SOME MULTISCALE, MULTIPHYSICS PHENOMENA IN MULTIPHASE REACTORS

Magnetoviscosity & Artificial Gravity

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Outline

- **Magnetic fluids & Magnetoviscosity**

  Ferrofluids
  Backgrounds on Magnetostatics
  Local Cauchy problem for ferrofluids
  (Porous media) volume-averaging theorems & upscaling
  Closures
  Simulation : Mitigation of wall bypass fraction

- **Non-magnetic fluids & Artificial gravity**

  Ground-based artificial gravisensing (Grad. magnetic field)
  Gas-liquid flows in porous media (e.g., Mars gravity)
  Liquid holdup, wetting efficiency, pressure drop
  Analysis using simplified modeling

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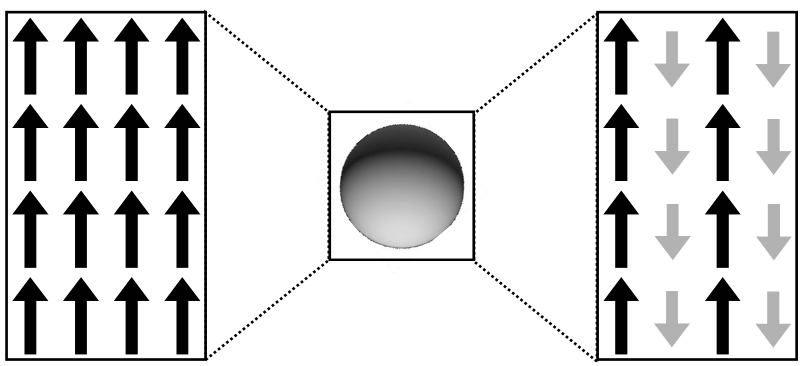
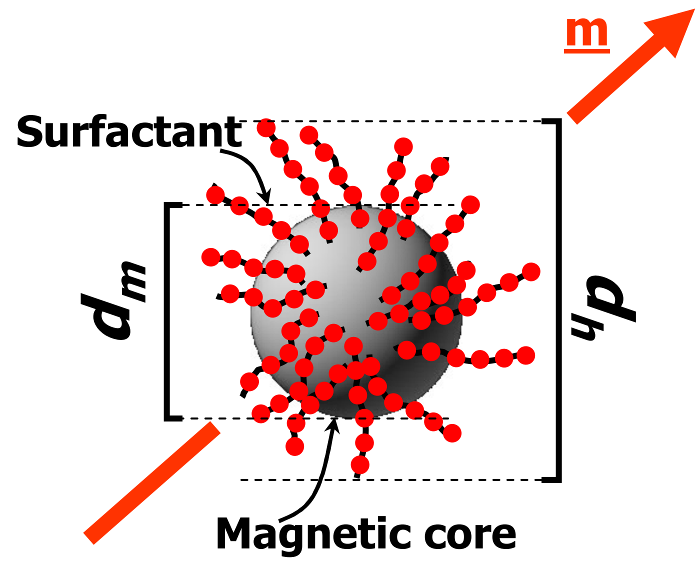
What is a ferrofluid (FF)?

- Single-domain superparamagnetic nanoparticles (*ca*. 10^4 μ_B,* $d_m \approx 10$ nm)
- Dipole-dipole interactions (long-range orientational correlation)
- Permanent magnetic dipole, **m**
- Stabilized ferro(i)colloidal suspension ($\leq$ 7% v/v, magnetic basis)



Parallel spin network (ferromagnetism, Fe)

Antiparallel spin network (ferrimagnetism, Fe_3O_4, γ-Fe_2O_3, Fe_3S_4)



$^*\mu_B = 0.9274\ 10^{-23}$ A.m²

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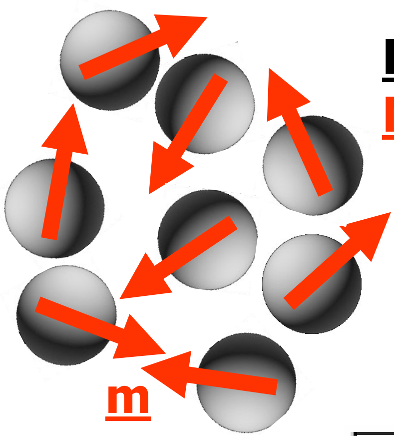
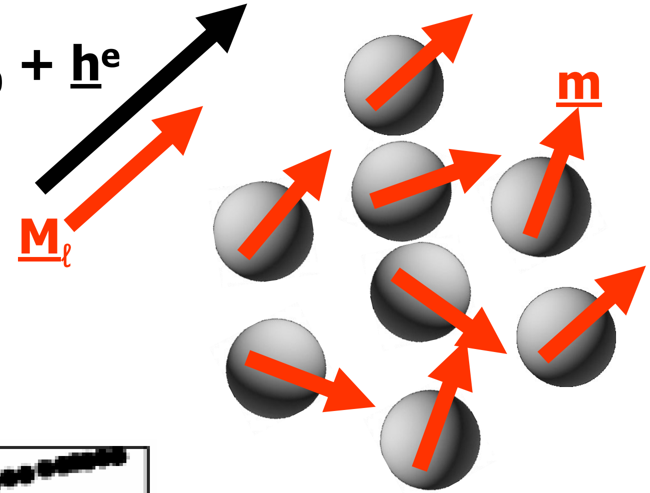
Magnetohydrostatics (Langevin magnetization law)

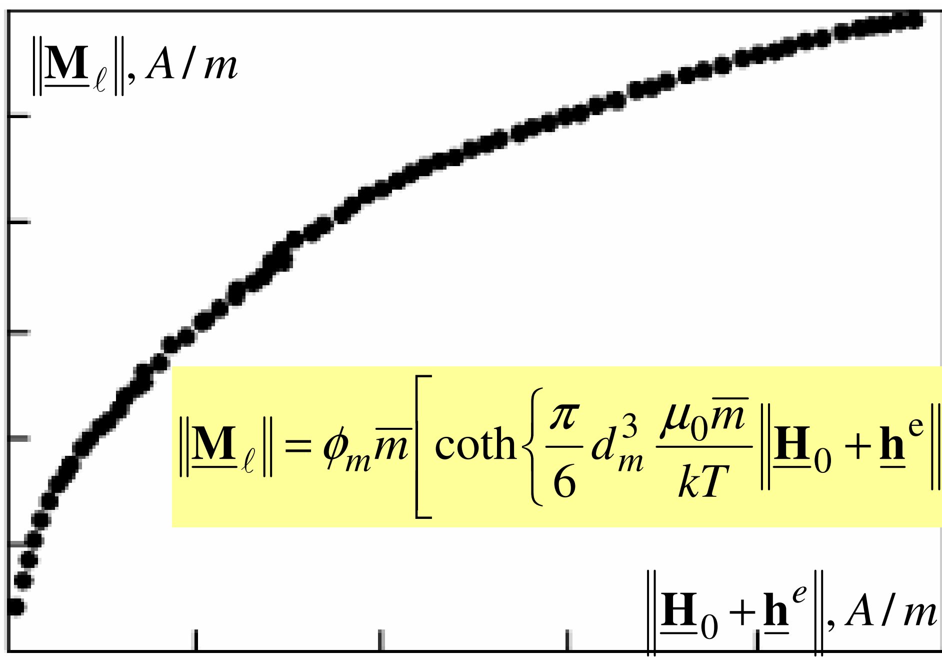


$\underline{\mathbf{H}}_0 = \underline{0}$

$\underline{\mathbf{M}}_\ell = \underline{0}$

$\underline{\mathbf{H}}_0 + \underline{\mathbf{h}}^e$





$$\left\| \underline{\mathbf{M}}_\ell \right\| = \phi_m \overline{m} \left[\coth\left\{ \frac{\pi}{6} d_m^3 \frac{\mu_0 \overline{m}}{kT} \left\| \underline{\mathbf{H}}_0 + \underline{\mathbf{h}}^{\mathrm{e}} \right\| \right\} - \frac{6}{\pi d_m^3} \frac{kT}{\mu_0 \overline{m}} \left\| \underline{\mathbf{H}}_0 + \underline{\mathbf{h}}^{\mathrm{e}} \right\|^{-1} \right]$$

