Closing the Loop: Forging High-Quality Virtual Enterprises in a Reverse Supply Chain through Solution Portfolios


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**Abstract**: Reverse supply chains are receiving increased attention in both academia and practice to address business and environmental sustainability opportunities. As few organizations are adept at both forward and reverse supply chains, subcontracting various activities is imperative. Forging temporary partnerships as a virtual enterprise to take advantage of a short-term market opportunity is one available avenue. Vendor selection that can best achieve combined expertise to complete the entire or strategic portions of a reverse supply chain, while simultaneously forming the virtual enterprise quickly to seize market opportunities, is an emerging and important issue. This paper presents a mixed-integer program that seeks to select vendors that minimize the maximum formation time of a reverse supply chain virtual enterprise, subject to a set of practical decision-making constraints. This model is then integrated into a novel algorithmic technique that generates a portfolio of high-quality and yet diverse solutions of optimal vendor choices, allowing managers to integrate intangible and subjective factors into their final decisions. Numerical examples and computational experiments on simulated data demonstrate the model’s efficiency for generating sets of high-quality solutions and the flexibility for accommodating a range of decision factors. Moreover, this paper sets the stage to further investigate the research nexus of reverse supply chains and virtual enterprises.

**Keyword**: Virtual Enterprise, Reverse Supply Chain, Sustainability, Integer Programming, Solution Portfolios, Diversity

1. **Introduction**

Reverse logistics and supply chains have taken on a broader meaning over the past two decades. What used to be a concern from a business perspective in managing product returns to manufacturers has expanded to take on environmental sustainability dimensions seeking to extend the life of products and materials (Zhu et al., 2008; Prahinski and Kocabasoglu, 2006). The multiple reasons for introducing reverse supply chains in order to ‘close-the-loop’ for supply chains has caused organizations to consider the introduction of reverse logistics operations (Srivastava, 2008). Since few organizations are competent at both forward and reverse supply
chains, subcontracting various reverse logistics activities has become a common practice, at times in response to regulatory take-back policy (e.g. Guarnieri, et al., 2014).

As part of this subcontracting practice, organizations may seek out a single broker or a fourth party to support their reverse supply chain activities (Krumwiede and Sheu, 2002). Sometimes the need for reverse supply chain support may be a very temporary requirement such as in the cases of humanitarian operations (Kovács and Spens 2011) or warranty returns. This type of temporary formation of an organization to provide service is similar to virtual organizations or virtual enterprises forming temporary partnerships to take advantage of a short-term market opportunity.

The virtual enterprise formation literature can prove useful for investigating reverse supply chain virtual enterprise formation. Although the idea of virtual reverse supply chains has been introduced (Browne and Zhang 1999, Meade et al, 2007), the issues and decision models associated with virtual enterprise formation, as well as fourth party reverse supply chain providers, has received scant attention in the literature. To contribute and advance the literature in both the reverse supply chains and virtual enterprise formation we seek to investigate the nexus of these two areas in this paper.

This investigation introduces a mixed-integer programming formulation that seeks to form a virtual reverse supply chain VRSC. The optimization model sets the stage for identifying a portfolio of providers that can form a virtual enterprise. Each group within the portfolio will be a feasible combination of VRSC partners. While an optimal solution, if one exists, is guaranteed to be included using this approach, near-optimal alternatives will also be identified. These alternative solutions are generated by balancing the desire for collective diversity, while simultaneously emphasizing optimal, or near-optimal solutions. The result will be a portfolio of solutions of VRSC partners from which decision makers can select, where a decision maker is an organization that wants to lead the formation of a virtual enterprise, such as a fourth-party reverse logistics providers who wishes to take advantage of a temporary market opportunity. The portfolio development methodology balances both the diversity and quality of solutions.
While there is clearly an argument for high quality (near-optimal) solutions in the context of optimization, the introduction of multiple, diverse solutions is compelled by practical reasons. By providing a decision-maker with a set of diverse solutions, they can use their experience to evaluate alternative, yet high-quality, solutions with respect to intangible factors such as propriety issues, fairness and equity considerations, and other factors that are difficult to quantify.

The contribution of this paper is manifold: (1) it advances the literature in both reverse supply chains and virtual enterprises by identifying opportunities for further investigation at their intersection; (2) it provides a mixed-integer programming formulation together with a solution method to generate a portfolio of near-optimal, yet mutually diverse, alternatives for virtual enterprise formation; and (3) it offers insights into the nature of the formulation and solution sets from both practice and research perspectives. Moreover, this work is the first demonstration of the technique to generate high-quality and diverse solutions for optimization problems with continuous variables.

Within this context we initially provide background on the issues facing VRSC in practice and research. The literature review summarizes various other techniques and approaches and identifies how the methodology introduced in this paper helps to fill an important gap in the literature and to advance knowledge. The model with various scenarios is then presented, followed by numerical illustrations, computational experimentation and discussions. The paper concludes with a summary of general observations, limitations associated with the study and directions for future research.

2. Literature Review

We classify the relevant literature into two categories. The first category defines the general structure of a reverse supply chain. Clearly identifying the structure and activities provides a practical foundation for the decision-making problem under study. The second category is pertinent to vendor selection and virtual enterprise formation techniques, which allows us to differentiate the approach presented in this paper from those reported in the literature.
2.1 General Structure of a Reverse Supply Chain

Blackburn et al. (2004) identify the following five key processes in a reverse supply chain:

1. **Product acquisition**: obtaining the used product from the user;
2. **Reverse logistics**: transporting the products to a facility for inspecting, sorting, and disposition;
3. **Inspection and disposition**: assessing the condition of the return and making the most profitable decision for reuse;
4. **Remanufacturing/Refurbishing**: returning the product to original specification;
5. **Marketing**: creating secondary markets for the recovered products.

