Error Analysis of a Precision Indoor Positioning System

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BIOGRAPHIES

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Daniel Breen Jr. recently completed his MS degree in Electrical Engineering at WPI where he served as a research assistant in the WPI Convergent Technology Center sponsored by the NIJ precision personnel locator project. His thesis examined multi-carrier locator performance in noise and with mismatched system clocks. Mr. Breen is currently employed by the Naval Undersea Warfare Center where he works on missile simulation systems for submarines.

Benjamin Woodacre recently completed his MS degree in Electrical Engineering at WPI, where he served as a research assistant in the WPI Convergent Technology Center sponsored by the NIJ precision personnel locator project. His thesis examined variation of TDOA location algorithm performance as a function of geometry and techniques for system auto-calibration. Mr. Woodacre is currently entering the Ph.D. program at WPI.

ABSTRACT

This paper quantifies the positioning accuracy of a new approach to indoor navigation, particularly with regard to errors in mutual clock synchronization and errors in knowledge of the locations of the reference stations. Two previous papers have described the overall system architecture and signaling format, and have developed analytical estimates of positioning accuracy in terms of transmitted signal power, bandwidth, and system geometry. This contribution quantifies the effects of the important aspects of clock misalignment and errors in the results of the algorithm by which the reference stations autonomously establish their mutual relative positions upon arrival and setup of the positioning system. The system performance is shown to be well-behaved with respect to timing errors. Data quantifying the performance with regard to both timing and reference positioning errors are presented.

INTRODUCTION

A novel means for precision personnel location based upon a multi-carrier signal constructed in a manner similar to orthogonal frequency division multiplex (OFDM) was described in a previous paper [1]. The individual carriers span a wide bandwidth (with very low occupancy of the total band), leading to the designation Multi-carrier Ultra-wideband (MC-UWB) for this technique. The MC-UWB signal structure and modern spectral analysis solution technique offer significant
advantages for the problem domain of precision personnel location. This signal structure offers high precision location at a cost of essentially infinitesimal bandwidth thanks to its sparse line spectral content. The ability of modern spectral analysis to determine the frequencies of arbitrarily-spaced components of a waveform translates into the ability to obtain a super-resolution solution for time difference of arrival information from a signal despite large-amplitude multi-path components.

This system is intended for precision, real-time personnel location for emergency first responders inside buildings. As is well-known, this environment is extremely challenging, but such a system would meet an urgent need. The problem to be solved is distinguished from the GPS situation in several ways, some of which simplify the problem (such as the relatively small range of operation and the restricted number of simultaneous users) and some of which complicate the situation (such as the indoor, multi-path environment, the need for precision on the order of 10 cm, the need for low overall system cost, and the ad hoc nature of each response situation).

Alternative approaches to precision indoor positioning in multi-path-rich environments have often proposed the use of pulse-type ultra-wideband signals. However, the spectral footprint of such signals is difficult to control and presents serious spectral allocation and/or inter-service interference problems. The signal structure and TDOA (time difference of arrival) recovery approach of this paper avoids these problems with a ranging signal that is amenable to spectral assignment. The result is an approach which retains the spatial precision and multi-path immunity of the time domain ultra-wideband solution while allowing control of the spectral space required.

The simple form that the mobile node takes, that of a transmitter of a single, periodic signal with no time synchronization requirements, immensely lowers the cost of equipping personnel and materiel as compared with systems that require complex receivers or transceivers. The entire system is amenable to simple software radio implementation as demonstrated by a prototype implemented acoustically. Since the signal structure and system implementations with an audio channel are one-to-one with those used for RF OFDM communications systems, this opens the opportunity to integrate precision location into existing OFDM systems and/or to provide OFDM communications channels in any such realization of a precision locator.

Previously [2], system performance aspects of the Multi-carrier Ultra-wideband approach to positioning were presented. The analysis of system performance as a function of the major parameters, including bandwidth, frequency component spectral spacing, transmitted signal power, and range were presented. These results facilitate the design of MC-UWB positioning systems for specific environments.