Magnetization curve from Odenbach S. Colloids & Surf. A 217 (2003) 171

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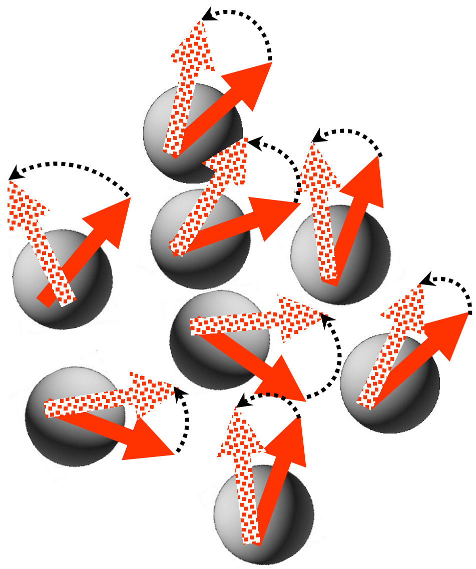
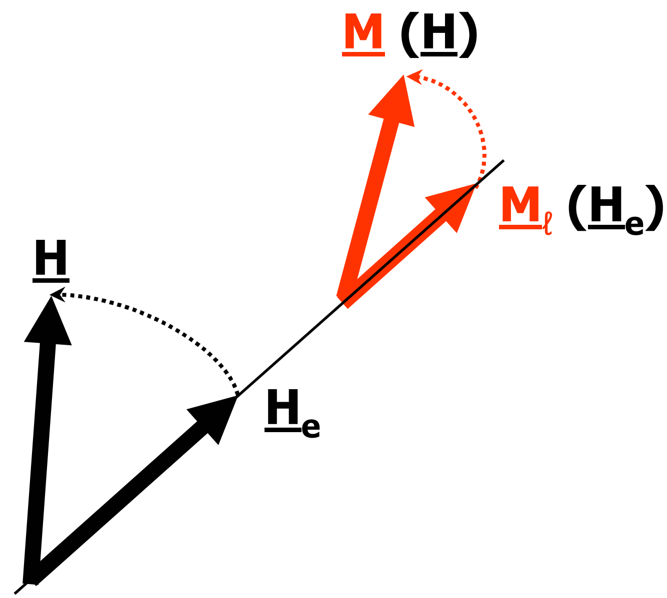
Magnetization relaxation (Shliomis equation)

Magnetic field &/or velocity field change



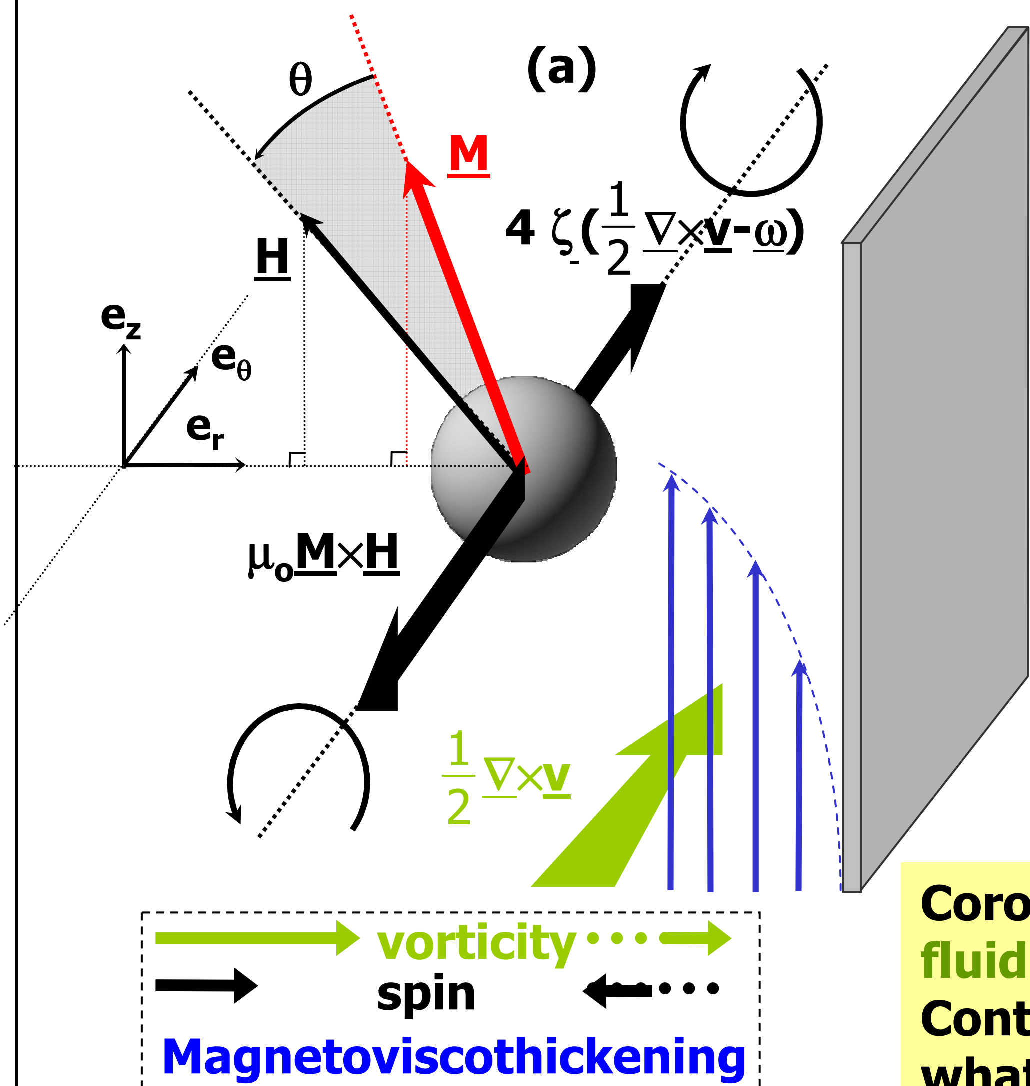


- Brownian relaxation
- Néel relaxation

Magnetostatic torque $\underline{\mathbf{M}} \times \underline{\mathbf{H}} \neq \underline{\mathbf{0}}$

$$\frac{\partial}{\partial t}\underline{\mathbf{M}} + \underline{\nabla} \cdot \underline{\mathbf{M}} \otimes \underline{\mathbf{v}} = \underline{\boldsymbol{\omega}} \times \underline{\mathbf{M}} - \tau^{-1}\left(\underline{\mathbf{M}}(\underline{\mathbf{H}}) - \underline{\mathbf{M}}_{\ell}\left(\underline{\mathbf{H}}^{e}\right)\right)$$

What is Magnetoviscosity?



Internal angular momentum balance

- **Magnetostatic torque** $\mu_0\, \underline{M} \times \underline{H}$
- **Mechanical torque**:
  vorticity-spin asynchrony
  $4\zeta\,(\frac{1}{2}\,\underline{\nabla\times v} - \underline{\omega})$

- ζ = vortex viscosity

Corotative :
fluid vorticity faster than particle spin
Contrarotative :
whatever

Rosenswieg R.E., Ferrohydrodynamics, Dover Publ., 1997

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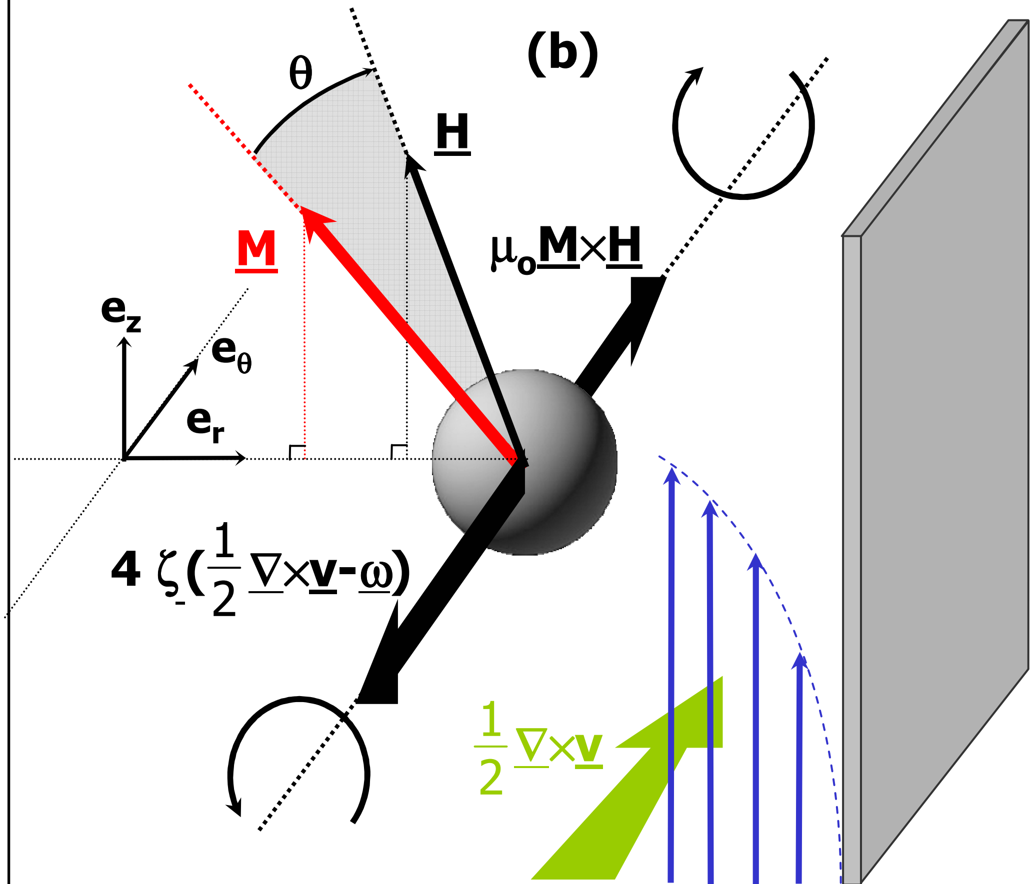
What is magnetoviscosity?