It is not difficult to see that the complexity of each process is determined by the composition of a product or material and that each process may need to be completed by a separate organization with specialized resources and expertise. Furthermore, other factors such as the regional locations, the number of possible vendors with needed capabilities and capacities, and the volume and quantity of returned items will inevitably complicate the formation and operations of each reverse supply chain.

Presley et al. (2007) take a step further to investigate a reverse supply chain structure by examining the inputs, outputs, activities, and supporting organizations responsible within this chain. Some of the details in a reverse supply chain network scheme are shown in Figure 1. Notice that at each process stage potentially different organizations will be involved and the stages are connected and the sequence of events are shown by the arrows flowing into the bottom of the activity boxes. These organizations may range from the original equipment manufacturer (OEM) to specialists in disassembling or transporting products and materials. Figure 1 is only an illustrative graphic with the stages and players changing depending on the product, market, and purpose of the reverse supply chain. For example, a reverse supply chain focusing on remanufacturing of electronics, may be very different than a reverse supply chain focusing on managing warranties associated with returned consumer products.
2.2 Vendor Selection Techniques

Vendor selection is essential to forging collaborative partnerships in strategic sourcing as well as important to completing daily operational functions. It is an important prerequisite for the formation of virtual enterprises (e.g. Meade, Liles and Sarkis, 1997). As such, this subject has received wide attention and has been studied extensively (de Boer, et al., 2001; Govindan et al, 2013; Igarashi, et al., 2013). Typically the supplier selection decision is made under the circumstance that a set of criteria, the importance of each criterion, and a pool of vendors with performance attributes or operational parameters are given, which are collectively referred to as discrete alternative multi-criteria decision making. Depending on the complexity and the strategic importance of the decision-making situation, the techniques range from a straightforward linear weighted sum approach, to more sophisticated stochastic modeling. We group the existing selection methods into the following three categories.

2.2.1 Ranking-based

This group of techniques aims to derive or calculate a score for each vendor candidate based on given set of criteria and their levels of importance. When the scores for all vendors under consideration are obtained, they can be ranked in a descending order, which indicates the best
choice. The simplest form of this category is scoring models where each vendor is evaluated based on linear weighted sum of its performance grades and multiple criteria with known levels of importance expressed by a set of weights. Several well-known methods fall into this category such as Total Cost of Ownership (TCO), Supplier Scorecard, Analytic Hierarchy Process (AHP), and the extended version of AHP – Analytic Network Process (ANP) (Degraeve et al, 2000; de Boer et al., 2001; Sarkis et al., 2007; Verdecho et al., 2012).

Some more sophisticated methods in this category not only integrate qualitative and quantitative factors into decision making, but also take numerous uncertainties in the evaluation process into consideration; for example, the consequences of the alternatives, problem definition, and decision makers’ preferences, which may not be completely known or precise. Such techniques with applications in vendor selection and evaluation are represented by Outranking Methods (e.g., de Boer et al., 1998) and Multi-Attribute Utility Theory (MAUT; e.g., Shaik and Abdul-Kader, 2011). Outranking Methods were first developed in France in the late sixties and aim to build a preference relation, usually called an outranking relation, among alternatives evaluated on several attributes (Pirlot, 1997). The MAUT approach, involving additive utility functions and preference modeling, has been researched and studied in great detail over a period of decades. The study by Shaik and Abdul-Kader (2011) is an example application of this technique in green supplier selection problem that considers environmental, green and organizational factors.

2.2.2 Deterministic optimization-based
These methods specify a managerial objective to achieve subject to a set of clearly defined constraints, which are formulated into a mathematical model with the selection choice as decision variables. Examples of this category include integer programming models (e.g., Trapp and Sarkis, 2014) and Data Envelopment Analysis (e.g., Liu et al., 2000). When there are multiple objectives to consider, the optimization problem may be formulated as a goal program.

2.2.3 Stochastic modeling-based
To capture the dynamic and uncertain elements inherent in the supplier selection decision-making process, a variety of techniques from not only operations research, but also the computer and mathematical sciences, can be used. For example, simulation (e.g., Ding et al., 2005), fuzzy
logic (e.g., Nakashima and Gupta, 2013), expert systems (Yigin et al., 2007), and artificial intelligence (e.g., de Boer et al., 2001) have been employed in supplier selection, and in some cases, a combination of the several methods is utilized to not only identify qualified suppliers but also to accommodate the diversity of procurement situations.

While many selection techniques exist, vendor selection decision analysis in the context of forging VRSC enterprises is still sparse. In addition to various situational factors concerning VRSC formation, the decision maker’s preferences may have some flexibility and are not necessarily rigid, and thus a portfolio of high-quality and mutually diverse solutions under various scenarios may be attractive. Consequently, a selection method allowing the user to choose from a set of near-optimal, diverse solutions in reasonable time is of great value. This paper’s model formulation and solution method for forging an efficient VRSC bridges this gap.

Table 1 provides a comparative analysis of various evaluation and selection techniques, many of which were mentioned above. The table also places and summarizes the relationship of the technique introduced in this paper (at the bottom of Table 1) to other existing approaches. Given that it is an optimization approach, it has similarities to other mathematical programming approaches, but includes some additional stages and complexity. However, the goals of the technique, as described previously in the paper and further detailed below, are quite different from any other proposed supplier selection or virtual enterprise formation technique.

