Frequency-Dependent Complex Permittivity Reconstruction of Layered Diaphragm in a Rectangular Waveguide: Comparison with Experiment

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Abstract – This study employs the technique developed in the earlier studies of the authors aimed at complex permittivity reconstruction of layered materials in the form of diaphragms (sections) in a single-mode waveguide of rectangular cross section from the transmission coefficient measured at different frequencies. The permittivity is assumed to be a function of frequency. We develop a numerical-analytical method of solution to this problem which guarantees accuracy of the result and can be easily implemented in view of practical measurements. Numerical results are presented for different types of materials.

Introduction

Reconstruction of electromagnetic parameters of nanocomposite and artificial materials is an urgent problem. In practice, as a rule, these parameters cannot be directly measured and methods of mathematical modeling must be applied. This work is a continuation of the series of papers [1, 2] devoted to the analysis of permittivity determination of layered materials. We set forth a specially developed mathematical and numerical technique that combines analytical and numerical approaches thus providing results with guaranteed accuracy. In the previous works we performed permittivity reconstruction for layered media in a waveguide when (a) plane-parallel layers are filled with homogeneous isotropic dielectric and their number may be as high as several hundred and (b) when layers are anisotropic. This study employs the earlier developed technique and deals with complex permittivity reconstruction of layered materials in the form of diaphragms (sections) in a single-mode waveguide of rectangular cross section from the transmission coefficient measured at different frequencies; the permittivity is assumed to be a function of frequency. We develop a numerical-analytical method which guarantees accuracy of the result and can be easily implemented in practical measurements using network analyzers. Numerical results are presented for different types of materials and compared with the measurement data. The developed technique can be applied in optics, nanotechnology, and design of microwave devices as well as in materials science when it is necessary to determine with prescribed accuracy electrophysical parameters of layered media having unknown properties using as little information about the media as possible.

Technique

Consider a waveguide of rectangular cross section with the perfectly conducting boundary surface containing a multi-layered medium in the form of a parallel-plane diaphragm Q: a parallelepiped separated into n sections adjacent to the waveguide walls (Fig. 1).

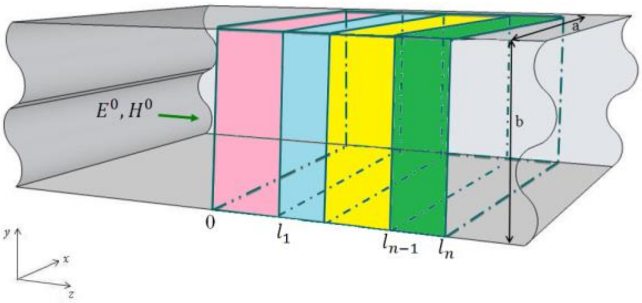

Fig. 1. The multi-sectional diaphragm.

The domain outside the diaphragm is filled with an isotropic and homogeneous medium having constant permeability μ_0 and constant permittivity ε_0, each section of the diaphragm is filled with a medium having frequency-dependent permittivity

$$\varepsilon_j(\omega) = \varepsilon_j^1 + i\frac{\sigma_j}{\omega},$$

where ε_j^1 is the real part of complex permittivity and σ_j is conductivity ($j = 1, 2, \ldots, n$).

The electromagnetic field inside and outside of the object in the waveguide is governed by Maxwell's equations. Assume that $\pi/a < k_0 < \pi/b$, where k_0 is the wavenumber, $k_0^2 = \omega^2\varepsilon_0\mu_0$. In this case, only one wave H_{10} propagates in the waveguide without attenuation (we have a single-mode waveguide ([8]). The incident electrical field is:

$$E^0 = \vec{e}_2 A \sin\left(\frac{\pi x_1}{a}\right) e^{-i\gamma_0(\omega)x_3}$$

with a known A and $\gamma_0(\omega) = \sqrt{\omega^2\varepsilon_0\mu_0 - \pi^2/a^2}$.

Inverse problem P: find the complex permittivity $\varepsilon_j(\omega) = \varepsilon_j^1 + i\frac{\sigma_j}{\omega}$ of each section from the known amplitude A of the incident wave and transmission coefficient F at different frequencies.