vorticity → spin
Magnetoviscothinning

Corotative :
particle spin faster than fluid vorticity

Cauchy stress tensor for (polar) ferrofluids

Symmetric pressure/viscous stress tensor

$$\underline{\underline{\mathbf{T}}} = \overbrace{-p\underline{\underline{\mathbf{I}}} + \eta\left(\underline{\underline{\nabla}}\,\underline{\mathbf{v}} + {}^{t}\underline{\underline{\nabla}}\,\underline{\mathbf{v}}\right) + \lambda\left(\nabla\cdot\underline{\mathbf{v}}\right)\underline{\underline{\mathbf{I}}}} + \underbrace{2\zeta\,\underline{\underline{\underline{\mathbf{E}}}}\cdot\left(\frac{1}{2}\underline{\nabla}\times\underline{\mathbf{v}} - \underline{\boldsymbol{\omega}}\right)}$$

Antisymmetric spin/vorticity tensor

Conversion of external angular momentun into internal angular momentum (antisymmetric stress tensor)

$$-\underline{\underline{\underline{\mathbf{E}}}}:\underline{\underline{\mathbf{T}}} = \overbrace{-\underline{\underline{\underline{\mathbf{E}}}}:\left(-p\underline{\underline{\mathbf{I}}} + \eta\left(\underline{\underline{\nabla}}\,\underline{\mathbf{v}} + {}^{t}\underline{\underline{\nabla}}\,\underline{\mathbf{v}}\right) + \lambda\left(\nabla\cdot\underline{\mathbf{v}}\right)\underline{\underline{\mathbf{I}}}\right)}^{=\underline{\mathbf{0}}} - 2\zeta\,\underline{\underline{\underline{\mathbf{E}}}}:\left(\underline{\underline{\underline{\mathbf{E}}}}\cdot\left(\frac{1}{2}\underline{\nabla}\times\underline{\mathbf{v}} - \underline{\boldsymbol{\omega}}\right)\right)$$

$$= \boxed{4\zeta\left(\frac{1}{2}\underline{\nabla}\times\underline{\mathbf{v}} - \underline{\boldsymbol{\omega}}\right)}$$

(Asynchrony) mechanical torque

$\underline{\underline{\underline{\mathbf{E}}}}$ **Unit antisymmetric orientation triadic**

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Magnetic stress tensor & Kelvin body force density

Magnetic stress tensor

$$\underline{\underline{\hat{\mathbf{M}}}} = \left(-\frac{\mu_0}{2}\left[\left(\underline{\mathbf{H}}_0 + \underline{\mathbf{h}}\right)\cdot\left(\underline{\mathbf{H}}_0 + \underline{\mathbf{h}}\right)\right]\underline{\underline{\mathbf{I}}} + \mu_0\left(\underline{\mathbf{H}}_0 + \underline{\mathbf{h}}\right)\otimes\left(\underline{\mathbf{H}}_0 + \underline{\mathbf{h}} + \underline{\mathbf{M}}\right)\right)$$

Neglect:
- **Magnetostriction of nanoparticles (incompressibility)**
- **Interparticle dipole-dipole**

Kelvin (magnetic) body force density

$$\underline{\nabla}\cdot\underline{\underline{\hat{\mathbf{M}}}} = \underline{\mathbf{F}}_{\mathrm{Kelvin}} = \mu_0 \underline{\underline{\nabla}}\left(\underline{\mathbf{H}}_0 + \underline{\mathbf{h}}\right)\cdot\underline{\mathbf{M}}$$

Kelvin force due to inhomogeneous magnetic field ≠ Lorentz force MHD (in)homogeneous magnetic field

$$\underline{\mathbf{F}}_{\mathrm{Lorentz}} = \mu_0 \underline{\mathbf{j}}\times\left(\underline{\mathbf{H}}_0 + \underline{\mathbf{h}}\right)$$

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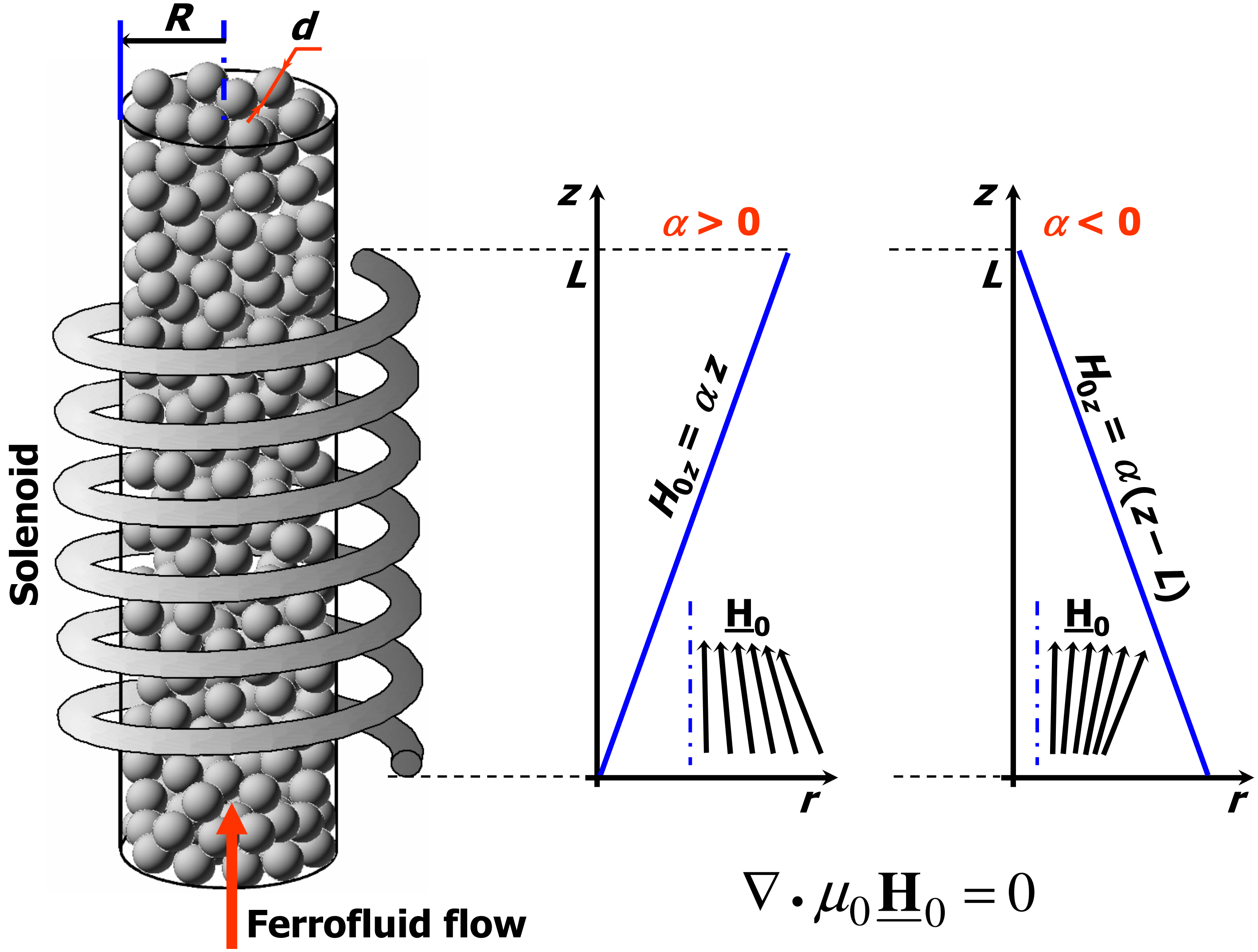
Ferrofluid flow through packed bed in linear-gradient d.c. magnetic field