Table 1: Summary of Multiple Criteria Evaluation Technique Characteristics.  
H = High, M = Medium, L = Low (Adapted from Sarkis and Sundarraj, 2000)

<table>
<thead>
<tr>
<th>Evaluation Technique</th>
<th>Cost of Implementation</th>
<th>Data Requirements</th>
<th>Ease of Sensitivity</th>
<th>Economic Rigor</th>
<th>Decision Maker Involvement</th>
<th>Management Understanding</th>
<th>Mathematical Complexity</th>
<th>Parameter Mixing - Flexibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scoring Models</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>H</td>
<td>H</td>
<td>L</td>
<td>L</td>
<td>H</td>
</tr>
<tr>
<td>AHP</td>
<td>M</td>
<td>M</td>
<td>L</td>
<td>L</td>
<td>H</td>
<td>H</td>
<td>L</td>
<td>H</td>
</tr>
<tr>
<td>Outranking</td>
<td>M</td>
<td>M</td>
<td>L</td>
<td>M</td>
<td>H</td>
<td>L</td>
<td>M</td>
<td>M</td>
</tr>
<tr>
<td>MAUT</td>
<td>H</td>
<td>H</td>
<td>M</td>
<td>M</td>
<td>H</td>
<td>M</td>
<td>M</td>
<td>H</td>
</tr>
<tr>
<td>DEA</td>
<td>M</td>
<td>M</td>
<td>L</td>
<td>M</td>
<td>L</td>
<td>L</td>
<td>H</td>
<td>M</td>
</tr>
<tr>
<td>Goal Program</td>
<td>M</td>
<td>M</td>
<td>M</td>
<td>H</td>
<td>M</td>
<td>L</td>
<td>L</td>
<td>H</td>
</tr>
<tr>
<td>Simulation</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>L</td>
<td>H</td>
<td>H</td>
<td>M</td>
</tr>
<tr>
<td>Expert Systems</td>
<td>H</td>
<td>H</td>
<td>L</td>
<td>M</td>
<td>M</td>
<td>M</td>
<td>M</td>
<td>H</td>
</tr>
<tr>
<td>Portfolio Technique</td>
<td>M</td>
<td>M</td>
<td>H</td>
<td>M</td>
<td>M</td>
<td>M</td>
<td>H</td>
<td>M</td>
</tr>
</tbody>
</table>
3. The Analytical Model and Methodology

A novel analytical model and methodology to help identify a portfolio of VRSC partners is introduced in this section. The environment of our exemplary decision-making process is end-of-life cell phones. This context sets the practical foundations for our methodology. Specifically, an initial model is presented, accompanied by a description of notation and the model formulation, followed by various transformations to make the model more tractable for solution. The technique to produce a portfolio of diverse and high-quality VRSC solutions, based on a reformulated mixed integer programming (MIP) formulation, is then presented.

3.1 The Decision-Making Context

We use the reverse supply chains of cell phones as a basis to exemplify the problem considered in this paper. The extremely short life cycles and rapid advent of new technologies are placing end-of-life cell phones at the forefront of reverse supply chain implementations (Franke, et al., 2006; Geyer and Blass, 2010). Original equipment manufacturers (OEMs) such as Motorola, Samsung and Apple, and network service providers such as AT&T, Verizon, and T-Mobile are actively taking back end-of-life handsets as a service to customers, and as part of their corporate environmental responsibility program, or for compliance reasons. Both OEMs and service carriers typically outsource the operations of reverse supply chains of phones to third-party enterprises (Geyer and Blass, 2010); for example, ReCellular (sold to ReCellular Acquisition Inc. in December 2013), PaceButler, and International Recycling Network (IRN) in the U.S. have identified the collection of end-of-use cell phones as a business opportunity. Apart from alliances with OEMs and network providers, these take-back enterprises team up with non-profit organizations and retailers to access the stock of retired handsets.

The structure of a cell phone reverse supply chain is similar to the general network illustrated in Figure 1 but has its own features, required processes and activities. Based on a comprehensive study (Neira et al., 2006), the major processes and activities of a reverse supply chain of end-of-life cell phones is shown in Figure 2. It can be observed that the entire chain involves multiple stakeholders and each process contains complex functions that require specialized resources, capabilities and technologies. Furthermore, multiple players may exist at each process; for example, the collection step has a range of participants, including OEMs, network service
providers, retailers, various collectors such as web-based collectors, non-governmental organizations and charities, and municipalities. Consequently, a great deal of coordination is imperative even at the collection point alone.

Unfortunately, not all organizations are proficient at managing the entire cell phone reverse supply chain, nor do they possess all the necessary expertise and resources. For instance, PaceButler, an Oklahoma based company, delivers a proven method for re-selling or recycling used cell phones, whereas IRN, headquartered in New Hampshire, specializes in recycling of a spectrum of electronic wastes. Therefore, to seize business opportunities as well as to protect the environment and promote sustainability, forging a virtual enterprise with carefully selected vendors/brokers at each stage to handle the entire end-of-life product supply chain is necessary. This situation represents a single product type environment. While many reverse supply chain organizations may be able to manage multiple products as well, thereby adding greater complexity to their management, the multi-product version is outside the scope of this paper, and we leave it as direction for future research considerations.

**Figure 2: Reverse Supply Chain of Cell Phones**
3.2 Virtual Reverse Supply Chain (VRSC) Problem Formulation

We next present the base optimization model to generate optimal vendor team selections for forming a virtual reverse supply chain. Following the example of cell phone reverse supply chains, we consider forming a single-product virtual enterprise (such as PaceButler, specializing solely in cell phones) and herein present our optimization model to select vendors to form such a single-product virtual reverse supply chain.

3.2.1 Definitions of Sets and Parameters

The basic context described above allows for explanation of the notation. The problem environment considers that a number of reverse supply chain stages (denoted by set $S$) exist. The entire supply chain is globally geographically dispersed in multiple regions ($R$). The regional aspect may be to help organizations develop more efficient processing of returned products or materials. The number of regions may be altered depending on the configuration of the VRSC. Within each region ($r$) and stage ($s$) we assume there are one or more service providers ($P_{rs}$) available. As in any supplier selection and evaluation approach, various business decisions, operations and supply chain strategic performance measures and parameters need to be considered. In the modeling effort here, investment cost or budget ($B$), cycle or delivery time ($T$), average quality ($Q$), and capacity (by meeting demand ($D$)) are all performance metrics and input parameters that are explicitly included. Specific parameters that contribute to these overall performance metrics are introduced for each provider in each region and stage.