Solving the forward problem for Maxwell's equations, we obtain explicit expressions for the field inside every section of diaphragm Q and outside the diaphragm. Then we apply continuity of the tangential field components on the interface of the diaphragm sections and obtain finally a recurrent formula that couples amplitudes A and F (general expressions were presented in [1, 2]). For a two-sectional diaphragm we obtain a system of two equations with two unknown permittivities:

$$\frac{A(\omega_1)}{F(\omega_1)} e^{i\gamma_0(\omega_1)l_1} = \frac{\gamma_2(\omega_1)\, p_3(\omega_1)}{2\gamma_0(\omega_1)\gamma_1(\omega_1)\gamma_2(\omega_1)} + \frac{\gamma_0(\omega_1) q_3(\omega_1)}{2\gamma_0(\omega_1)\gamma_1(\omega_1)\gamma_2(\omega_1)},$$

$$\frac{A(\omega_2)}{F(\omega_2)} e^{i\gamma_0(\omega_2)l_1} = \frac{\gamma_2(\omega_2)\, p_3(\omega_2)}{2\gamma_0(\omega_2)\gamma_1(\omega_2)\gamma_2(\omega_2)} + \frac{\gamma_0(\omega_2) q_3(\omega_2)}{2\gamma_0(\omega_2)\gamma_1(\omega_2)\gamma_2(\omega_2)},$$

where

$$p_3 = \gamma_1 p_2 \cos\alpha_2 + \gamma_2 q_2 i \sin\alpha_2,$$
$$p_2 = \gamma_0 p_1 \cos\alpha_1 + \gamma_1 q_1 i \sin\alpha_1, \quad p_1 = 1,$$
$$q_3 = \gamma_1 p_2 i \sin\alpha_2 + \gamma_2 q_2 \cos\alpha_2,$$
$$q_2 = \gamma_0 p_1 i \sin\alpha_1 + \gamma_1 q_1 \cos\alpha_1, \quad q_1 = 1.$$

$$\gamma_j(\omega) = \sqrt{\omega_k^2 \varepsilon_j^1 \mu_0 + \omega_k^2 \sigma_j \mu_0 - \pi^2/a^2}, \ \alpha_j = \gamma_j\left(l_j - l_{j-1}\right), \ j = 1,2, \ k = 1,2.$$

The system is solved numerically in the complex domain using fixed-point iterations to obtain the sought-after permittivities.

Numerical solution provides robust accurate results under the conditions that the problem parameters belong to a compact set; namely, real and imaginary permittivities (and other parameters) vary in distinct prescribed intervals; e.g. $1 < \varepsilon_j^1 < E_0$, where in practical applications E_0 ~ 20. In that case we prove [1] that there is a one-to-one correspondence (in the form of an invertible complex mapping) between the complex values of permittivity and transmission coefficient in a predefined frequency interval.

We also prove that the proposed numerical method provides calculations of permittivities with arbitrary prescribed accuracy and is stable with respect to initial data perturbations. Therefore, limitations on the accuracy of computing (reconstructing) permittivity from the transmission data are solely based on how accurate the values of the transmission coefficient are measured by the available equipment (network analyzers): the permittivity will be determined numerically with the same accuracy (or inaccuracy).

Results

For a two-sectional diaphragm we perform comparison of our numerical results with experimental data using the experimental results (measurements) obtained with the help of a setup employing network analyzers. Parameters of the diaphragm are $a = 2.286$ cm, $b = 1.02$ cm, $c = 2$ m, $l_1 = 0.995$ cm, and $l_2 = 2.004$ cm, and the excitation frequency is $f \in \left(8,2;12,4\right)$ GHz (see Figs. 2 and 3).

The reconstructed effective permittivity values shown in Figs. 2 and 3 correspond well to the statistical average (mean value, expectation) of the measurement data. The latter is characterized by a rather high typical measurement inaccuracy (oscillations) caused in particular by limitations of the measurement facilities and other factors; however, analysis of a big variety of similar results shows that the deviation from the calculated average value of the effective permittivity and that of the measurement data does not exceed 1%. Calculations are performed with much higher prescribed accuracy which is not visible on the graphs. Similar results obtained using numerical optimization are reported e.g. in [3].

We see that the reported results of computations show high efficiency of the proposed approach. A clear advantage of the method is that it has a solid mathematical background; the problems under study are proved to be well posed (uniquely solvable and stable with respect to small perturbations in the initial data and problem parameters) which guarantees the accuracy of the results. Also it was checked that the technique is easy to implement when using standard measurement equipment and network analyzers. The widely used methods of permittivity determination, like numerical minimization of error (test) functions or somewhat similar split-post dielectric resonator techniques [4], do not allow one, unlike the present approach, to estimate the accuracy of the permittivity calculation and prove uniqueness.