$$\nabla \cdot \mu_0 \underline{\mathbf{H}}_0 = 0$$

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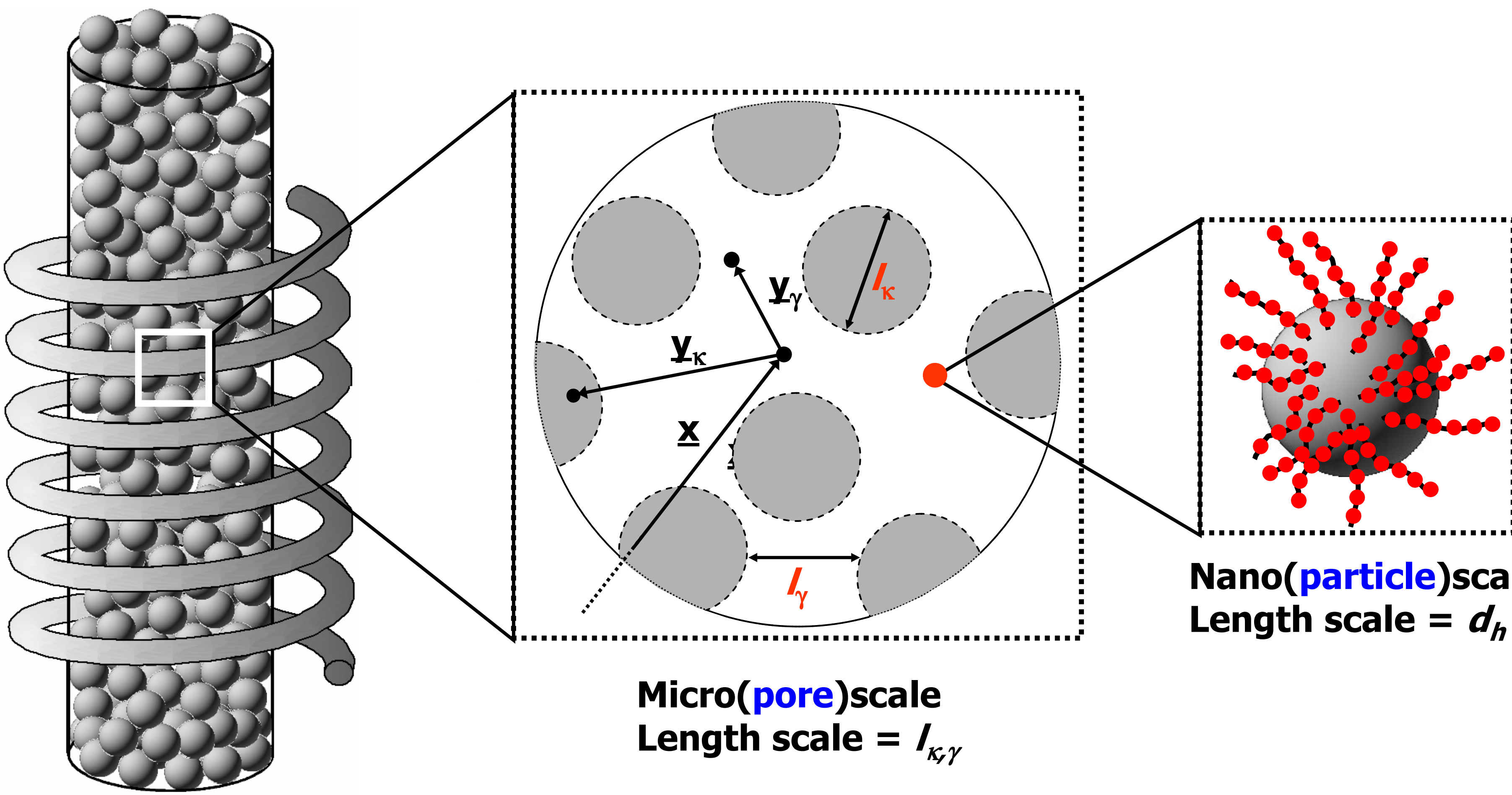
Multiple length scales: Ferrofluid flow in porous media



Nano(particle)scale
Length scale = d_h

Micro(pore)scale
Length scale = $l_{\kappa,\gamma}$

Macro(reactor)scale
Length scale = L

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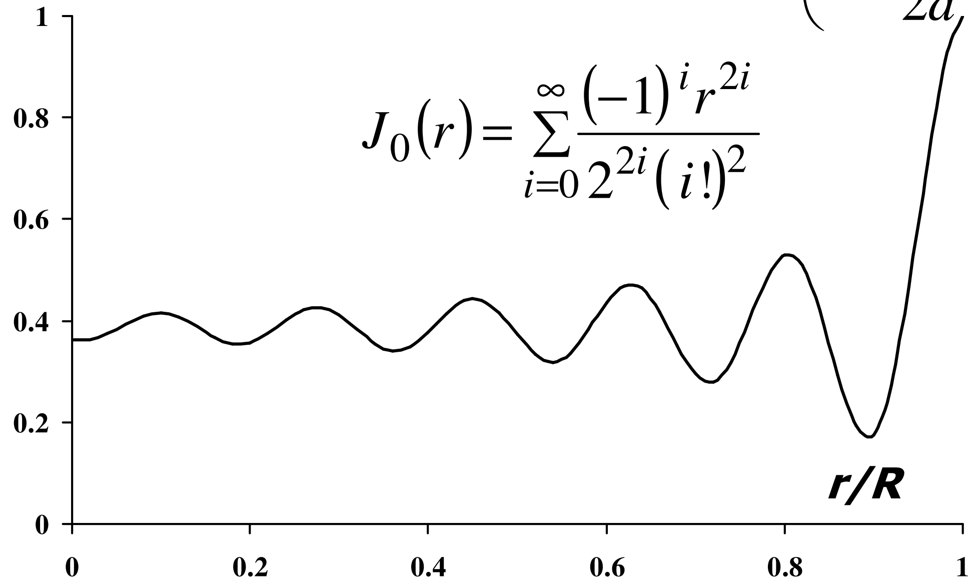
Is magnetoviscosity of help to cope with wall maldistribution in packed beds for low column/particle diameter ratio ?

Müller model porous medium

$$\varepsilon(r)=\varepsilon_b+(1-\varepsilon_b)J_0\left(\frac{a(D-2r)}{2d}\right)e^{-\frac{b(D-2r)}{2d}}$$

$$J_0(r)=\sum_{i=0}^{\infty}\frac{(-1)^i r^{2i}}{2^{2i}(i!)^2}$$



Radial porosity profile in bed (D/d=10)

$$\varepsilon_b=0.365+0.22\frac{d}{D}$$

$$a=\begin{cases}7.45-3.15\dfrac{d}{D} & \dfrac{D}{d}\in[2.02-13.0]\\ 7.45-11.25\dfrac{d}{D} & \dfrac{D}{d}\geq 13.0\end{cases}$$

$$b=0.315-0.725\frac{d}{D}$$

Wall bypass fraction

$$BP_w=\frac{\int_{D/2-d/2}^{D/2} r\,\varepsilon(r)v_z(r)dr}{\int_{D/2-d/2}^{D/2} r\,U_0dr}$$

Müller GE. Chem. Eng. Sci. 46 (1991) 706

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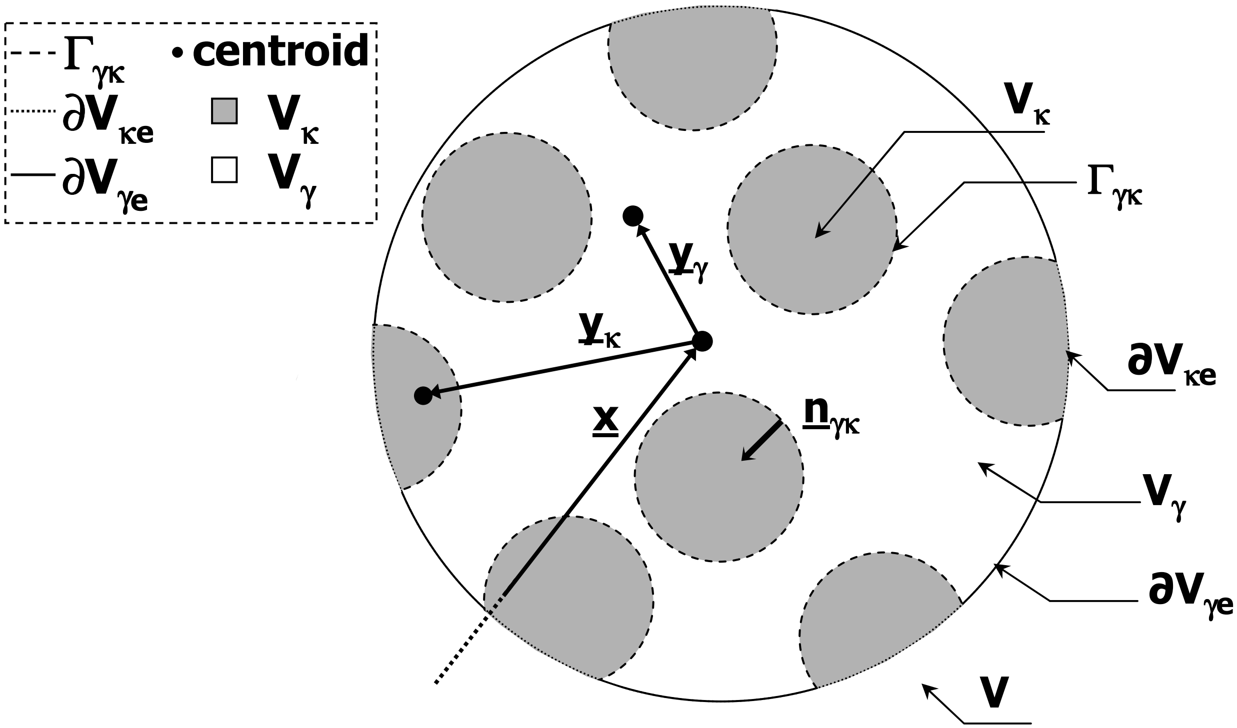
Microscale Cauchy problem (pore scale)

Details of averaging volume V



$$V = V_\gamma \cup V_\kappa \cup \Gamma_{\gamma\kappa}$$

$$\Gamma_{\gamma\kappa} = \left(\overline{V}_\gamma / V_\gamma\right) / \partial V_{\gamma e}$$

$$\partial V = \partial V_{\gamma e} \cup \partial V_{\kappa e}$$

Phase indicator function

$$\xi_\gamma(\underline{\mathbf{r}}) = \begin{cases} 1 & \text{if} \quad \underline{\mathbf{r}} \in V_\gamma \cup \Gamma_{\gamma\kappa} \\ 0 & \text{otherwise} \end{cases}$$

Whitaker S., The method of volume averaging, Kluwer, 1999
Drew DA., Passman SL., Theory of multicomponent fluids, Springer, 1999

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Microscale Cauchy problem (pore scale model)