While any of the above performance metrics may take priority, a crucial issue in (agile) virtual enterprise formation, to which we give precedence in our model below, is how quickly the VRSC can be formed. This is especially critical if the marginal value of time of the product is high, such as electronics. Thus, the responsiveness and effectiveness of the provider of a service for a specific stage in a region ($f_{rsp}$) plays an especially important role in this model. Table 2 summarizes the definitions of sets and parameters used in our optimization model.

3.2.2 Definition of Variables

Binary variables $x_{rsp}$ indicate which stages and regions are assigned to which providers:
\[ x_{rsp} = \begin{cases} 1 & \text{if region } r, \text{stage } s \text{ is assigned to provider } p; \\ 0 & \text{otherwise.} \end{cases} \]  

(1)

Table 2: Sets and Parameters Used in the Mathematical Programming Formulation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S )</td>
<td>Set of stages (e.g., collection, sorting, storage, disassembly, reintegration), indexed by ( s )</td>
</tr>
<tr>
<td>( R )</td>
<td>Set of regions (e.g., local, regional, national), indexed by ( r )</td>
</tr>
<tr>
<td>( P_{rs} )</td>
<td>Set of providers for each region ( r ) and stage ( s ), indexed by ( p )</td>
</tr>
<tr>
<td>( B )</td>
<td>Investment budget for the reverse supply chain formation process</td>
</tr>
<tr>
<td>( T )</td>
<td>Threshold for total cycle time, over all stages, for completing reverse supply chain process</td>
</tr>
<tr>
<td>( D_{rs} )</td>
<td>Demand for region ( r ) and stage ( s )</td>
</tr>
<tr>
<td>( Q_{rs} )</td>
<td>Quality threshold for completing reverse supply chain process in region ( r ) and stage ( s )</td>
</tr>
<tr>
<td>( c_{rsp} )</td>
<td>Cost of assigning region ( r ), stage ( s ) to provider ( p )</td>
</tr>
<tr>
<td>( a_{rsp} )</td>
<td>The capacity available in region ( r ), stage ( s ) for provider ( p )</td>
</tr>
<tr>
<td>( q_{rsp} )</td>
<td>Quality rating of region ( r ), stage ( s ) for provider ( p )</td>
</tr>
<tr>
<td>( t_{rsp} )</td>
<td>Cycle time necessary to complete operations in region ( r ), stage ( s ) for provider ( p )</td>
</tr>
<tr>
<td>( f_{rsp} )</td>
<td>Formation time necessary to &quot;start up&quot; operations in region ( r ), stage ( s ) for provider ( p )</td>
</tr>
</tbody>
</table>

3.2.3 Mathematical Programming Formulation

The problem of selecting vendors for forging an efficient virtual reverse supply chain can now be given below.

\[
\text{Minimize } \max_{r,s,p} \{f_{rsp} x_{rsp}\} \\
\text{subject to } \sum_{r \in R} \sum_{s \in S} \sum_{p \in P_{rs}} c_{rsp} x_{rsp} \leq B, \\
\sum_{p \in P_{rs}} a_{rsp} x_{rsp} \geq D_{rs} \ \forall \ r \in R, s \in S, \\
\sum_{p \in P_{rs}} (q_{rsp} - Q_{rs}) x_{rsp} \geq 0 \ \forall \ r \in R, s \in S, \\
\sum_{s \in S} \max_{r \in R_p} \{t_{rsp} x_{rsp}\} \leq T, \\
x_{rsp} \in \{0,1\} \ \forall \ r \in R, s \in S, p \in P_{rs}. 
\]  

(2)–(7)

Objective (2) seeks to minimize the maximum formation time to assemble the virtual enterprise. Constraint (3) enforces budgetary limitations on the entities chosen in the virtual enterprise, while constraint sets (4) and (5) ensure that demand and average desired quality, respectively, are met in every region-stage. Constraint set (6) ensures that the total cycle (delivery) time over all stages does not exceed a certain threshold, \( T \). Together, (2)–(7) form a binary integer formulation.
which is nonlinear due to the max expressions in (2) and (6). These expressions can, however, be linearized, as we next discuss.

3.2.4 Linearizing Formulation (2)–(7)

It is not difficult to see that the minimax form present in objective function (2) can be linearized by introducing a single continuous variable, $q$, and simultaneously requiring $q$ to upper bound the formation time for every selected provider in region $r \in \mathcal{R}$, stage $s \in \mathcal{S}$, and provider $p \in \mathcal{P}_{rs}$.

Moreover, the max function in the left-hand side of constraint (6) can also be linearized by introducing additional auxiliary variables and constraints. Specifically, for each $s \in \mathcal{S}$ introduce a continuous variable, $y_s$, as well as binary variables $d_{rsp}$ for each region $r \in \mathcal{R}$, stage $s \in \mathcal{S}$, and provider $p \in \mathcal{P}_{rs}$, for a total of $\sum_{r \in \mathcal{R}} \sum_{s \in \mathcal{S}} |\mathcal{P}_{rs}|$ binary variables.

For each stage $s \in \mathcal{S}$, define constants $\bar{t}_s = \max \{t_{rsp}\}$, and let $y_s = \max \{t_{rsp} x_{rsp}\}$. These latter equality restrictions will be enforced implicitly via the following three constraint sets:

\begin{align*}
y_s &\geq t_{rsp} x_{rsp} \quad \forall \ r \in \mathcal{R}, s \in \mathcal{S}, p \in \mathcal{P}_{rs}, \quad (8) \\
y_s &\leq t_{rsp} x_{rsp} + \bar{t}_s (1 - d_{rsp}) \quad \forall \ r \in \mathcal{R}, s \in \mathcal{S}, p \in \mathcal{P}_{rs}, \quad (9) \\
\sum_{r \in \mathcal{R}} \sum_{p \in \mathcal{P}_{rs}} d_{rsp} &= 1 \quad \forall \ s \in \mathcal{S}. \quad (10)
\end{align*}