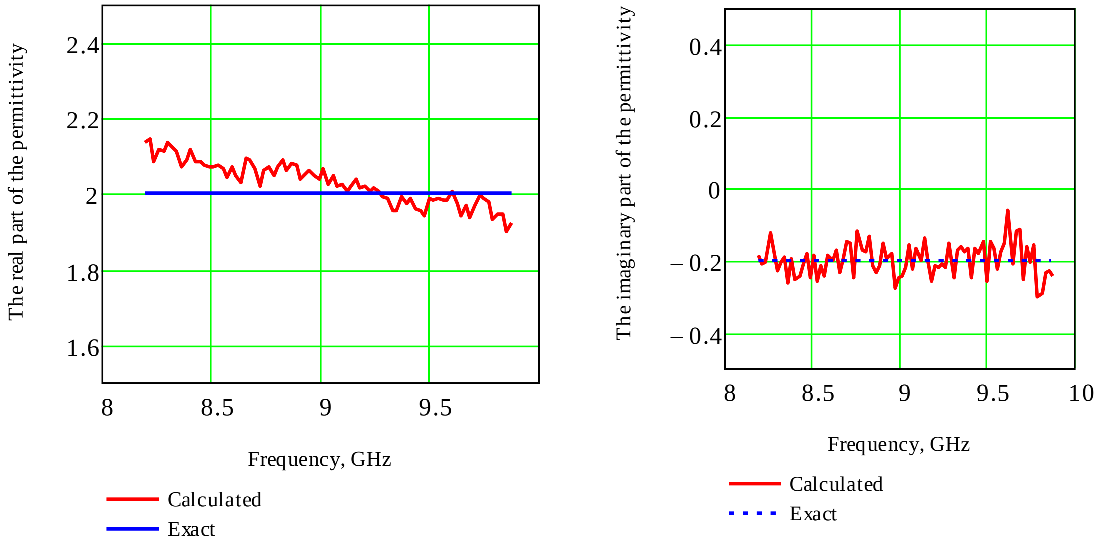

Fig. 2. The real (a) and imaginary, σ_j, (b) parts of the permittivity of the first section.

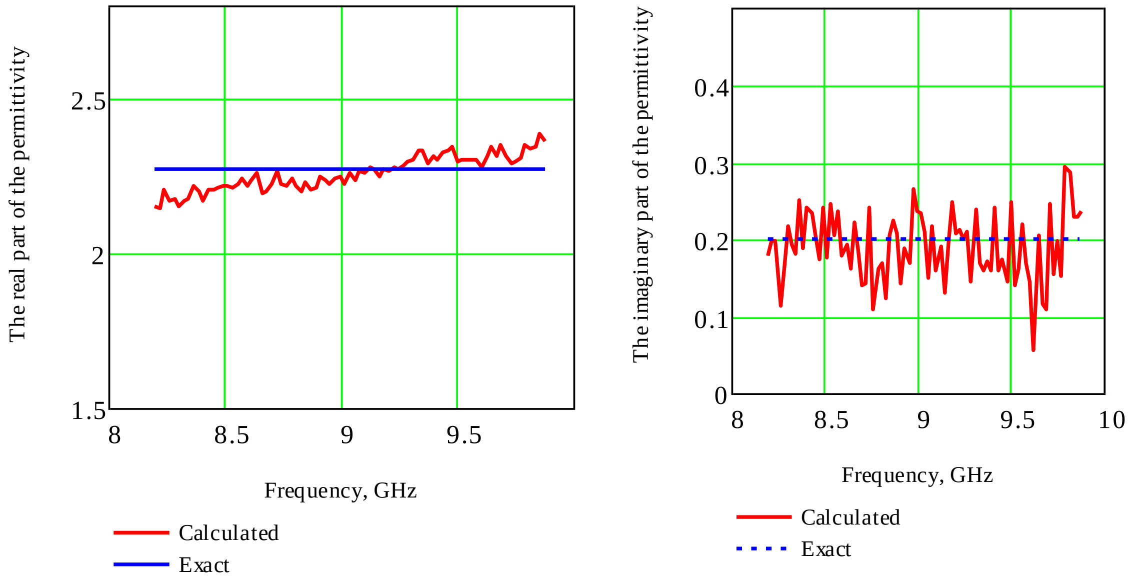

Fig. 3. The real (a) and imaginary, σ_j, (b) parts of the permittivity of the second section.

Conclusion

In this work, we have proposed a numerical-analytical method of reconstructing complex permittivity of a two-sectional diaphragm from the transmission coefficient when the permittivities of the layered medium are frequency-dependent. An important feature of the method is its robustness and stability with respect to the initial data (well-posedness). Numerical results and their comparison with experimental data confirm high efficiency of the method. The technique can be directly developed to multi-sectional diaphragms with several tens of sections; the use of parallel computations will enable extension of the results to layered media consisting of many hundreds and even thousands of layers.

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