Continuity

$$\nabla \cdot \xi_\gamma \underline{\mathbf{v}} = 0$$

Linear momentum

$$\frac{\partial}{\partial t}\xi_\gamma \rho \underline{\mathbf{v}} + \underline{\nabla} \cdot \xi_\gamma \rho \underline{\mathbf{v}} \otimes \underline{\mathbf{v}} = \underline{\nabla} \cdot \xi_\gamma \underline{\underline{\mathbf{T}}} + \underline{\nabla} \cdot \xi_\gamma \underline{\underline{\hat{\mathbf{M}}}} + \xi_\gamma \rho \underline{\mathbf{g}}$$

Internal angular momentum (spin)

$$\frac{\partial}{\partial t}\xi_\gamma \rho I \underline{\boldsymbol{\omega}} + \underline{\nabla} \cdot \xi_\gamma \rho I \underline{\boldsymbol{\omega}} \otimes \underline{\mathbf{v}} = \underline{\nabla} \cdot \xi_\gamma \underline{\underline{\mathbf{C}}} - \underline{\underline{\underline{\mathbf{E}}}} : \xi_\gamma \underline{\underline{\mathbf{T}}} + \xi_\gamma \underline{\mathbf{G}}$$

Magnetization relaxation

$$\frac{\partial}{\partial t}\xi_\gamma \underline{\mathbf{M}} + \underline{\nabla} \cdot \xi_\gamma \underline{\mathbf{M}} \otimes \underline{\mathbf{v}} = \xi_\gamma \underline{\boldsymbol{\omega}} \times \underline{\mathbf{M}} - \tau^{-1} \xi_\gamma \left(\underline{\mathbf{M}}(\underline{\mathbf{H}}) - \underline{\mathbf{M}}_\ell \left(\underline{\mathbf{H}}^e \right) \right)$$

Maxwell flux law

$$\nabla \cdot \xi_\gamma \mu_0 \left(\underline{\mathbf{H}} + \underline{\mathbf{M}} \right) = 0$$

$$\nabla \cdot \xi_\kappa \mu_0 \left(1 + \chi_\kappa \right) \underline{\mathbf{H}} = 0$$

$$\nabla \cdot \mu_0 \underline{\mathbf{H}}_0 = 0$$

Ampère-Maxwell law

$$\underline{\nabla} \times \xi_\gamma \left(\underline{\mathbf{H}}_0 + \underline{\mathbf{h}} \right) = \underline{\mathbf{0}}$$

$$\underline{\nabla} \times \xi_\kappa \left(\underline{\mathbf{H}}_0 + \underline{\mathbf{h}} \right) = \underline{\mathbf{0}}$$

$$\underline{\nabla} \times \underline{\mathbf{H}}_0 = \underline{\mathbf{0}}$$

Slipless no-penetration on $\Gamma_{\gamma\kappa}$

$$\underline{\mathbf{n}}_{\gamma\kappa} \cdot \underline{\mathbf{v}} = 0 \qquad \underline{\mathbf{n}}_{\gamma\kappa} \times \underline{\mathbf{v}} = \underline{\mathbf{0}}$$

Spin equation extension onto $\Gamma_{\gamma\kappa}$

Magnetization relax. extension onto $\Gamma_{\gamma\kappa}$

Continuity induced field tangent components on $\Gamma_{\gamma\kappa}$

$$\lim_\kappa \underline{\mathbf{h}} \times \underline{\mathbf{n}}_{\gamma\kappa} - \lim_\gamma \underline{\mathbf{h}} \times \underline{\mathbf{n}}_{\gamma\kappa} = \underline{\mathbf{h}}^\kappa \times \underline{\mathbf{n}}_{\gamma\kappa} - \underline{\mathbf{h}}^\gamma \times \underline{\mathbf{n}}_{\gamma\kappa} = \underline{\mathbf{0}}$$

Discontinuity induced field normal components on $\Gamma_{\gamma\kappa}$

$$\lim_\kappa \underline{\mathbf{h}} \cdot \underline{\mathbf{n}}_{\gamma\kappa} - \lim_\gamma \underline{\mathbf{h}} \cdot \underline{\mathbf{n}}_{\gamma\kappa} = \underline{\mathbf{h}}^\kappa \cdot \underline{\mathbf{n}}_{\gamma\kappa} - \underline{\mathbf{h}}^\gamma \cdot \underline{\mathbf{n}}_{\gamma\kappa} = \underline{\mathbf{M}} \cdot \underline{\mathbf{n}}_{\gamma\kappa}$$

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Microscale Cauchy problem (constitutive equations)

Cauchy stress tensor

$$\xi_\gamma \underline{\underline{\mathbf{T}}} = \xi_\gamma \left(-p\underline{\underline{\mathbf{I}}} + \eta\left(\underline{\nabla}\,\underline{\mathbf{v}} + {}^t\underline{\nabla}\,\underline{\mathbf{v}}\right) + \lambda\left(\nabla\cdot\underline{\mathbf{v}}\right)\underline{\underline{\mathbf{I}}} + \zeta\,\underline{\underline{\underline{\mathbf{E}}}}\cdot\left(\underline{\nabla}\times\underline{\mathbf{v}} - 2\underline{\boldsymbol{\omega}}\right)\right)$$

Magnetic stress tensor

$$\xi_\gamma \underline{\underline{\hat{\mathbf{M}}}} = \xi_\gamma \left(-\frac{\mu_0}{2}\left[\left(\underline{\mathbf{H}}_0 + \underline{\mathbf{h}}\right)\cdot\left(\underline{\mathbf{H}}_0 + \underline{\mathbf{h}}\right)\right]\underline{\underline{\mathbf{I}}} + \mu_0\left(\underline{\mathbf{H}}_0 + \underline{\mathbf{h}}\right)\otimes\left(\underline{\mathbf{H}}_0 + \underline{\mathbf{h}} + \underline{\mathbf{M}}\right)\right)$$

Couple stress dyadic (short-range diffusive exchange of angular momentum)

$$\xi_\gamma \underline{\underline{\mathbf{C}}} = \xi_\gamma \left(\eta'\left(\underline{\nabla}\,\underline{\boldsymbol{\omega}} + {}^t\underline{\nabla}\,\underline{\boldsymbol{\omega}}\right) + \lambda'\nabla\cdot\underline{\boldsymbol{\omega}}\,\underline{\underline{\mathbf{I}}}\right)$$

Body couple density (magnetic torque)

$$\xi_\gamma \underline{\mathbf{G}} = \xi_\gamma\, \mu_0 \underline{\mathbf{M}}\times\left(\underline{\mathbf{H}}_0 + \underline{\mathbf{h}}\right)$$

Langevin law

$$\xi_\gamma \underline{\mathbf{M}}_\ell = \xi_\gamma\, \phi_m \overline{m}\left[\coth\left\{\frac{\pi}{6} d_m^3 \frac{\mu_0 \overline{m}}{kT}\left\|\underline{\mathbf{H}}_0 + \underline{\mathbf{h}}^{\mathrm{e}}\right\|\right\} - \frac{6}{\pi d_m^3}\frac{kT}{\mu_0 \overline{m}}\left\|\underline{\mathbf{H}}_0 + \underline{\mathbf{h}}^{\mathrm{e}}\right\|^{-1}\right]\frac{\underline{\mathbf{H}}_0 + \underline{\mathbf{h}}^{\mathrm{e}}}{\left\|\underline{\mathbf{H}}_0 + \underline{\mathbf{h}}^{\mathrm{e}}\right\|}$$

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Volume Averaging

ϕ bounded piecewise continuous scalar/vector/tensor field (of $\underline{x}$ + $\underline{y}$) in $\overline{V}$

Superficial average

$$\left\langle \xi_\gamma \varphi\big|_{\underline{x}+\underline{y}} \right\rangle\Big|_{\underline{x}} = \frac{1}{|V|}\int_V \xi_\gamma \varphi\big|_{\underline{x}+\underline{y}}\, d\tau = \frac{1}{|V|}\int_{V_\gamma} \varphi\big|_{\underline{x}+\underline{y}}\, d\tau$$

Intrinsic average

$$\left\langle \xi_\gamma \varphi\big|_{\underline{x}+\underline{y}} \right\rangle_\gamma\Big|_{\underline{x}} = \frac{1}{|V_\gamma|}\int_{V_\gamma} \xi_\gamma \varphi\big|_{\underline{x}+\underline{y}}\, d\tau = \frac{1}{|V_\gamma|}\int_{V_\gamma} \varphi\big|_{\underline{x}+\underline{y}}\, d\tau$$

Local volume fraction

$$\left\langle \xi_\gamma\big|_{\underline{x}+\underline{y}} \right\rangle\Big|_{\underline{x}} = \frac{1}{|V|}\int_V \xi_\gamma\big|_{\underline{x}+\underline{y}}\, d\tau = \frac{|V_\gamma|}{|V|} = \varepsilon_\gamma$$

Gray (1975) spatial decomposition

$$\xi_\gamma \varphi\big|_{\underline{x}+\underline{y}} = \xi_\gamma \langle \varphi \rangle_\gamma\big|_{\underline{x}} + \xi_\gamma \tilde{\varphi}\big|_{\underline{y}}$$

Scale separation between pore level microscale (realm of $\underline{y}$) and porous medium macroscocpic scale (realm of $\underline{x}$) : *$l \ll L$*