In this paper we examine the performance of multi-carrier based TOA estimation error as a function of transmitter and receiver frequency mismatch. Also, preliminary results of our research regarding automatic receiver location calibration are presented.

**MULTI-CARRIER SIGNAL STRUCTURE**

As described in the previous papers [1,2], the new method is based upon transmission of a continuous multi-carrier signal of the form

\[ s_c(t) = \sum_{m=0}^{M-1} A e^{2\pi j (f_0 + m\Delta f) t \pm \phi_m} \]

comprising M sinusoidal carriers with frequency spacing \( \Delta f \) and arbitrary phases \( \phi_m \). For simplicity we will consider only the case of baseband signal generation at the transmitter and a direct conversion (similarly baseband) receiver. Discrete Fourier Transform (DFT) processing may be used to implement this architecture. Let \( N \) be the number of time samples that are generated by the transmitter’s inverse DFT (IDFT) and then repeatedly transmitted to form the continuous output waveform and \( f_s \) be the sampling rate. Of course, the DFT may be implemented via a Fast Fourier Transform (FFT) algorithm.

Furthermore, we will consider only the case of real signals and hence describe our signal by

\[ s(t) = s_c(t) + s_c^*(t) \]

\[ = \sum_{m=0}^{M-1} A \cos(2\pi (M_0 + mK)\delta f t + \phi_m) \]

where we can express the parameters of the previous equations in terms of the sampled signal system as \( \Delta f = K\delta f \) and \( f_0 = M_0\delta f \), where \( \delta f = \frac{f_s}{N} \) is the DFT frequency sample separation.

If this signal is received at reference sites, with distances \( d_k \) from the source, giving rise to propagation delays \( \tau_k = d_k/c \), then each carrier component is shifted by a phase shift which depends both upon \( \tau_k \) and the carrier’s frequency.

It was previously shown [1] that if we assume that the reference sites have synchronized clocks, while the source clock has some unknown offset \( t_0 \), this offset induces
another unknown phase shift dependent on carrier index

$$\psi_m = 2\pi(f_0 + m\Delta f)t_0.$$ 

Let’s now assume that the sampled received signal plus noise at the $k_{th}$ reference is of the form

$$r_k(n) = s_k(n) + n_k(n)$$

for $n = 0..N-1$ with DFT

$$R_k(m) = S_k(m) + N_k(m).$$

The phase difference between adjacent carriers for the signal received at reference $k$, $S_k(M_0 + mK)$ for $m = 0..M-1$, corrected for the known phases $\phi_m$ (Eq. 1) satisfies

$$\Delta \theta_k = \Phi_{mk} - \Phi_{m-1,k} - \phi_m + \phi_{m-1}$$

$$= -2\pi\Delta f \tau_k + \psi_m - \psi_{m-1}$$

$$= -2\pi\Delta f \tau_k + m\Delta \psi$$

modulo $2\pi$.

Finally, the difference of the phases obtained as above for carrier $m$ at two sites, $q$ and $r$, is $\theta_{qr} = \Delta \theta_r - \Delta \theta_q$, from which we can recover the Time Difference of Arrival (TDOA) of the signals at those sites,

$$\Delta \tau_{qr} = -\theta_{qr} - 2\pi\Delta f = \frac{d_2 - d_1}{c}$$

where $c$ is the velocity of the wave in our medium.

Now, since our signal is periodic, with period $T = \frac{1}{\Delta f}$, the solution suffers a time (range) aliasing ambiguity. However, thanks to the limited spatial scope of the location problem we defined, by choosing $\Delta f$ sufficiently small, we can make our TDOA solution unambiguous throughout a ranging cell which is defined by the locus of points within distance $R = \frac{c}{2\Delta f}$ of any receiving site.

Thus the problem of TDOA estimation for this signal reduces to the estimation of the value of phase change between adjacent carriers in the received signal, $\Delta \theta_k$ that best fits the given noisy data. That is, the phasor amplitudes of the $M$ carriers (after compensation for the known phases $\phi_m$) have the form $Be^{2\pi\tau f_0}$ and we seek the frequency sample phase progression coefficient $\Omega = \Delta \theta_k$. This problem is effectively a sinusoidal frequency estimation problem applied to the phasor valued signal spectrum.

