Modeling Glioblastoma Multiforme

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Abstract –
Glioblastoma multiforme (GBM), a highly lethal brain cancer, accounts for over 30% of brain tumors in adult patients. Patients typically survive only 12-18 months after diagnosis. Our model describes the dynamics of GBM via a system of partial differential equations for tumor cells, nutrients, toxins, and mechanical resistance of brain matter. Using a 2D conservative Alternating Direction Implicit scheme, we numerically approximate the solution to our model and implement it in C for simulation on a parallel computer.

Simulation Results
The following figures show results from an example computer simulation of the model. The figure to the right shows the initial conditions of the system. Tumor cells and nutrients are distributed as illustrated and no toxins exist. The resistance is a constant throughout.

Mathematical Model
Our continuum model for the growth of Glioblastoma multiforme describes the dynamic changes in four quantities: tumor cell concentration, nutrient concentration, toxin concentration, and mechanical resistance. Each quantity has its own governing equation together forming a coupled system of four non-linear partial differential equations.

\[
\begin{align*}
\frac{\partial \rho}{\partial t} & = \nabla \cdot (D_r \nabla \rho) + \beta \rho - \gamma \nabla \cdot \rho \nabla \phi \\
\frac{\partial \phi}{\partial t} & = \nabla \cdot (D_\phi \nabla \phi) + \beta \phi - q_\phi \phi \\
\frac{\partial \tau}{\partial t} & = -k \max \left( \frac{\partial \phi}{\partial \tau}, 0 \right) \\
\frac{\partial r}{\partial t} & = -\kappa \max \left( \frac{\partial \rho}{\partial t}, 0 \right)
\end{align*}
\]

Coefficients

- **Diffusion Coefficient**
  \[ D_r(r) = \sigma_r \left( 1 + \eta_r \frac{\rho}{\rho_b} \right) \]
  \[ \sigma_r, \eta_r > 0 \]
- **Fractional Rate of Net Proliferation**
  \[ \beta_\rho \left( \frac{\phi}{\rho} \right) = \frac{A \phi}{B + \frac{\phi}{\rho}} - C \tau \]
- **Fractional Rate of Nutrient Consumption**
  \[ \beta_\phi(\phi) = \begin{cases} 
\alpha_\phi \phi, & \phi < b_\phi \\
0, & \phi \geq b_\phi
\end{cases} \]
- **Fractional Rate of Toxin Production**
  \[ \beta_\phi \phi \]

Biological Assumptions
- Tumor cells, nutrients, and toxins **diffuse** through brain matter.
- Tumor cells **consume** nutrients and **produce** toxins.
- Toxins **kill** tumor cells.
- Migrating tumor cells **erode** mechanical resistance.

**References:**

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