Gray WG. Chem. Eng. Sci. 30 (1975) 229

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Volume Averaging Theorem & Corollaries

Spatial derivative

$$\left\langle \underline{\nabla}\, \xi_\gamma\, \varphi\big|_{\underline{\mathbf{x}}+\underline{\mathbf{y}}} \right\rangle\Big|_{\underline{\mathbf{x}}} = \underline{\nabla} \left\langle \xi_\gamma \varphi \right\rangle\Big|_{\underline{\mathbf{x}}} + \frac{1}{|V|} \int_{\Gamma_{\gamma\kappa}} \varphi\big|_{\underline{\mathbf{x}}+\underline{\mathbf{y}}}\, \underline{\mathbf{n}}_{\gamma\kappa}\, d\sigma$$

Corollaries

$$\left\langle \underline{\underline{\nabla}}\, \xi_\gamma\, \underline{\boldsymbol{\varphi}}\big|_{\underline{\mathbf{x}}+\underline{\mathbf{y}}} \right\rangle\Big|_{\underline{\mathbf{x}}} = \underline{\underline{\nabla}} \left\langle \xi_\gamma \underline{\boldsymbol{\varphi}} \right\rangle\Big|_{\underline{\mathbf{x}}} + \frac{1}{|V|} \int_{\Gamma_{\gamma\kappa}} \underline{\boldsymbol{\varphi}}\big|_{\underline{\mathbf{x}}+\underline{\mathbf{y}}} \otimes \underline{\mathbf{n}}_{\gamma\kappa}\, d\sigma$$

$$\left\langle \nabla \cdot \xi_\gamma\, \underline{\boldsymbol{\varphi}}\big|_{\underline{\mathbf{x}}+\underline{\mathbf{y}}} \right\rangle\Big|_{\underline{\mathbf{x}}} = \nabla \cdot \left\langle \xi_\gamma \underline{\boldsymbol{\varphi}} \right\rangle\Big|_{\underline{\mathbf{x}}} + \frac{1}{|V|} \int_{\Gamma_{\gamma\kappa}} \underline{\boldsymbol{\varphi}}\big|_{\underline{\mathbf{x}}+\underline{\mathbf{y}}} \cdot \underline{\mathbf{n}}_{\gamma\kappa}\, d\sigma$$

$$\left\langle \underline{\nabla} \cdot \xi_\gamma\, \underline{\underline{\boldsymbol{\varphi}}}\big|_{\underline{\mathbf{x}}+\underline{\mathbf{y}}} \right\rangle\Big|_{\underline{\mathbf{x}}} = \underline{\nabla} \cdot \left\langle \xi_\gamma \underline{\underline{\boldsymbol{\varphi}}} \right\rangle\Big|_{\underline{\mathbf{x}}} + \frac{1}{|V|} \int_{\Gamma_{\gamma\kappa}} \underline{\underline{\boldsymbol{\varphi}}}\big|_{\underline{\mathbf{x}}+\underline{\mathbf{y}}} \cdot \underline{\mathbf{n}}_{\gamma\kappa}\, d\sigma$$

$$\left\langle \underline{\nabla} \times \xi_\gamma\, \underline{\boldsymbol{\varphi}}\big|_{\underline{\mathbf{x}}+\underline{\mathbf{y}}} \right\rangle\Big|_{\underline{\mathbf{x}}} = \underline{\nabla} \times \left\langle \xi_\gamma \underline{\boldsymbol{\varphi}} \right\rangle\Big|_{\underline{\mathbf{x}}} + \frac{1}{|V|} \int_{\Gamma_{\gamma\kappa}} \underline{\mathbf{n}}_{\gamma\kappa} \times \underline{\boldsymbol{\varphi}}\big|_{\underline{\mathbf{x}}+\underline{\mathbf{y}}}\, d\sigma$$

General derivative-integral interchange theorem

$$\left\langle \frac{\partial}{\partial t} \xi_\gamma \varphi \right\rangle\Big|_{\underline{\mathbf{x}}} = \frac{\partial}{\partial t} \left\langle \xi_\gamma \varphi \right\rangle\Big|_{\underline{\mathbf{x}}}$$

Whitaker S., The method of volume averaging, Kluwer, 1999

Drew DA., Passman SL., Theory of multicomponent fluids, Springer, 1999

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Upscaling (General Macroscopic FF Model)

Continuity

$$\nabla \cdot \varepsilon_\gamma \langle \mathbf{v} \rangle_\gamma = 0$$

Linear momentum

$$\rho \frac{\partial}{\partial t} \varepsilon_\gamma \langle \mathbf{v} \rangle_\gamma + \rho\, \varepsilon_\gamma \nabla \langle \mathbf{v} \rangle_\gamma \cdot \langle \mathbf{v} \rangle_\gamma + \boxed{\rho \nabla \cdot \varepsilon_\gamma \langle \tilde{\mathbf{v}} \otimes \tilde{\mathbf{v}} \rangle_\gamma} = -\varepsilon_\gamma \nabla \langle p \rangle_\gamma + \varepsilon_\gamma \rho \mathbf{g} + (\eta + \zeta)\varepsilon_\gamma \left(\Delta \langle \mathbf{v} \rangle_\gamma + \nabla \langle \mathbf{v} \rangle_\gamma \cdot \frac{\nabla \varepsilon_\gamma}{\varepsilon_\gamma} + \frac{\Delta \varepsilon_\gamma}{\varepsilon_\gamma} \langle \mathbf{v} \rangle_\gamma \right)$$

$$+ \boxed{\frac{1}{|\mathrm{V}|} \int_{\Gamma_{\gamma\kappa}} \left\{ -\tilde{p} \mathbf{I} + (\eta + \zeta) \nabla \left(\xi_\gamma \tilde{\mathbf{v}} \right) \right\} \cdot \mathbf{n}_{\gamma\kappa} \, d\sigma} + 2\zeta \varepsilon_\gamma \nabla \times \langle \boldsymbol{\omega} \rangle_\gamma + \varepsilon_\gamma \nabla \left(\mathbf{H}_0 + \langle \mathbf{h} \rangle_\gamma \right) \cdot \langle \mathbf{M} \rangle_\gamma$$

$$+ \boxed{2\zeta \frac{1}{|\mathrm{V}|} \int_{\Gamma_{\gamma\kappa}} \mathbf{n}_{\gamma\kappa} \times \tilde{\boldsymbol{\omega}} \, d\sigma} + \boxed{\left(\frac{1}{|\mathrm{V}|} \int_{\Gamma_{\gamma\kappa}} \left(\tilde{\mathbf{H}}_0 + \tilde{\mathbf{h}}^\gamma \right) \otimes \mathbf{n}_{\gamma\kappa} \, d\sigma \right) \cdot \langle \mathbf{M} \rangle_\gamma} + \boxed{\varepsilon_\gamma \left\langle \nabla \left(\xi_\gamma \tilde{\mathbf{H}}_0 + \xi_\gamma \tilde{\mathbf{h}} \right) \cdot \left(\tilde{\mathbf{M}} \right) \right\rangle_\gamma}$$

Internal angular momentum (spin)

$$\rho I \frac{\partial}{\partial t} \varepsilon_\gamma \langle \boldsymbol{\omega} \rangle_\gamma + \rho I \varepsilon_\gamma \nabla \langle \boldsymbol{\omega} \rangle_\gamma \cdot \langle \mathbf{v} \rangle_\gamma + \boxed{\rho I \nabla \cdot \varepsilon_\gamma \langle \tilde{\boldsymbol{\omega}} \otimes \tilde{\mathbf{v}} \rangle_\gamma} = 2\zeta \varepsilon_\gamma \left(\nabla \times \langle \mathbf{v} \rangle_\gamma + \frac{\nabla \varepsilon_\gamma}{\varepsilon_\gamma} \times \langle \mathbf{v} \rangle_\gamma - 2 \langle \boldsymbol{\omega} \rangle_\gamma \right) + \mu_0 \varepsilon_\gamma \langle \mathbf{M} \rangle_\gamma \times \left(\mathbf{H}_0 + \langle \mathbf{h} \rangle_\gamma \right)$$

$$+ \boxed{\mu_0 \varepsilon_\gamma \left\langle \tilde{\mathbf{M}} \times \left(\tilde{\mathbf{H}}_0 + \tilde{\mathbf{h}} \right) \right\rangle_\gamma} + \eta' \varepsilon_\gamma \Delta \langle \boldsymbol{\omega} \rangle_\gamma + (\eta' + \lambda') \varepsilon_\gamma \nabla \left(\nabla \cdot \langle \boldsymbol{\omega} \rangle_\gamma \right) + \boxed{\eta' \nabla \cdot \left(\frac{1}{|\mathrm{V}|} \int_{\Gamma_{\gamma\kappa}} \tilde{\boldsymbol{\omega}} \otimes \mathbf{n}_{\gamma\kappa} \, d\sigma \right)} + \boxed{\frac{\eta'}{|\mathrm{V}|} \int_{\Gamma_{\gamma\kappa}} \left(\nabla \xi_\gamma \tilde{\boldsymbol{\omega}} \right) \cdot \mathbf{n}_{\gamma\kappa} \, d\sigma}$$

$$+ \boxed{(\eta' + \lambda') \nabla \left(\frac{1}{|\mathrm{V}|} \int_{\Gamma_{\gamma\kappa}} \tilde{\boldsymbol{\omega}} \cdot \mathbf{n}_{\gamma\kappa} \, d\sigma \right)} + \boxed{\frac{(\eta' + \lambda')}{|\mathrm{V}|} \int_{\Gamma_{\gamma\kappa}} \left(\nabla \cdot \xi_\gamma \tilde{\boldsymbol{\omega}} \right) \mathbf{n}_{\gamma\kappa} \, d\sigma}$$