The problem of estimating the frequencies of multiple, arbitrarily spaced, sinusoidal signals from a noisy linear combination has a long history. Modern analytic and computational methods are now known collectively as Modern Spectral Analysis [3]. The approach implemented in this work follows the State Space Estimator approach first outlined by Rao and Arun [4].

**SAMPLING CLOCK FREQUENCY SKEW**

For purposes of constructing a practical system based upon the TDOA recovery method outlined above, some considerations outside ideal signal theory must enter. The theoretical treatment, above and in preceding papers, treats the sampling clocks used for the discrete signal generation in the transmitter and the sampling clocks in all the receivers as synchronized. However, to reduce the cost, weight, size and power consumption of the transmitter, we wish to avoid the inclusion of a receiver in the mobile device that would be needed to achieve synchronization with some master clock reference signal. If the transmitters simply transmitted a periodic signal without external synchronization, the required signal generation system is reduced to a waveform-bearing read-only memory, D/A converter and associated clock, RF modulator and amplifier stages.

Synchronization of the receivers to a single master reference is easily accomplished and given their small number, stationary nature and size insensitivity (due to vehicular mounting,) also easily accommodated in terms of cost, size and power consumption.

The question that must be answered is whether the lack of synchronization between a transmitter’s sampling clock and that of the synchronized receivers has a significant impact? In this section we will examine this analytically from a perturbation analysis and then experimentally to explore the limits of the perturbation approach.

When the transmitter and receiver clocks are perfectly matched, then the transmitted carriers will fall at the center of the receiver’s carrier channels as illustrated in Fig. 1. Frequency skew results if the frequency of the transmitter’s sampling clock and the receiver’s sampling clock are not exactly the same, but differ by some factor, $\gamma$. In this circumstance, the spacing between received signal carriers exhibits a stretching by $\gamma = 1 + \varepsilon$ such that the $n^{th}$ carrier is offset from the receiver channel centers by $n\varepsilon\Delta \omega$ as depicted in Fig. 2.

**PERTURBATION ANALYSIS**

To prosecute our analytic perturbation analysis of the effect of sampling clock frequency skew we shall make a number of simplifying assumptions, without loss of
generality. We shall assume a baseband signal with carriers occupying only a subset of the possible carrier frequencies between \( n_1 \delta \omega \) and \( n_2 \delta \omega \), where \( n_1 > 0 \) and \( n_2 < \frac{1}{2} - 1 \). We shall also assume a channel gain of unity for any placement of the transmitter.

Given that the sampling clock frequency at the transmitter is skewed by a factor, \( \gamma \), and the propagation delay is \( t_0 \), we obtain the continuous time, multi-carrier signal at the receiver:

\[
s(t) = \sum_{n=n_1}^{n_2} A_n e^{j \gamma \delta \omega (t-t_0) + \phi_n} \quad (10)
\]

\[
= \sum_{n=n_1}^{n_2} \frac{A_n}{2} \left[ e^{j(\gamma \delta \omega (t-t_0) + \phi_n)} + e^{-j(\gamma \delta \omega (t-t_0) + \phi_n)} \right] \quad (11)
\]

Now let

\[
A_n = \begin{cases} 
\frac{A e^{i \phi_n}}{2} & \text{if } n > 0 \\
\frac{A e^{-j \phi_n}}{2} & \text{if } n < 0
\end{cases} \quad (12)
\]

and

\[
T = \frac{1}{N \delta f} = \frac{2\pi}{N \delta \omega} \quad (13)
\]

then the time sampled signal \( s(m) = s(mT) \) can be expressed as

\[
s(t) = \sum_{n=n_1}^{n_2} \tilde{A}_n e^{j 2 \pi \frac{m \gamma \tau}{N T}} \quad (14)
\]

where \( \tau = \frac{t_0}{N T} \).