Thus, nonlinear constraint set (6) can be replaced with (linear) constraint sets (8)–(10) together with the following linear constraint:

\begin{equation}
\sum_{s \in \mathcal{S}} y_s \leq T. \quad (11)
\end{equation}

The final, linearized formulation for forming a virtual enterprise with minimal formation time is:

\begin{align*}
\text{Minimize} & \quad q \\
\text{subject to} & \quad q \geq f_{rsp} x_{rsp} \quad \forall \ r \in \mathcal{R}, s \in \mathcal{S}, p \in \mathcal{P}_{rs}, \quad (12) \\
& \quad \sum_{r \in \mathcal{R}} \sum_{s \in \mathcal{S}} \sum_{p \in \mathcal{P}_{rs}} c_{rsp} x_{rsp} \leq B, \quad (13) \\
& \quad \sum_{p \in \mathcal{P}_{rs}} a_{rsp} x_{rsp} \geq D_{rs} \quad \forall \ r \in \mathcal{R}, s \in \mathcal{S}, \quad (14) \\
& \quad \sum_{p \in \mathcal{P}_{rs}} (q_{rsp} - Q_{rs}) x_{rsp} \geq 0 \quad \forall \ r \in \mathcal{R}, s \in \mathcal{S}, \quad (15)
\end{align*}
Formulation (12)–(21) is a mixed (binary) integer linear program that is equivalent to (2)–(7), containing \(|S| + 1\) continuous variables, \(2 \cdot \left[ \sum_{r \in \mathcal{R}} \sum_{s \in \mathcal{S}} |\mathcal{P}_{rs}| \right] \) binary variables, and \(3 \cdot \left[ \sum_{r \in \mathcal{R}} \sum_{s \in \mathcal{S}} |\mathcal{P}_{rs}| \right] + 2 \cdot |\mathcal{R}| |S| + |S| + 2 \) constraints. This is advantageous because (12)–(21) can now be passed directly to state-of-the-art mixed-integer programming solvers such as CPLEX (IBM ILOG CPLEX, 2014) or Gurobi (Gurobi Optimization, 2014). We refer to the formulation (12)–(21) as VRSC.

Some advantages to modeling provider decisions with binary variables include that the same provider can appear in multiple stages and/or regions, simply by adding side constraints enforcing such relationships. Further, if minimum levels of activity, logical implications, etc. exist, they could also be readily added in our proposed formulation (12)–(21).

### 3.3 Methodology for Generating High-Quality and Diverse Solutions

Having linearized the VRSC formulation, it can now be solved using an off-the-shelf optimization solver, which generate a single optimal solution (assume one exists, and can be found). Given the uncertain planning environment surrounding the formation of virtual reverse supply chains; however, it may be of interest to have a collection of high-quality solutions – particularly if they are simultaneously diverse. It is the identification of high-quality and yet diverse solutions to VRSC that we now pursue.

#### 3.3.1 Rationale for Portfolios of High-Quality and Diverse Solutions

A portfolio of high-quality and mutually diverse solutions can empower managers with greater flexibility in their decision making, as the additional solutions can be used to weigh subtle decision factors that are not easily modeled. Some of these uncertainties may arise from intangible factors and managerial preferences or perceptions that may not be easily modeled. For
example, there may be situations where a decision maker has a differing historical perspective of some service providers (e.g. some may be poorer collaborators) that would be difficult to express analytically. The standard output of an optimization solver, being only a single solution, cannot offer such flexibility. Thus, a portfolio of high-quality and yet mutually diverse solutions can give broader alternatives with respect to provider selection, and thereby provides the rationale for such portfolios.

Given two binary vectors of the same dimension, diversity between the two can be viewed as the sum of the number of vector indices whose values are not in agreement. For example, the diversity of vector $A = [0 \ 0 \ 1 \ 0]$ and vector $B = [1 \ 1 \ 1 \ 0]$ is 2. This measure can be expressed, for example, using the $L_1$ (taxicab) norm, and it naturally extends to a collective diversity measure when there are more than two binary vectors, as we soon discuss. In general, however, it can be challenging to obtain a portfolio of solutions that are both high in quality and yet collectively diverse. There are two reasons for this. First, solutions that score relatively high in quality tend to evaluate poorly with respect to diversity, due to structural similarities. Second, diverse solutions typically come from disparate areas of the feasible region, and are not likely share similar quality characteristics. When both high quality and diversity are emphasized, this situation produces tension. We now provide a general, yet brief, outline of the methodology, followed by a discussion of implementation details specific to VRSC. For a more complete review of the methodology, we refer to Trapp and Konrad (2013).

### 3.3.2 Outline of Methodology

For ease of reference, let us denote by $S$ the set of constraints (13)–(20) together with the variable space (21). For the sake of exposition, assume VRSC has multiple feasible solutions, and further suppose that we have an optimal solution vector $\mathbf{x}^* = (x_{rsp}^*, d_{rsp}^*, y_s^*, q^* \ \forall \ r \in R, s \in S, p \in P_{rs})$ and optimal objective function value $z^*$. As previously mentioned, the VRSC formulation can be solved to obtain such an optimal solution using an off-the-shelf solver such as CPLEX (IBM ILOG CPLEX 2014) or Gurobi (Gurobi Optimization, 2014). Let $X$ be the set containing all identified solutions, i.e., presently $X = \{x^*\}$. Armed with this $x^*$ and $z^*$, consider the same feasible region but now using the following modified (fractional) objective:
\[ R(x) = \frac{N(x)}{D(x)} = \frac{\text{Relative Solution Diversity}}{\text{Relative Deterioration in Objective Quality}}. \] (22)

Objective (22) expresses the ratio of the relative solution diversity to the relative deterioration in objective function quality (i.e., formation time). Now assume we have any other feasible solution \( \bar{x} = (\bar{x}, \bar{d}, \bar{y}, \bar{q}) \) that is distinct to \( x^* \). The denominator representing the objective quality deterioration can be easily obtained using:

\[ \bar{q} - z^*. \] (23)

Moreover, it is possible to express the diversity of \( \bar{x} \) with respect to elements in \( X \) via a linear metric. It is important to note that this diversity is measured \textit{precisely over the} \( x^* \_rs \text{p components} \) of the solutions in \( X \); the other \( (d^* \_rs, y^*_s, q^*) \) variables are simply used for bookkeeping, and their interpretation contributes nothing meaningful with respect to diversity. The diversity metric will be covered in greater detail shortly.

