for transmission engagement at the desired engine speed of 8500rpm.

19. \[ m_{fw} g = \frac{2 \times 453 \times F_{PS}}{3 \times \left(\frac{2 \pi r_{fw} \omega_{eng} ft}{720}ight)^2} = \frac{2 \times 453 \times 123}{3 \times 8500} = 46 \text{ grams} \]

**Equation 54: Flyweight Mass in Terms of Pressure Spring Force and Flyweight Velocity with Conversion Factors Included**

Installed for the 2012 tune were 70g flyweights which result in an engagement speed found in the following calculation and reflected by the actual dynamic profile in Fig. 14.

20. \[ \omega_{eng} = \frac{2 \times 453 \times F_{PS}}{3 \times m_{fw} g \left(\frac{2 \pi r_{fw} \omega_{eng} ft}{720}ight)^2} = 1,827 \text{ rpm} \]

For straight shift operation, the 65% increase in flyweight mass results in a torque spring half as stiff as necessary to actuate the secondary pulley at the desired 9000 rpm. Instead, the torque spring compresses at 4000rpm as shown in Fig. 14.

21. \[ F_{b} lb = F_{fw} lb - F_{PS} lb \]

**Equation 55: Belt Force as Flyweight Force overcomes Pressure Spring Force**

22. \[ F_{TS} lb = F_{b} lb \]

**Equation 56: Torque Spring Engagement as Torque Spring Force and Belt Force Approach Equivalence**

23. \[ F_{TS} lb = \frac{3}{2} \times \frac{m_{fw}}{453} \times \left(\frac{r_{fw} \omega_{opt} ft}{720}\right)^2 - F_{PS} lb \]

**Equation 57: Torque Spring Force in Terms of Flyweight Mass, Operating Engine Speed, and Pressure Spring Force**

Table 7 lists the components of the components that should have been installed to achieve the desired 2012 tune if the flyweights had been changed followed by if the spring forces had been changed.

**Table 7: 2012 Active Adjustable Components**

<table>
<thead>
<tr>
<th>Component</th>
<th>Notation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure Spring Force</td>
<td>( F_{PS} )</td>
<td>123</td>
<td>Pounds (lb)</td>
</tr>
<tr>
<td>Flyweight Mass</td>
<td>( m_{fw} )</td>
<td>46</td>
<td>Grams (g)</td>
</tr>
<tr>
<td>Torque Spring Force</td>
<td>( F_{TS} )</td>
<td>140</td>
<td>Pounds (lb)</td>
</tr>
</tbody>
</table>
Pressure Spring Force \( F_{ps} \) | 320 | Pounds (lb) 
Flyweight Mass \( m_{fw} \) | 70 | Grams (g) 
Torque Spring Force \( F_{ts} \) | 350 | Pounds (lb) 

Observations of the dynamic operation of the improperly tuned 2012 FSAE vehicle elicited a number of inefficiencies that made the vehicle perform unpredictably and unfit to meet competition demands. Field observations were made as follows:

- Vehicle had a slow launch which appeared to lack aggressive application of power to the track.
- Engine audibly bogs down during initial engagement leading to poor take off and acceleration characteristics
  - The flyweight mass is too high causing the primary pulley to close at lower engine speeds and grip the belt. When this happens, the engine does not produce enough power to overcome the standing inertia of the vehicle and the gear ratio becomes too high for torque to be effectively applied to the track.
  - When the CVT engages at too low of an engine speed, extra time and power are required to raise the speed of the system to efficient operating standards thus rendering the tune incompatible.
  - Although the power curve used by the 2012 tune depicts an aggressive engine engagement profile, the transition of this power through the CVT system is not supported by the components in use.
- Effective power band for straight shift operation is not reached as observed through slow vehicle advancement and low top speed.
  - The high flyweight masses result in engagement of the secondary pulley too soon thus utilizing an engine speed for straight shift that is lower than optimal. Power production is decreased and dynamic potential is lost. Lower power production and lower engine speeds translate into slower advancement of the vehicle toward top speed as well as a limit on the top speed that can be reached. Lower flyweight masses or an increased torque spring force will delay the secondary pulley engagement thus increasing the range of engine performance in use.
- Back shifting control is limited leading to loss of effective power and acceleration recovery or responsiveness following course obstacles such as deceleration and turns.
  - The flyweight masses and torque spring characteristics are also the leading causes in inefficient back shifting control. As the flyweight masses are too high, when the vehicle reduces speed, the torque spring stiffness is not sufficient to down shift the transmission and reduce the gear ratio. Due to this, the gear ratio remains too high to transfer needed torque to the track for vehicle speed recovery.
Per the observations and the supporting speed profile that was achieved in Fig. 14, improper flyweight mass led to premature transmission actuation during engagement and straight shift. This caused the vehicle to have poor launch characteristics and to utilize a low engine speed with poor power production during the majority of vehicle advancement. The top speed was limited because the high ratio transmission phase was active early during operation at low engine speeds.

Neither the desired tune nor the achieved tune for the 2012 set up was fit for competition demands. The 2013 tune aimed to establish a more effective dynamic profile as well as achieve that profile through installation of the appropriate components.
4.2 2013 Tune

The 2013 WPI FSAE tune hopes to achieve an effectively operating vehicle fit for competition by being more conservative in its approach to aggressive operation. A conservative tune that operates smoothly and in a predictable manner is more effective and useful than an aggressive tune that leads to uncontrolled operation and inefficiencies. Table 8 lists the new input parameters for the 2013 tune.

**Table 8: 2013 Input Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine Speed at Engagement</td>
<td>$\omega_{\text{eng}}$</td>
<td>5000</td>
<td>Rpm</td>
</tr>
<tr>
<td>Engine Speed at Straight Shift</td>
<td>$\omega_{\text{opt}}$</td>
<td>9000</td>
<td>Rpm</td>
</tr>
<tr>
<td>Engine Speed at Peak</td>
<td>$\omega_{\text{peak}}$</td>
<td>11000</td>
<td>Rpm</td>
</tr>
<tr>
<td>Engine Power at Engagement</td>
<td>$P_{\text{eng}}$</td>
<td>40</td>
<td>Hp</td>
</tr>
<tr>
<td>Engine Power at Optimal Straight Shift</td>
<td>$P_{\text{opt}}$</td>
<td>68</td>
<td>Hp</td>
</tr>
<tr>
<td>Engine Power at Peak</td>
<td>$P_{\text{peak}}$</td>
<td>90</td>
<td>Hp</td>
</tr>
<tr>
<td>Low Ratio Primary Radius</td>
<td>$r_{Pl}$</td>
<td>1</td>
<td>Inches</td>
</tr>
<tr>
<td>1:1 Ratio Primary Radius</td>
<td>$r_{Ps}$</td>
<td>3</td>
<td>Inches</td>
</tr>
<tr>
<td>High Ratio Primary Radius</td>
<td>$r_{Ph}$</td>
<td>5</td>
<td>Inches</td>
</tr>
<tr>
<td>Low Ratio Secondary Radius</td>
<td>$r_{Sl}$</td>
<td>7</td>
<td>Inches</td>
</tr>
<tr>
<td>1:1 Ratio Secondary Radius</td>
<td>$r_{Ss}$</td>
<td>3</td>
<td>Inches</td>
</tr>
<tr>
<td>High Ratio Secondary Radius</td>
<td>$r_{Sh}$</td>
<td>1</td>
<td>Inches</td>
</tr>
<tr>
<td>Chain Reduction Teeth Primary</td>
<td>$n_{pint}$</td>
<td>13</td>
<td>Constant</td>
</tr>
<tr>
<td>Chain Reduction Teeth Secondary</td>
<td>$n_{sint}$</td>
<td>42</td>
<td>Constant</td>
</tr>
<tr>
<td>Output Shaft/Tire Radius</td>
<td>$r_{o}$</td>
<td>10</td>
<td>Inches</td>
</tr>
<tr>
<td>Pressure Spring Force</td>
<td>$F_{PS}$</td>
<td>123</td>
<td>Pounds (lb)</td>
</tr>
</tbody>
</table>