Our receiver forms the DFT of this signal to extract the carrier phase information as described earlier. Again, to simplify the manipulations but with no loss of generality, we shall consider the DFT of \( \text{N} \) time samples distributed so that sample zero in a centered DFT time index system lies at \( t = 0 \). In this case the DFT sequence is given by

\[
S(k) = \sum_{m=-\frac{N}{2}+1}^{\frac{N}{2}} \sum_{n=n_1}^{n_2} \tilde{A}_n e^{j 2 \pi \frac{m \gamma \tau}{N T}} e^{-j \frac{2 \pi m n \gamma}{N}}. \quad (15)
\]

To obtain a perturbation analysis we truncate all terms of the Taylor series expansion in epsilon of the exponential factor free of \( \tau \) above first order, yielding,

\[
S(k) = \sum_{m=-\frac{N}{2}+1}^{\frac{N}{2}} \sum_{n=n_1}^{n_2} \tilde{A}_n \left[ 1 + j \frac{2 \pi n m \epsilon}{N} \right] e^{j 2 \pi \frac{m n \epsilon}{N} - j \frac{2 \pi m n \gamma}{N}} \quad (16)
\]

which can be placed in the form

\[
S_0(k) + S_1(k) = N \tilde{A}_n e^{-j \frac{2 \pi n \gamma}{N}} + \frac{j 2 \pi \epsilon}{N} \sum_{m=-\frac{N}{2}+1}^{\frac{N}{2}} \sum_{n=n_1}^{n_2} n m \tilde{A}_n e^{j 2 \pi \frac{m n \epsilon N}{N}} e^{-j \frac{2 \pi m n \gamma}{N}}. \quad (17)
\]

This last equation has two components, a zero order term which matches that of a clock matched recovery of carrier amplitudes with a linear phase factor proportional to time delay scaled by \( \gamma \), and, another perturbation term that scales as \( \epsilon \). Thus, to order zero in \( \epsilon \), the time delays estimated by the receivers, \( \hat{t}_0 \) are related to the true time delays \( t_0 \) by

\[
\hat{t}_0 = \gamma \tau \text{NT} = \gamma t_0. \quad (18)
\]

The first order perturbation results in a distortion of the recovered carrier amplitudes which is a periodic function of the propagation delay, \( t_0 \). That this scales gradually with increasing epsilon and has period \( P_{\text{dist}} = \frac{N \tau}{\gamma} \) is clear by noting that \( \gamma \) periods of the transmitted signal fit inside a single \( NT \) receiver sample window period and will lie in this interval in precisely the same fashion for a

**Figure 1.** When transmitter and receiver sampling clocks are synchronized, carrier line spectral components fall on the receiver channel centers, \( \omega_n = n \Delta \omega \).

**Figure 2.** Carriers are offset by \( n \epsilon \Delta \omega \) due to sampling clock frequency skew factor of \( \gamma = 1 + \epsilon \).
shift of the frequency skewed waveform by the given \( d_{\text{dist}} \). For small perturbations,

\[
N - \frac{1}{2} \ll \gamma \ll N + \frac{1}{2}
\]

it is clear that the receiver sample values are highly correlated with the intended values, and differ by a distortion that scales linearly in \( \varepsilon \), by visualization of the Whittaker construction of the transmitted signal in terms of sinc functions with transmitter sample rate spacing. From Eq. 19 it is also clear that the distortion, for fixed \( \gamma \), increases with \( N \). This offsets the benefits of large sample windows associated with additive noise suppression previously noted [2].

Thus, for small clock mismatch, the only effect on recovered TDOA estimates is a scaling by the skew factor and a small “noise” term which grows with increasing skew. Given the threshold behavior with respect to signal-to-noise ratio associated with modern spectral analysis, we would expect the simple scaling effect to dominate for small skew factors with an abrupt breakdown at some larger skew factor value. In the next section we examine some simulation results that illustrate this qualitative behavior.