The set \( X \) can be populated in a constructive manner to generate \( P \) high-quality and diverse solutions (assuming such solutions exist, and can be found). First, replace the objective of VRSC with fractional objective (22); refer to the resulting formulation as VRSC\(_M\). While objective (22) is fractional and so nonlinear in nature, the algorithm of Dinkelbach (Dinkelbach, 1967) is able to solve such a nonlinear fractional binary integer program. It does so by first transforming the problem to a simpler, linearized program, and then solving a sequence of such problems that quickly converges to a global optimum.

When seeking additional solutions, it is important that previously identified solutions, such as the vector \( x^{(0)*} = x^* \) in \( X \), not be rediscovered. It can be explicitly excluded by adding to VRSC\(_M\) the following constraint (Balas and Jeroslow, 1972):

\[ \sum_{(r,s,p):x_{rsp}=0} x_{rsp} + \sum_{(r,s,p):x_{rsp}=1} (1 - x_{rsp}) \geq 1, \] (24)

Solving VRSC\(_M\) with constraint (24) will generate a solution \( x^{(1)*} \) distinct from \( x^{(0)*} \) that maximizes the ratio in (22); i.e., it simultaneously emphasizes solution quality and diversity (with respect to all elements in \( X \)).
Now consider that a sequential solution-finding process after $X$ contains at least two solutions. The collective diversity can be computed as the distance from the centroid of all solutions in $X$, where the centroid is the vector composed of the component-wise average of each element:

$$c_{rsp} = \left( c_{rsp} = \frac{1}{h} \sum_{j=0}^{h-1} x_{rsp}^{(j)} \right).$$

(25)

The centroid diversity metric uses the centroid $c$ to compute the distance of any vector $x \in \{0,1\}^n$ from the elements of $X$ as follows:

$$\sum_{r \in R} \sum_{s \in S} \sum_{p \in Prs} c_{rsp} + \sum_{r \in R} \sum_{s \in S} \sum_{p \in Prs} (1 - 2c_{rsp})x_{rsp}. \quad (26)$$

As linear expression (26) increases, so does the mutual diversity from all elements of $X$. For additional algorithmic details, including numerator and denominator normalization to eliminate any predisposed bias in magnitudes, we refer to Trapp and Konrad (2013). So long as feasible solutions remain, this entire process can be repeated as often as desired to generate a set of solutions $X$ that balance quality and diversity. Figure 3 depicts an overview of the entire process.

![Figure 3: Illustration of Algorithm to Find $P$ Diverse and High-quality Solutions to VRSC](Trapp and Konrad, 2013)

### 3.3.3 Evaluating Collective Diversity

The collective diversity of the solutions in $X$ can be assessed in a manner similar to that used in related studies (Danna and Woodruff 2009, Prokopyev et al. 2009):
The $D_{bin}(X)$ metric expressed in (27) takes a value between 0 and 1, and provides the average (scaled) pairwise distance between all solutions in $X$.

### 3.4 Illustrative Example

To further explain our methodological discussion and motivate the impending computational study, we provide an illustrative example on a small test instance with two regions, three stages, and three or four providers in each region-stage. We characterize four types of solutions to illustrate the contribution of our methodology. Specifically, the four solutions are: an optimal solution (i.e., $\mathbf{x}^{(0)*}$); a high-quality, but not very diverse solution from $\mathbf{x}^{(0)*}$; a solution that is diverse from $\mathbf{x}^{(0)*}$, but not overly high-quality; and finally, a solution that is both high quality, as well as relatively diverse from $\mathbf{x}^{(0)*}$ – one generated by the method introduced in Section 3.3. To keep the example manageable, we depict each solution featuring the capacity that selected providers can make available, leaving out additional complicating details such as meeting budget, quality, and cycle time restrictions. Highlighting the available capacity of each solution is sufficient to characterize the four types of solutions. These four solutions are depicted in Figures 4, 5, 6, and 7, respectively.

Figure 4 shows an optimal solution $\mathbf{x}^{(0)*}$ with a minimized formation time of 40 (days – though this metric is not visually displayed). Figure 5 depicts a second solution that shares with $\mathbf{x}^{(0)*}$ the same minimized formation time of 40 days; however, it can be seen that its solution structure is not very diverse from that of $\mathbf{x}^{(0)*}$. In Figure 6, a solution is illustrated that is quite diverse from $\mathbf{x}^{(0)*}$ – in fact, greater than 70% pairwise diversity, as measured by (27). However, its quality lags behind, as the formation time is 44 days. Finally, in Figure 7 we present a solution that is both high in quality (indeed, it is another optimal solution, having formation time of 40 days), and yet diverse from $\mathbf{x}^{(0)*}$; metric (27) gives over 35% pairwise diversity with respect to $\mathbf{x}^{(0)*}$.
Figure 4: Illustrative Example: Optimal Solution $x^{(0)*}$

Figure 5: Illustrative Example: High-Quality Solution, yet not Diverse from $x^{(0)*}$
Figure 6: Illustrative Example: *Diverse Solution from $\chi^{(0)}$, yet not High-Quality*

Figure 7: Illustrative Example: *High-Quality and Diverse Solution from $\chi^{(0)}$*
4. Computational Experiments

It is useful to investigate the behavior of the methodology to find multiple high-quality and diverse solutions to the optimization model under varying conditions. We propose test classes for VRSC in a cell phone context and subsequently discuss the computational performance of the methodology.

