Interpolation Algorithms for Interfacing FDTD and FEM Meshes in Multiphysics Modeling of Microwave Sintering

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This paper is focused on the development of a particular element of a numerical tool for comprehensive simulation of electromagnetic, thermal, and mechanical phenomena occurring during microwave sintering. Combination of the relevant solvers requires interchange of numerical data between FDTD and the FEM grids. Here we outline two techniques interfacing the FDTD mesh of Cartesian cells and the FEM mesh of rectangular hexahedra, including cubic spline interpolation and the Shamos technique for determination of areas of convex polyhedra respectively. Computational experiments with these algorithms implemented in MATLAB show satisfactory accuracy of interfacing with the average error not more than 5%.

Introduction

Microwave (MW) sintering of powders is gaining attention in the interdisciplinary engineering community, but the practice of microwave sintering is as yet underutilized in industry, because it is practically difficult to control and is still relatively poorly understood. Modeling of MW sintering could help to clarify many issues and suggest engineering solutions for designing efficient MW sintering systems.

Creation of such a model faces challenges stemming from the very nature of MW sintering which is a coupled electromagnetic, thermal, and structural process. Coupling physical deformation and changes in material characteristics due to temperature and density changes creates a nonlinear modeling problem. In order to accurately simulate this entire process, our model represents this coupling on a macro scale, using an explicit routine to couple *QuickWave-3D* [1] and *Abaqus FEA* [2] software traditionally considered in academia and industry as accurate and adequate modeling tools for electromagnetic/thermal and mechanical processes, respectively.

In practice, we achieve this coupling via an interface which allows the transfer of data between *QuickWave-3D* (*QW*) and *Abaqus*, where changes in the temperature field due to MW treatment are calculated using *QW* and then input to *Abaqus*, which in turn calculates mechanical deformation and change in density due to the temperature change. The role of the interface is to prepare temperature field data from *QW* for input to *Abaqus*, and to prepare geometrical data from *Abaqus* for input into *QW*, according to a scheme for comprehensive modeling of metal powder sintering.

The main challenge in developing this interface is that not only must the interface translate data from the syntax of one software package into that of another, but it must also allow conversion of the data from one type of load discretization to another. *QuickWave-3D* uses conformal Finite Difference Time Domain (FDTD) numerical methods to perform full-wave analysis and model electromagnetic and thermal processes, which relies on discretization of the load (and the entire MW cavity) into regular Cartesian cells, while *Abaqus* uses the Finite Element Method (FEM) to model mechanical changes due to heat, a method centered around the ability to discretize the load using a mesh of nodes which form hexahedral elements. In this paper, we report the state of the project dedicated to the conversion of data between the cells of FDTD methods and the elements of FEM that is approached with the use of interpolation or intersection algorithms to assign the correct field value to the location of the corresponding node, cell, or element.

Technique

In the construction of our interface, we make assumptions to retain robustness and physical meaning, and to simplify the interfacing problem, enabling us to create a fast and physically accurate interface. First, we assume that the temperature field is smooth and can be approximated piecewise with cubic functions; we also assume that temperature flows smoothly across boundaries of cells and elements. Second, we assume that the elements of the FEM mesh are convex, and that they are similar in aspect ratio and size to the cells of the FDTD mesh, whose dimensions are set by *QW* to be approximately $\lambda/20$ on each side, where λ is the wavelength of incident radiation.

These assumptions are motivated to correspond to either an actual physical scenario (smoothness of temperature field), or actual restrictions that *Abaqus* sets on the mesh (i.e., that it tends to not use concave elements, and that the lengths and angles of each element must not be so small that they cause numerical error). At the present stage of the work, we also make another assumption to aid in quick creation of an operational model; namely, in the intersection algorithm we assume that the FEM elements are constant in the z-direction – that is, they are extended 2D convex polygons.

Under these assumptions, we developed two techniques of bringing data between FDTD and FEM meshes, which are described below. Both techniques take into account that field values on the FDTD mesh are the average values over the entire portion of the load inside a cell, and that field values on the FEM mesh are the node temperatures – i.e., the values correspond to temperatures taken at the locations of the nodes of each element.

The first algorithm we use to implement this mesh switch is a simple interpolation algorithm using three-dimensional cubic splines with MATLAB's built-in `interp1` function. This algorithm fits piecewise cubic functions to the input field data, and evaluates these functions at the appropriate points in the load to obtain the field values for reporting in the output data. This technique, while fast to implement and relatively accurate, does not account for conservation of enthalpy when transferring temperature fields.