The most notable change in the input parameters is the engagement speed which is 5000rpm and produces 40hp. The 2012 team aimed to engage at 8500rpm producing 62hp but ended up with a speed of 2200rpm and a power production of 15hp. Another alteration that significantly impacts the vehicle profile is the alteration of the intermediate gear set sprockets. The change in teeth
number drastically lowers the final gear ratio at each transmission phase as shown in the calculations of equations 4-7 below. This effect, as will be noticed in the resulting speed diagram, will alter the vehicle profile by reducing torque production and thus the slope of vehicle acceleration but extend the overall range of vehicle top speed.

1. \( G_{cl} = \frac{r_{sl}}{r_{pl}} = \frac{7\text{in}}{1\text{in}} = 7 \)

Equation 58: Gear Ratio of CVT in Low Ratio

2. \( G_{cs} = \frac{r_{ss}}{r_{ps}} = \frac{3\text{in}}{3\text{in}} = 1 \)

Equation 59: Gear Ratio of CVT in Straight Shift

3. \( G_{ch} = \frac{r_{sh}}{r_{ph}} = \frac{1\text{in}}{5\text{in}} = .2 \)

Equation 60: Gear Ratio of CVT in High Ratio

4. \( G_{int} = \frac{n_{sint}}{n_{pint}} = \frac{42}{13} = 3.23 \)

Equation 61: Gear Ratio of Secondary Sprocket Chain Reduction

5. \( G_{FL} = G_{cl} * G_{int} * r_o = 7 * 3.23 * 10 = 226.1 \)

Equation 62: Total Vehicle Gear Ratio at Low CVT Engagement

6. \( G_{FS} = G_{cs} * G_{int} * r_o = 1 * 3.23 * 10 = 32.3 \)

Equation 63: Total Vehicle Gear Ratio at Straight Shift CVT Operation

7. \( G_{FH} = G_{ch} * G_{int} * r_o = .2 * 3.23 * 10 = 6.46 \)

Table 9 lists the above gear ratio calculations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low CVT Ratio</td>
<td>( G_{cl} )</td>
<td>7</td>
<td>Unitless</td>
</tr>
<tr>
<td>Straight Shift CVT Ratio</td>
<td>( G_{cs} )</td>
<td>1</td>
<td>Unitless</td>
</tr>
<tr>
<td>High CVT Ratio</td>
<td>( G_{ch} )</td>
<td>.2</td>
<td>Unitless</td>
</tr>
<tr>
<td>Intermediate Gear Ratio</td>
<td>( G_{int} )</td>
<td>3.23</td>
<td>Unitless</td>
</tr>
<tr>
<td>Final Gear Ratio Low</td>
<td>( G_{FL} )</td>
<td>226.1</td>
<td>Unitless</td>
</tr>
<tr>
<td>Final Gear Ratio</td>
<td>( G_{FS} )</td>
<td>32.3</td>
<td>Unitless</td>
</tr>
</tbody>
</table>
Straight Shift

| Final Gear Ratio High | $G_{Fh}$ | 6.46 | Unitless |

With the change in active gear ratio, the torque produced by the 2013 tune is less than that of the 2012 tune at each transmission phase. This means that the possibility of a higher rate of vehicle acceleration is lost but the likelihood of maintaining tire traction is improved and vehicle control is improved. As will be discovered, the reduction in torque also extends the possible range of vehicle speed.