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Upscaling (General Macroscopic FF Model), Cont'd

Magnetization relaxation

$$\frac{\partial}{\partial t}\varepsilon_\gamma\langle\underline{\mathbf{M}}\rangle_\gamma+\underline{\underline{\nabla}}\varepsilon_\gamma\langle\underline{\mathbf{M}}\rangle_\gamma\cdot\langle\underline{\mathbf{v}}\rangle_\gamma+\varepsilon_\gamma\langle\underline{\mathbf{M}}\rangle_v\left(\nabla\cdot\langle\underline{\mathbf{v}}\rangle_v\right)+\underline{\nabla}\cdot\varepsilon_\gamma\left\langle\underline{\tilde{\mathbf{M}}}\otimes\underline{\tilde{\mathbf{v}}}\right\rangle_\gamma=\varepsilon_\gamma\langle\underline{\boldsymbol{\omega}}\rangle_\gamma\times\langle\underline{\mathbf{M}}\rangle_\gamma+\varepsilon_\gamma\left\langle\underline{\tilde{\boldsymbol{\omega}}}\times\underline{\tilde{\mathbf{M}}}\right\rangle_\gamma-\tau^{-1}\varepsilon_\gamma\left(\langle\underline{\mathbf{M}}\rangle_\gamma-\langle\underline{\mathbf{M}}_\ell\rangle_\gamma\right)$$

Maxwell flux law

$$\nabla\cdot\left(\varepsilon_\gamma\langle\underline{\mathbf{h}}\rangle_\gamma+\varepsilon_\gamma\langle\underline{\mathbf{M}}\rangle_\gamma+\varepsilon_\kappa\langle\underline{\mathbf{h}}\rangle_\kappa\right)=0$$

$$\varepsilon_\kappa\nabla\cdot\langle\underline{\mathbf{h}}\rangle_\kappa-\frac{1}{|V|}\int_{\Gamma_{\gamma\kappa}}\underline{\tilde{\mathbf{h}}}^\kappa\cdot\underline{\mathbf{n}}_{\gamma\kappa}d\sigma=0$$

Ampère-Maxwell law

$$\underline{\nabla}\times\left(\varepsilon_\gamma\langle\underline{\mathbf{h}}\rangle_\gamma+\varepsilon_\kappa\langle\underline{\mathbf{h}}\rangle_\kappa\right)=\underline{\mathbf{0}}$$

$$\varepsilon_\kappa\nabla\times\langle\underline{\mathbf{h}}\rangle_\kappa-\frac{1}{|V|}\int_{\Gamma_{\gamma\kappa}}\underline{\tilde{\mathbf{h}}}^\kappa\times\underline{\mathbf{n}}_{\gamma\kappa}d\sigma=\underline{\mathbf{0}}$$

Volume conservation

$$\varepsilon_\gamma+\varepsilon_\kappa=1$$

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Closure Problem

15 closure equations to be set

Drag force closure (Ergun equation)

$$\frac{1}{|\mathrm{V}|}\int_{\Gamma_{\gamma\kappa}}\left\{-\tilde{p}\underline{\underline{\mathbf{I}}}+(\eta+\zeta)\underline{\underline{\nabla}}\left(\xi_\gamma\,\underline{\tilde{\mathbf{v}}}\right)\right\}\cdot\underline{\mathbf{n}}_{\gamma\kappa}d\sigma=-\left(\frac{150(\eta+\zeta)}{d^2}\left(\frac{1-\varepsilon_\gamma}{\varepsilon_\gamma}\right)^2+\frac{1.75\rho}{d}\frac{1-\varepsilon_\gamma}{\varepsilon_\gamma}\left\|\left\langle\underline{\mathbf{v}}\right\rangle_\gamma\right\|\right)\varepsilon_\gamma\left\langle\underline{\mathbf{v}}\right\rangle_\gamma$$

Shear(η')/bulk(λ') spin viscosity =0*

$$\eta'\underline{\nabla}\cdot\left(\frac{1}{|\mathrm{V}|}\int_{\Gamma_{\gamma\kappa}}\underline{\tilde{\boldsymbol{\omega}}}\otimes\underline{\mathbf{n}}_{\gamma\kappa}d\sigma\right)=0$$

$$\frac{\eta'}{|\mathrm{V}|}\int_{\Gamma_{\gamma\kappa}}\left(\underline{\underline{\nabla}}\xi_\gamma\underline{\tilde{\boldsymbol{\omega}}}\right)\cdot\underline{\mathbf{n}}_{\gamma\kappa}d\sigma=0$$

$$(\eta'+\lambda')\underline{\nabla}\left(\frac{1}{|\mathrm{V}|}\int_{\Gamma_{\gamma\kappa}}\underline{\tilde{\boldsymbol{\omega}}}\cdot\underline{\mathbf{n}}_{\gamma\kappa}d\sigma\right)=0$$

$$\frac{(\eta'+\lambda')}{|\mathrm{V}|}\int_{\Gamma_{\gamma\kappa}}\left(\nabla\cdot\xi_\gamma\underline{\tilde{\boldsymbol{\omega}}}\right)\underline{\mathbf{n}}_{\gamma\kappa}d\sigma=0$$

***Schumacher KR. et al. Phys. Rev. E 67 (2003) 026308**

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Closure Problem

Simplified mean field estimation theory:
$-\mu_\kappa / \mu_\gamma < 5$: negligible intergrain contact points

- **Linear magnetic material behavior (linear Langevin limit)**

$$\xi_\gamma \underline{\mathbf{M}}_\ell = \xi_\gamma \chi_0 \left(\underline{\mathbf{H}}_0 + \underline{\mathbf{h}}^{\mathrm{e}}\right)$$

- **Effective medium magnetic permeability <μ> of FF/porous medium composite***

$$\langle \mu(r) \rangle = \mu_\gamma \left(1 + 2(1-\varepsilon(r)) \frac{\mu_\kappa - \mu_\gamma}{\mu_\kappa + 2\mu_\gamma} \right) \left(1 - (1-\varepsilon(r)) \frac{\mu_\kappa - \mu_\gamma}{\mu_\kappa + 2\mu_\gamma} \right)^{-1}$$

- **Intrinsic volume-average demagnetizing and granular induced magnetic fields**

$$\Xi = \chi_0 (3 + 2\chi_0)^{-1}$$

$$\langle \underline{\mathbf{h}} \rangle_\kappa = (1 + \Xi) \langle \underline{\mathbf{h}} \rangle_\gamma$$

- **Eight closure terms need to be closed:**

$$\rho \underline{\nabla} \cdot \varepsilon_\gamma \langle \tilde{\underline{\mathbf{v}}} \otimes \tilde{\underline{\mathbf{v}}} \rangle_\gamma, \quad 2\zeta \frac{1}{|\mathrm{V}|} \int_{\Gamma_{\gamma\kappa}} \underline{\mathbf{n}}_{\gamma\kappa} \times \tilde{\underline{\boldsymbol{\omega}}} \, d\sigma, \quad \left(\frac{1}{|\mathrm{V}|} \int_{\Gamma_{\gamma\kappa}} \left(\tilde{\underline{\mathbf{H}}}_0 + \tilde{\underline{\mathbf{h}}}^\gamma \right) \otimes \underline{\mathbf{n}}_{\gamma\kappa} d\sigma \right) \cdot \langle \underline{\mathbf{M}} \rangle_\gamma, \quad \varepsilon_\gamma \left\langle \underline{\underline{\nabla}} \left(\xi_\gamma \tilde{\underline{\mathbf{H}}}_0 + \xi_\gamma \tilde{\underline{\mathbf{h}}} \right) \cdot \left(\tilde{\underline{\mathbf{M}}} \right) \right\rangle_\gamma$$

$$\rho I \underline{\nabla} \cdot \varepsilon_\gamma \langle \tilde{\underline{\boldsymbol{\omega}}} \otimes \tilde{\underline{\mathbf{v}}} \rangle_\gamma, \quad \mu_0 \varepsilon_\gamma \left\langle \tilde{\underline{\mathbf{M}}} \times \left(\tilde{\underline{\mathbf{H}}}_0 + \tilde{\underline{\mathbf{h}}} \right) \right\rangle_\gamma, \quad \underline{\nabla} \cdot \varepsilon_\gamma \left\langle \tilde{\underline{\mathbf{M}}} \otimes \tilde{\underline{\mathbf{v}}} \right\rangle_\gamma, \quad \varepsilon_\gamma \left\langle \tilde{\underline{\boldsymbol{\omega}}} \times \tilde{\underline{\mathbf{M}}} \right\rangle_\gamma$$