EXPERIMENTAL CLOCK SKEW RESULTS

In order to experimentally determine the effect of frequency skew on time delay estimation, a simulator was implemented with Matlab. The simulator uses an \( N \)-sample period, multi-carrier signal. Each of the \( M < \frac{N}{2} - 1 \) carriers are assigned a random phase, \( \phi_n \) as explained earlier.

In the following we will use a set of parameters chosen only for simplicity of the description. Our test signal is a periodic waveform with 8192 samples comprising 101 carriers, the first at the 400th channel (that is, FFT frequency index) and each separated by 10 channels. With a sampling rate of 8192 MHz, those parameters translate to a signal bandwidth of 1 GHz centered at 900 MHz with each channel having width 1 MHz and hence the carriers lie at 10 MHz intervals beginning at 400 MHz and ending at 1.4 GHz.

Simulation involves construction of the signal with the chosen sampling clock skew factor, \( \gamma \), and some random phase for the carriers. The resulting signal is passed through an FFT which recovers the carrier phases, from which the initial random phase values are subtracted. The state space estimator described in [2] is applied to these coefficients to extract the Time Of Arrival (TOA) associated with this signal from an unknown but fixed offset common to the receivers. Differences of these offset time delays provide the TDOA information needed to prosecute location solution [1,2].

Since the presence of frequency skew, to zero perturbation order, introduces a scaling of the measured TOA, the TOA estimate offset is in this case a function of the true propagation delay, that is,

\[
t_\Delta = \tilde{t}_0 - t_0 = \varepsilon t_0.
\]

(20)
For all the tests that follow we choose to fix \( t_0 = \frac{NT}{2} \), that is, at half the period of the waveform, or, half the interval over which TOA estimates must repeat because of the periodicity of the signal.

Based upon Eq. 19 we would expect the zero order term to dominate for \( |\epsilon| < 0.00006 \). This behavior is demonstrated in Fig. 3 in which \( \epsilon \) is swept over the range \( \pm 0.00001 \). Using the expression for \( t_0 \), above, we would expect the estimate error to range over \( \pm 5 \) ps, which it does.

By expanding the range of \( \gamma \) values exercised, we can observe the linear increase of the noise-like distortion introduced by frequency skew. The accuracy of the zero order prediction is evidenced by the correspondence between these results and the linear function in Fig. 4.

Finally, in Fig. 5, by slightly extending the range of \( \gamma \) values tested, we observe the sudden breakdown of the modern spectral analysis when the inherent signal-to-noise performance threshold has been exceeded. These results indicate that for expected values of clock skew the TOA errors which this skew introduces will represent no hindrance to meeting the desired performance specifications.

**AUTOCALIBRATION AND TOA ERRORS**

In a previous paper, [2], we introduced bounds and estimates for location error for various TDOA based solvers [5, 6] as a function of receiver array geometry and signal to noise ratio. Another factor influencing the overall performance of a precision personnel locator system is the accuracy with which the receiver sites are known. High precision is possible in a time-unconstrained setting or in a fixed infrastructure based system. However in an ad hoc and rapidly emergent situation this “calibration” step must be carried out rapidly and without human intervention. In this paper we introduce some results of preliminary efforts within the context of our work towards finding practical and effective techniques to perform calibration and to estimate ultimate performance of the resulting system.

By placing a transmitter at every reference node (RN) in addition to the receiver needed for mobile transmitter location, each of which is activated only once during system calibration, sufficient information is obtained to estimate the distance separating each RN from each other RN. Thus the RN geometry could be obtained if it were possible to do so from inter-node distances.