4.1 Computational Setup

We propose three test classes. The number of stages, $|\mathcal{S}|$, is likely to be rather static, and we take it to be $|\mathcal{S}| \in \{2,3,4,5\}$ in all experiments. Similarly, for any given region $r$ and stage $s$, we take the number of potential providers $|\mathcal{P}_{rs}| \in \{3,4,5\}$. The three test classes fundamentally differ in the number of regions $\mathcal{R}$; we separate them into small, medium, and large. The small test class features $\mathcal{R} \in \{2,3,4,5\}$, the medium class has $\mathcal{R} \in \{6,7,8,9,10\}$, while the large test class has $\mathcal{R} \in \{10,50\} \cap \mathbb{Z}$, so that the classes experience increasing number of regions from small, to medium, to large. For each of the three test classes we randomly generated 1,000 test instances. The largest of these was a test instance of $\mathcal{R} = 50$, $\mathcal{S} = 5$, and over 1,000 total potential providers, leading to a formulation (12)–(21) with more than 2,000 binary variables and 3,500 constraints.

Other key parameters include formation time $f_{rsp}$, which was set to between 15 and 45 days for each provider, cycle time $t_{rsp}$, taken to be between 20 and 36 days for each stage, and demand $D_{rs}$, which after some careful calculations was taken to be in the range of 654 and 1,308 kilograms available for collection per day, or between 19,620 and 39,240 per month.\(^1\) All of the parameters were independently generated and uniformly distributed over their respective domains.

4.2 Computational Environment


Our approach was coded in C++ and compiled using g++ version 4.4.7 20120313 (Red Hat 4.4.7-4) using 2 Intel(R) Xeon(R) E5-2690 CPUs each with 8 cores running at 2.90GHz and 64GB RAM. All optimization was performed using the callable library of IBM ILOG CPLEX 12.5.1 (IBM ILOG CPLEX, 2014). We set a one-hour limit to solve any optimization problem, and prioritized numerical stability by setting the CPX_NUMERICAL_EMPHASIS parameter to CPX_ON. We set \( P = 10 \) to return, where possible, ten high-quality and mutually diverse solutions.

4.3 Summary of Computational Results

Table 3 summarizes key performance metrics on the 1,000 test instances for each of the three classes. In particular, the algorithm either returned infeasible, or returned a full set of \(|X| = 10\) solutions, for all 3,000 instances. The second column of Table 3 details the number of instances that returned (\(|X| = 10\)) solutions, while the other four columns provide, for each respective class, mean values for four measures over those instances that returned a full \(|X| = 10\) solutions. Standard deviations are recorded in parentheses, and mean runtimes are measured in CPU time.

| Test Class | Count of \(|X| = 10\) | Mean Runtimes (seconds) | Mean Iterations | Mean Gap | Mean Dbin(\(X\)) |
|------------|------------------------|-------------------------|-----------------|----------|-----------------|
| Small      | 770                    | 50.74 (83.90)           | 1.88 (0.33)     | 0.09 (0.09) | 0.44 (0.09)     |
| Medium     | 385                    | 33.01 (72.01)           | 1.78 (0.41)     | 0.05 (0.04) | 0.43 (0.09)     |
| Large      | 99                     | 92.34 (148.58)          | 1.59 (0.49)     | 0.02 (0.03) | 0.48 (0.06)     |

The small test class featured the least number of infeasible test instances \((1,000 – 770 = 230)\); as the randomly generated test instances grew in size, so did the tendency for the resulting models to be infeasible. Over all test classes, the average runtime was 48.6 seconds, which is a relatively modest amount of time in our estimation. Even for the largest of test classes, the algorithmic runtime was in our opinion well within the time needed to make a critical, yet strategic, decision on constructing a reverse virtual supply chain. Over all test classes and instances, the maximum runtime was 624.6 seconds, amounting to about 10 minutes of CPU time, though practically it was much shorter than this given that the computation can be conducted via parallel processing. These short runtimes were directly related to the low iteration counts (on average, less than two) of the implementation of Dinkelbach’s algorithm, which is known to have super-linear
convergence properties (Schaible, 1976). This bodes well if the model were to be extended in the future; while likely larger in size, it may still be amenable to solution via state-of-the-art mixed-integer programming solvers.

4.4 Discussions on Quality and Diversity of Solutions to VRSC
We now address the performance of the methodology with respect to the diversity and quality of generated solutions.

4.4.1 Analysis of Solution Diversity
As discussed in Section 3.3.2, recall that the collective diversity calculated in (27) and reported in Table 3 is measured over precisely the $x_{rsp}$ components of all solutions in $X$, as they are the only variables over which diversity is meaningful. Table 3 displays some rather high observations for mean diversity metrics across all three test classes, specifically with respect to other studies (Trapp and Konrad (2013); Trapp and Sarkis, (2014)). This means that the solutions in the set $X$ are quite diverse, and specifically there is significant flexibility in choice of suitable providers to meet the specified demand in all region-stage pairs. Looking across the three test classes individually, there does not appear to be a significant trend towards more or less diversity as the test instances grow in size from small to medium to large. This implies that there is adequate flexibility inherent in the model with respect to the choice of providers.

4.4.2 Analysis of Solution Quality
The quality of any solution $x$ can be quantified by determining its gap from optimality (i.e., distance from the minimum objective function value $z^*$). The optimality gap is expressed in the following manner, where $\epsilon$ is taken to be a very small but positive number:

$$\frac{q - z^* + \epsilon}{z^* + \epsilon}.$$  (28)

In all cases, the mean optimality gap was within 10% of the global optimal solution, indicating high quality of the solutions, especially in light of the fact that, as in many real-life scenarios, many of the data appearing in formulation (12)–(21) are likely to come from best-guess approximations. Across the three test classes, the mean optimality gap appears to be largest in
the small test class, where a mean gap of 0.09 was observed, with the medium test class coming in at 0.05, and the large at 0.02.