We accounted for conservation of enthalpy by developing a second technique, based on the area of intersecting polyhedra. For this technique, we assume that the *Abaqus* elements form layers which correspond to the z-slices in the *QW* cells. In this case, determining the volume of the element which intersects different cells in the FDTD mesh can be reduced to a 2D problem, which consists of calculating pairwise the areas of intersection of respective FDTD cells and FEM elements; from these areas we obtain intersected volumes, which multiply the enthalpy densities of the corresponding cells and, finally, sum to the enthalpy of the element. Performing calculations using mass density and specific heat, we find the element temperature from this calculation. Implementation of this algorithm is based on the intersection technique given in [4].

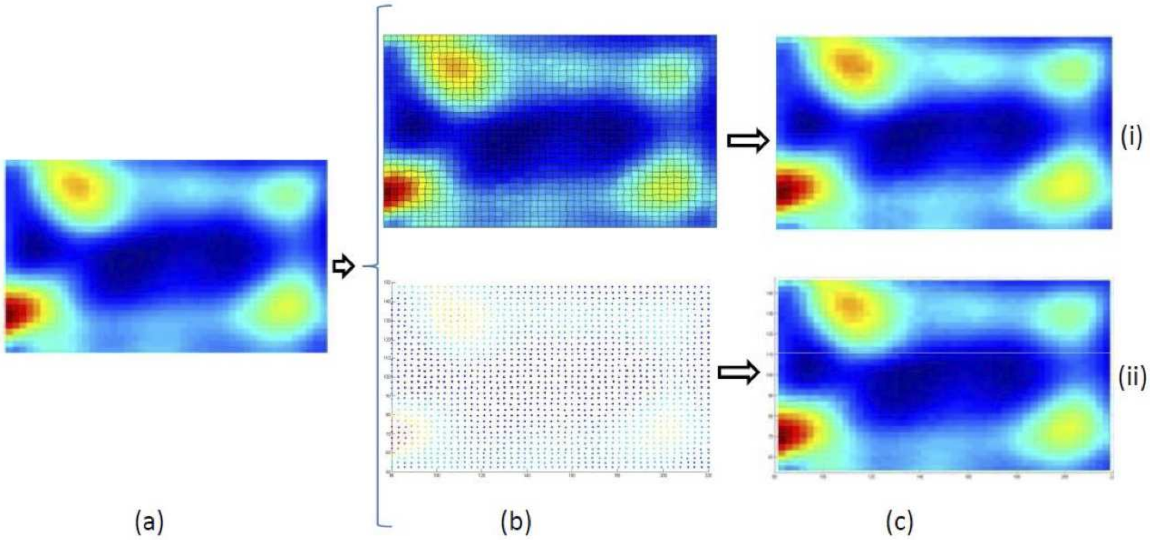

Fig. 1. (a) Original cell-centered temperature distribution from *QW*. (b) Interpolated temperature distribution: (i) element temperature distribution; (ii) node temperature distribution. (c) Recovered *QW* temperature distribution: (i) polygon intersection algorithm; (ii) using `griddata3` in MATLAB.

Results

The results presented in Fig. 1 show an initial layer of the load in *QW*, then the element and node temperatures for an *Abaqus*-like mesh created in MATLAB, and finally, the *QW* layer regenerated from these values. The *Abaqus* element temperatures are obtained through the intersection method while the node temperatures are obtained through the interpolation method.

The error in the algorithms was calculated for several different input temperature distributions, and for several different types of *Abaqus*-like mesh generated by MATLAB, ranging from Cartesian-like elements to highly non-Cartesian elements. For those meshes generated to more closely resemble a Cartesian mesh, the error is less.

Conclusion and Future Work

The present contribution has taken first steps to develop an interface between *QuickWave-3D* and *Abaqus-FEA* software packages, and in doing so has advanced the status of an important element of a comprehensive model of MW sintering. Using this interface as a starting point, the comprehensive model will be further developed to achieve the goal of accurately representing the coupled process of MW sintering. The interface itself will also be further developed to account for the full range of convex polyhedral elements in FEM, and to support the conformal mesh capabilities of *QW*. To extend the intersection technique to one which allows for a range of shapes of convex polyhedra, we will follow [4] in using geometric dual space to decrease the cost of the computation to $O(n \log n)$, where n is the total number of faces of polyhedra.

References

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