By way of example, suppose that in a \( N = 4 \) RN example as shown in Fig. 6, the range between each pair of RNs has been estimated. Each range, \( d_{ij} \), designates the distance between the \( i^{th} \) and \( j^{th} \) RNs. This set of distances form a \( N \times N \) positive symmetric distance matrix

\[
D_{ij} = \begin{bmatrix}
d_{11} & d_{12} & d_{13} & d_{14} \\
d_{21} & d_{22} & d_{23} & d_{24} \\
d_{31} & d_{32} & d_{33} & d_{34} \\
d_{41} & d_{42} & d_{43} & d_{44}
\end{bmatrix}.
\]  

(21)

Clearly, \( d_{ii} = 0 \) and \( d_{ij} = d_{ji} \), so \( D_{ij} \) becomes
components of the eigenvalue decomposition in Eq. 24. The usual algorithm can be most easily expressed by first translating \( D_{ij} \) so as to place its centroid at the origin, resulting in \( B_{ij} \):

\[
B_{ij} = \left( A_{ij} - A_\cdot \cdot - A_{\cdot j} + A_{\cdot \cdot} \right), \quad \text{where} \quad A_{ij} = -\frac{1}{2} D_{ij}^2.
\]  

(23)

Where \( \cdot \) designates the mean across the specified index. Using the eigenvalues \( \Gamma \) and eigenvectors \( \Lambda \) of an eigenvalue decomposition,

\[
B_{ij} = \Gamma \Lambda \Gamma^*,
\]

(24)

the RN locations may be extracted. The coordinates of the RNs in \( p \) dimensions are contained in each row of a matrix \( X \), where \( X \) can be constructed from the components of the eigenvalue decomposition in Eq. 24.

\[
X = \Gamma \Lambda^{1/2} = \begin{bmatrix}
i_1 & j_1 & k_1 & l_1 \\
i_2 & j_2 & k_2 & l_2 \\
i_3 & j_3 & k_3 & l_3 \\
i_4 & j_4 & k_4 & l_4
\end{bmatrix}.
\]  

(25)

Because the dimensionality of the space in our example is \( M = 2 \), only two columns of \( X \) are significant and contain the desired coordinates. The \( M \) largest eigenvalues from \( \Lambda \) correspond to the columns of interest, yielding the solution coordinates. If the eigenvalues are assumed to be ordered left to right from largest to smallest along the diagonal of the \( \Gamma \) matrix then it is the first two columns that yield the \( x \) and \( y \) coordinates of each node.

In classic MDS problems, all the distances are treated as approximate. However, in the case of deployment of RNs attached to large vehicles, such as fire trucks, it becomes possible to have exact knowledge of the distances between some of the nodes, those attached to the same vehicle. We wished to improve calibration estimation through the use of the \textit{a priori} information about any known inter-RN ranges. Optimal application of this information would involve two parts: replacing imperfect distance measures in the distance matrix by the \textit{a priori} known values, and, solving a highly non-linear structure specific matrix decomposition in place of the eigenvalue decomposition to obtain solutions that also reflect these \textit{a priori} relationships exactly. In this phase of the work we have only applied the first step.

Fig. 7 depicts a particular scenario involving \textit{a priori} information. Three trucks, each carrying three RNs are parked along two sides of an operations area. Only the distances between the three nodes on each truck are well known. The distances between any nodes not residing on the same truck are assumed to be obtained by TOA estimates and are assumed to have a standard deviation in...
centimeters as given by the abscissa in Fig. 8. The ordinate in this graph shows the resulting root mean square error in the location of the RNs as obtained both with and without the use of the a priori information. As can be seen, the advantage obtained from the additional information is very small. This may be attributed to one of two factors: either that little advantage is to be had from a priori information in general, or, that any substantial advantage will only be obtained by implementing the difficult second stage of the modified MDS procedure described above. From the analysis presented in [2] the level of geometric calibration degradation as a function of TOA distance estimate error is compatible with practical application within the context under study. Further work will combine these performance components and analyze overall system performance.

CONCLUSIONS
Analysis of TOA estimation error as a function of transmitter and receiver frequency mismatch was conducted. The results demonstrate the effectiveness of a simple linear model for the induced time offset. For practical FFT size, consistent with the design being pursued in our overall endeavor, clock skews on the order of 1% can be accommodated.

Preliminary results of our research, regarding automatic receiver location calibration, have been presented. While well known MDS algorithms are effective, the advantages of and means to fully exploit a priori information requires further investigation.

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