Thus, there is a clear decreasing trend in the mean optimality gap as the test class sizes grow larger; equivalently, it can be observed that the quality of the solution set $X$ improves with the size of the test class. For RVSC, this is likely due to the objective function and its relation to the randomly generated values for the $f_{rup}$ parameter. For small test instances, there are at most 25 region-stage pairs, allowing for a greater fluctuation in the minimum possible formation time. For test instances in the largest class, however, there can be up to 250 region-stage pairs; for such an instance it is very unlikely to find an optimal objective function value much below the maximum possible value of the $f_{rup}$ parameter, hence the gaps for alternate solutions are likely to be smaller.

Figures 8, 9, and 10 depict the mean optimality gap distribution for each instance, for the small, medium, and large test classes, respectively. Collectively, they are another representation of the high-quality of the obtained diverse solution sets.

![Figure 8: Histogram of Mean Optimality Gap in Solution Sets: Small Test Class](image)
Figure 9: Histogram of Mean Optimality Gap in Solution Sets: Medium Test Class

Figure 10: Histogram of Mean Optimality Gap in Solution Sets: Large Test Class
5. Discussions and Conclusions

Given the many sustainability and business concerns facing organizations and their product streams, the issue with “closing-the-loop” in supply chains has received increased attention over the past two decades. An important set of activities for this goal to close the loop is the introduction of reverse supply chains. Given that many organizations do not have the necessary capabilities to effectively and efficiently manage the entire reverse supply chain, turning to fourth party service providers for expertise and resources is a prudent strategy. Sometimes the need is short-term or focused on one particular product or material, and thus a rapid formation of a virtual enterprise for reverse supply chains will be needed. These were the motivations for this paper and study. To help address this important and emergent issue, decision tools and models can prove value to OEMs and others whose products need to be managed in an extended producer responsibility or related context.

We have developed a mathematical programming representation for the problem of constructing a virtual reverse supply chain enterprise in the context of end-of-life cell phones. The model has explicitly incorporated a variety of infrastructural and performance characteristics. For each region and stage in the reverse supply chain process, the formulation seeks to choose a set of providers that can meet the specified demand and quality, and time limitations, with the aim of minimizing formation time for the entire virtual reverse supply chain. The original, nonlinear formulation is reformulated as a linearized version through the inclusion of additional variables and constraints, making it amenable to state-of-the-art mixed-integer linear programming solvers such as CPLEX (IBM ILOG CPLEX 2014) or Gurobi (Gurobi Optimization, 2014).

There are considerable challenges and uncertainty inherent in assembling a virtual reverse supply chain enterprise. To this end, we discuss how to obtain a portfolio of optimal and near-optimal solutions, while specifically emphasizing that they are mutually diverse, so as to provide flexibility in managerial decision-making. This flexibility is particularly useful in situations like forming a virtual enterprise, where there exists intangible and unquantifiable dynamics that affect the decision making. Moreover, the present context is the first application to extend the portfolio technique of Trapp and Konrad (2013) to a mixed binary integer linear program, i.e. one that contains continuous decision variables; indeed, the insight that formulation transformations
that involve only auxiliary continuous variables is a significant contribution to the general portfolio approach and the diversity/quality solution paradigm. Our test results demonstrate that portfolios of solutions can be quickly generated that feature optimal, near-optimal, and mutually diverse solutions.

Although the contributions of this work are manifold, limitations to the application and model do exist. At the same time, these limitations provide opportunities for further development and research. First, the modeling effort included only a single product and had an objective of formation time minimization. Extending this effort to include multiple products, multiple time periods, and alternate or multiple objectives, are all possibilities for future research directions. Although experimentation with realistic data and size were incorporated, actual data and implementation in a real-world setting is needed. Issues in real world settings such as data incompleteness, the feasibility of acquiring all necessary information, and acceptance by management may be limitations of the model and its practical application, and should be further investigated. Finally, the solution set is a portfolio of high-quality and diverse solutions for a given set of constraints and performance metrics. There may be additional managerial, perceptual and intangible data (e.g. trust, image, legitimacy, relationships) that are not easy to model within the context of an optimization program. To overcome some of these limitations, linkage of the outcomes from the model introduced here to other multiple criteria decision approaches, such as AHP/ANP that can incorporate a broader set of intangible measures and decision factors, can enhance the methodology.

Supply chain management researchers have ample opportunities to help develop analytical models to solve emergent concerns of various sustainable, social, and regulatory pressures. This paper builds not only on the foundations for virtual enterprises and reverse supply chains, but also provides insight into additional research avenues where some of these pressing issues can be more fully understood and addressed.

References


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\[\text{In the diverse and high-quality solution approach of Trapp and Konrad (2013), there is a normalization procedure that requires knowledge of the maximum formation time (that is still feasible) to assemble the reverse logistics virtual enterprise (see discussion around equation (6) in Trapp and Konrad (2013)). This requires a maximax formulation, which uses disjunctive programming, and can be done by modifying formulation (12)–(21) via:}\]

1. The addition of \(\sum_{r \in R} \sum_{s \in S} |\mathcal{P}_{rs}|\) auxiliary binary variables \(g_{rsp}\);
2. Removing constraint set (13), and replacing it with a slightly modified one:
\[
\left(f_{rsp}^{\text{Max}} - f_{rsp}\right)x_{rsp} + \left(f_{rsp}^{\text{Max}} - f_{rsp}^{\text{Min}}\right)g_{rsp} + q \leq f_{rsp}^{\text{Max}} + \left(f_{rsp}^{\text{Max}} - f_{rsp}\right) \quad \forall \ r \in R, s \in S, p \in \mathcal{P}_{rs};
\]
3. And adding a single constraint:
\[
\sum_{r \in R} \sum_{p \in \mathcal{P}_{rs}} d_{rsp} = 1, \quad \forall \ s \in S.
\]