8. $\tau_{eng \ ft-lb} = \frac{P_{eng \ hp} \cdot G_{FL} \cdot 5252 \ rev}{\omega_{eng \ min} \ rev} = \frac{40 \ hp \cdot 2.26 \cdot 5252 \ rev}{5000 \ rev} = 95 \ ftlb$

Equation 64: Output Torque at Engine Engagement Speed and Phase

9. $\tau_{opt \ ft-lb} = \frac{P_{opt \ hp} \cdot G_{FS} \cdot 5252 \ rev}{\omega_{opt \ min} \ rev} = \frac{68 \ hp \cdot 32 \cdot 5252 \ rev}{9000 \ rev} = 12.7 \ ftlb$

Equation 65: Output Torque at Optimal Engine Speed and Straight Shift Phase

10. $\tau_{peak \ ft-lb} = \frac{P_{peak \ hp} \cdot G_{Fh} \cdot 5252 \ rev}{\omega_{peak \ min} \ rev} = \frac{90 \ hp \cdot .06 \cdot 5252 \ rev}{11000 \ rev} = 2.79 \ ftlb$

Equation 66: Output Torque at Peak Engine Speed and High Ratio Phase

Though torque is lost in the new tune during engagement, 25 percent higher speeds are attained during the straight shift and high ratio phases as shown in equations 15 and 16 below.

11. $\omega_{outeng \ rev \ min} = \frac{\omega_{eng \ min \ rev}}{G_{FL}} = \frac{5000 \ rev}{226} = 22.12 \ rev \ min$

Equation 67: Output Angular Velocity at Engine Engagement Speed and Low Gear Ratio

12. $\omega_{outopt \ rev \ min} = \frac{\omega_{opt \ rev \ min}}{G_{FS}} = \frac{9000 \ rev}{32.3} = 278.63 \ rev \ min$

Equation 68: Output Angular Velocity at Optimal Engine Speed and Straight Shift Gear Phase

13. $\omega_{outpeak \ rev \ min} = \frac{\omega_{peak \ rev \ min}}{G_{Fh}} = \frac{11000 \ rev}{6.5} = 1692.3 \ rev \ min$

Equation 69: Output Angular Velocity at Peak Engine Speed and High Gear Ratio
14. \( v_{FL} \text{mph} = \frac{r_{out \text{in}}}{63,360 \text{in}} \text{miles} \times \frac{573.25 \omega_{out \text{eng rev}}}{10 \times 573.25 + 22.12} \frac{\text{hour}}{63360} = 2 \text{mph} \)

Equation 70: Vehicle Velocity During Low Ratio Engagement

15. \( v_{FS} \text{mph} = \frac{r_{out \text{in}}}{63,360 \text{in}} \text{miles} \times \frac{573.25 \omega_{out \text{opt rev}}}{\text{hour}} = \frac{573.25 \times 278.6}{63360} = 25.2 \text{mph} \)

Equation 71: Vehicle Velocity During Straight Shift Engagement

16. \( v_{FH} \text{mph} = \frac{r_{out \text{in}}}{63,360 \text{in}} \text{miles} \times \frac{573.25 \omega_{out \text{peak rev}}}{\text{hour}} = \frac{573.25 \times 1692.3}{63360} = 153.11 \text{mph} \)

Equation 72: Vehicle Velocity at During High Ratio Operation

The speed diagram in Figure 15 depicts the 2013 dynamic profile and its desire to operate between the 2012 tuning goals and the implemented tune.

![Figure 15: 2013 Dynamic Profile in Comparison to Previous Year. 2013 is shown in red.](image-url)
As previously described, the engagement speed of the 2013 tune is reduced from the 2012 tune and the rate of gain in engine speed between engagement and straight shift is reduced as shown in Fig. 15. This however does result in more fluid and controlled launch and acceleration of the vehicle by allowing the CVT to more slowly adjust gear ratio. The extended range of the vehicle’s speed is also noticeable in Fig. 15. Though this in itself is an interesting quality, in racing situations, it is unlikely that the vehicle will encounter an opportunity to reach such high speeds. Future tuning adjustments may consider altering the intermediate gear set again so as to trade some of the unusable vehicle speed for increased torque and improved acceleration characteristics.

In order to achieve the tuning profile described above and shown in Fig. 15, the following calculations for component characteristics are used. As in table 8, the pressure spring force is unchanged from the previous tune while the engine engagement speed is changed so as to elicit a new flyweight mass.

\[
m_{fw}g = \frac{2 \times 453 \times F_{PS}}{3 \times (\frac{2 \pi r_{fw} \omega_{eng} ft}{720})^2} = \frac{2 \times 453 \times 123}{3 \times 5000} = 55 \text{grams}
\]

Equation 73: Flyweight Mass in Terms of Pressure Spring Force and Flyweight Velocity with Conversion Factors Included

During straight shift, the 22% decrease in flyweight mass while the optimal engine speed remains the same results in a comparable increase in torque spring force as shown by the full calculation below.

\[
F\text{TS} lb = \frac{3}{2} \times \frac{m_{fw}}{453} \times \left(\frac{r_{fw} \omega_{opt} ft}{720}\right)^2 - F_{PS} lb = 158 \text{pounds}
\]

Equation 74: Torque Spring Force in Terms of Flyweight Mass, Operating Engine Speed, and Pressure Spring Force

Table 9 lists the regulatory components that were installed to achieve the tuning profile desired by the 2013 team and depicted in Fig. 15.

<table>
<thead>
<tr>
<th>Component</th>
<th>Notation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure Spring Force</td>
<td>$F_{PS}$</td>
<td>123</td>
<td>Pounds (lb)</td>
</tr>
<tr>
<td>Flyweight Mass</td>
<td>$m_{fw}$</td>
<td>55</td>
<td>Grams (g)</td>
</tr>
<tr>
<td>Torque Spring Force</td>
<td>$F_{TS}$</td>
<td>158</td>
<td>Pounds (lb)</td>
</tr>
</tbody>
</table>

At the end of the day, the new 2013 tune corrects the previous bogging problems by reducing flyweight mass thereby increasing the active engagement speed of the system which allows for higher power production and attainment of the desired engagement speed. A reduced overall gear ratio decreases torque production slightly reducing the rate at which vehicle speed is gained but increasing the top speed range and smooth vehicle control. Further tuning is always possible to continually optimize the dynamic operation of the vehicle for changing course conditions.
Chapter 5. Conclusion

Continuously variable transmissions can be an effective gear reduction systems when applied to racing applications due to the wide range of adjustability as well as the ease with which they can be tuned. This study was performed to provide tuners with an understanding of the CVT tuning methodology so as to allow them to make substantiated tuning decisions quickly for competition applications. With the tuning program provided here as a basis to begin with, an accurate tuning profile can be produced which provides a preemptive dynamic strategy for competition and recommendations for components to realize that strategy.

The scope of this study was limited to CVT tuning for common component adjustments and tuning decisions for competition applications. Further studies may consider application of this approach for use of a CVT in common road vehicles which utilize a lower but constant engine speed to attain higher fuel efficiency. In order to achieve further adjustment and control over the engagement and back shifting characteristics of the vehicle, future studies may also consider delving into the adjustment of flyweight profiles as well as delineation of the helical torque ramp calculations. Alterations to these components would act to extend the range of variability in engagement aggressiveness as well as the responsiveness of torque feedback while allowing the tuner to maintain the previous tune characteristic of the extent of this study.

Always remember that tuning is a process that never ends and constantly develops the effectiveness of the vehicle in operation. Simulation facilitates this process by narrowing the range of desired tuning adjustments and reducing time, but the gain attained from field operation and observations makes the process of tuning fun and rewarding.
References