***Kuzhir P. et al. European J. Mechanics B/Fluids 22 (2003) 331**

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Post facto justification for hydrodynamic/magnetostatics decoupling

$$\left\| \underline{\mathbf{H}}_0 \right\| >> \left\| \underline{\mathbf{h}} \right\|$$

$$\left\| \underline{\underline{\nabla}}\, \underline{\mathbf{H}}_0 \cdot \underline{\mathbf{M}} \right\| >> \left\| \underline{\underline{\nabla}}\, \underline{\mathbf{h}} \cdot \underline{\mathbf{M}} \right\|$$

Induced magnetic field and induced magnetic force neglected w/r to corresponding applied fields

$$\left| H_{0z} \right| >> \left| h_z \right|$$

$$\left| H_{0r} \right| >> \left| h_r \right|$$

$$\left| m_z H_{0r} - m_r H_{0z} \right| >> \left| m_z h_r - m_r h_z \right|$$

$$\left| m_r \frac{dH_{0r}}{dr} \right| >> \left| m_r \frac{\partial h_r}{\partial r} + m_z \frac{\partial h_r}{\partial z} \right|$$

$$\left| m_z \frac{dH_{0z}}{dz} \right| >> \left| m_z \frac{\partial h_z}{\partial z} + m_r \frac{\partial h_z}{\partial r} \right|$$

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Steady-state 0-order axisymmetric volume-average FF model
Decoupled HYDRODYNAMIC Submodel

Continuity

$$\int_0^{D/2} \varepsilon v_z r dr = \frac{U_0 D^2}{8}$$

Linear momentum (radial)

$$\rho\left(v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z}\right) = (\eta+\zeta)\left(\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r}\frac{\partial v_r}{\partial r} - \frac{v_r}{r^2} + \frac{\partial^2 v_r}{\partial z^2} + \frac{v_r}{\varepsilon}\left(\frac{d^2\varepsilon}{dr^2} + \frac{1}{r}\frac{d\varepsilon}{dr}\right) + \frac{1}{\varepsilon}\frac{d\varepsilon}{dr}\frac{\partial v_r}{\partial r}\right)$$

$$-2\zeta\frac{\partial \omega_\theta}{\partial z} + \frac{\mu_0}{\varepsilon}\left(m_r\left(\frac{dH_{0r}}{dr} + \xcancel{\frac{\partial h_r}{\partial r}}\right) + m_z \xcancel{\frac{\partial h_r}{\partial z}}\right) - \frac{v_r}{d}\frac{1-\varepsilon}{\varepsilon}\left(150\frac{\eta+\zeta}{d^2}\frac{1-\varepsilon}{\varepsilon} + 1.75\frac{\rho}{d}\sqrt{v_r^2 + v_z^2}\right)$$

Linear momentum (axial)

$$\rho\left(v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z}\right) = (\eta+\zeta)\left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} + \frac{\partial^2 v_z}{\partial z^2} + \frac{v_z}{\varepsilon}\left(\frac{d^2\varepsilon}{dr^2} + \frac{1}{r}\frac{d\varepsilon}{dr}\right) + \frac{1}{\varepsilon}\frac{d\varepsilon}{dr}\frac{\partial v_z}{\partial r}\right) + 2\zeta\left(\frac{\partial \omega_\theta}{\partial r} + \frac{\omega_\theta}{r}\right)$$

$$+\frac{\mu_0}{\varepsilon}\left(m_z\left(\frac{dH_{0z}}{dz} + \xcancel{\frac{\partial h_z}{\partial z}}\right) + m_r \xcancel{\frac{\partial h_z}{\partial r}}\right) - \frac{v_z}{d}\frac{1-\varepsilon}{\varepsilon}\left(150\frac{\eta+\zeta}{d^2}\frac{1-\varepsilon}{\varepsilon} + 1.75\frac{\rho}{d}\sqrt{v_r^2 + v_z^2}\right) - \frac{dp}{dz} - \rho g$$

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Decoupled HYDRODYNAMIC Submodel, Cont'd

Linear momentum (azimuthal)

$$v_\theta = 0$$

Internal angular momentum (radial)

$$\omega_r = 0$$

Internal angular momentum (axial)

$$\omega_z = 0$$

Internal angular momentum (azimuthal)

$$\rho I\left(v_r \frac{\partial \omega_\theta}{\partial r} + v_z \frac{\partial \omega_\theta}{\partial z} \right) = 2\zeta\left(\frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r} - \frac{v_z}{\varepsilon}\frac{d\varepsilon}{dr} - 2\omega_\theta \right) + \frac{\mu_0}{\varepsilon}\left(m_z\left(H_{0r} + \cancel{h_r}\right) - m_r\left(H_{0z} + \cancel{h_z}\right)\right)$$

Magnetization relaxation (radial)

$$v_r \frac{\partial m_r}{\partial r} + v_z \frac{\partial m_r}{\partial z} + m_r\left(\frac{v_r}{r} + \frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z} \right) = -\omega_z m_\theta - \tau^{-1}\left(m_r - \varepsilon\, m_{\ell r} \right)$$

Magnetization relaxation (axial)

$$v_r \frac{\partial m_z}{\partial r} + v_z \frac{\partial m_z}{\partial z} + m_z\left(\frac{v_r}{r} + \frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z} \right) = -\omega_\theta m_r - \tau^{-1}\left(m_z - \varepsilon\, m_{\ell z} \right)$$

Magnetization relaxation (azimuthal)

$$m_\theta = 0$$

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Steady-state 0-order axisymmetric volume-average FF model
Decoupled MAGNETOSTATIC submodel

Maxwell flux law, FF demagnetizing field, *h_r*

$$\frac{m_r}{r}+\frac{\partial m_r}{\partial r}+\frac{\partial m_z}{\partial z}=-\varepsilon\left(\frac{h_r}{r}+\frac{\partial h_r}{\partial r}+\frac{\partial h_z}{\partial z}\right)+\Xi\left(H_{0r}+h_r\right)\frac{d\varepsilon}{dr}$$

$$-(1-\varepsilon)\left(\Xi\left(\frac{H_{0r}}{r}+\frac{\partial H_{0r}}{\partial r}+\frac{\partial H_{0z}}{\partial z}\right)+(\Xi+1)\left(\frac{h_r}{r}+\frac{\partial h_r}{\partial r}+\frac{\partial h_z}{\partial z}\right)\right)$$

Ampère-Maxwell law (azimuthal), *h_z*

$$\left(1+\Xi\left(1-\varepsilon\right)\right)\left(\frac{\partial h_r}{\partial z}-\frac{\partial h_z}{\partial r}\right)+\Xi\left(H_{0z}+h_z\right)\frac{d\varepsilon}{dr}=0$$

Azimuthal FF demagnetizing field, *h_θ*

$$h_\theta=0$$

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Boundary conditions

$\frac{d}{dz}\frac{dp}{dz}(L)=0$	$\frac{d^2}{dz^2}\frac{dp}{dz}(0)=0$		**Pressure field (continuity)**
$v_r(r,0)=0 \quad r\in[0;D/2]$	$\frac{\partial v_r}{\partial z}(r,L)=0 \quad r\in[0;D/2]$		**v_r**
$v_r(0,z)=0 \quad z\in\,]0;L[$	$v_r(D/2,z)=0 \quad z\in\,]0;L[$		
$v_z(r,0)=U_o \quad r\in[0;D/2]$	$\frac{\partial v_z}{\partial z}(r,L)=0 \quad r\in[0;D/2]$		**v_z**
$\frac{\partial v_z}{\partial r}(0,z)=0 \quad z\in\,]0;L[$	$v_z(D/2,z)=0 \quad z\in\,]0;L[$		
$\omega_\theta(r,0)=0 \quad r\in[0;D/2]$	$\frac{\partial^2\omega_\theta}{\partial z^2}(r,L)=0 \quad r\in[0;D/2]$		**ω_θ**
$\omega_\theta(0,z)=0 \quad z\in\,]0;L[$	$\omega_\theta(D/2,z): \quad z\in\,]0;L[$	Continuity extension	
$m_r(r,0)=0 \quad r\in[0;D/2]$	$m_r(r,L): \quad r\in\,]0;D/2]$	Continuity extension	**m_r**
$m_r(0,z)=0 \quad z\in\,]0;L]$	$m_r(D/2,z): \quad z\in\,]0;L[$	Continuity extension	
$m_z(r,0)=0 \quad r\in[0;D/2]$	$m_z(r,L): \quad r\in\,]0;D/2]$	Continuity extension	**m_z**
$\frac{\partial m_z}{\partial r}(0,z)=0 \quad z\in\,]0;L]$	$m_z(D/2,z): \quad z\in\,]0;L[$	Continuity extension	
$h_r(r,0)=0 \quad r\in[0;D/2]$	$h_r(r,L): \quad r\in\,]0;D/2]$	Continuity extension	**h_r**
$h_r(0,z)=0 \quad z\in\,]0;L]$	$h_r(D/2,z): \quad z\in\,]0;L[$	Continuity extension	
$h_z(r,0)=0 \quad r\in[0;D/2]$	$h_z(r,L): \quad r\in\,]0;D/2]$	Continuity extension	**h_z**
$\frac{\partial h_z}{\partial r}(0,z)=0 \quad z\in\,]0;L]$	$h_z(D/2,z): \quad z\in\,]0;L[$	Continuity extension	

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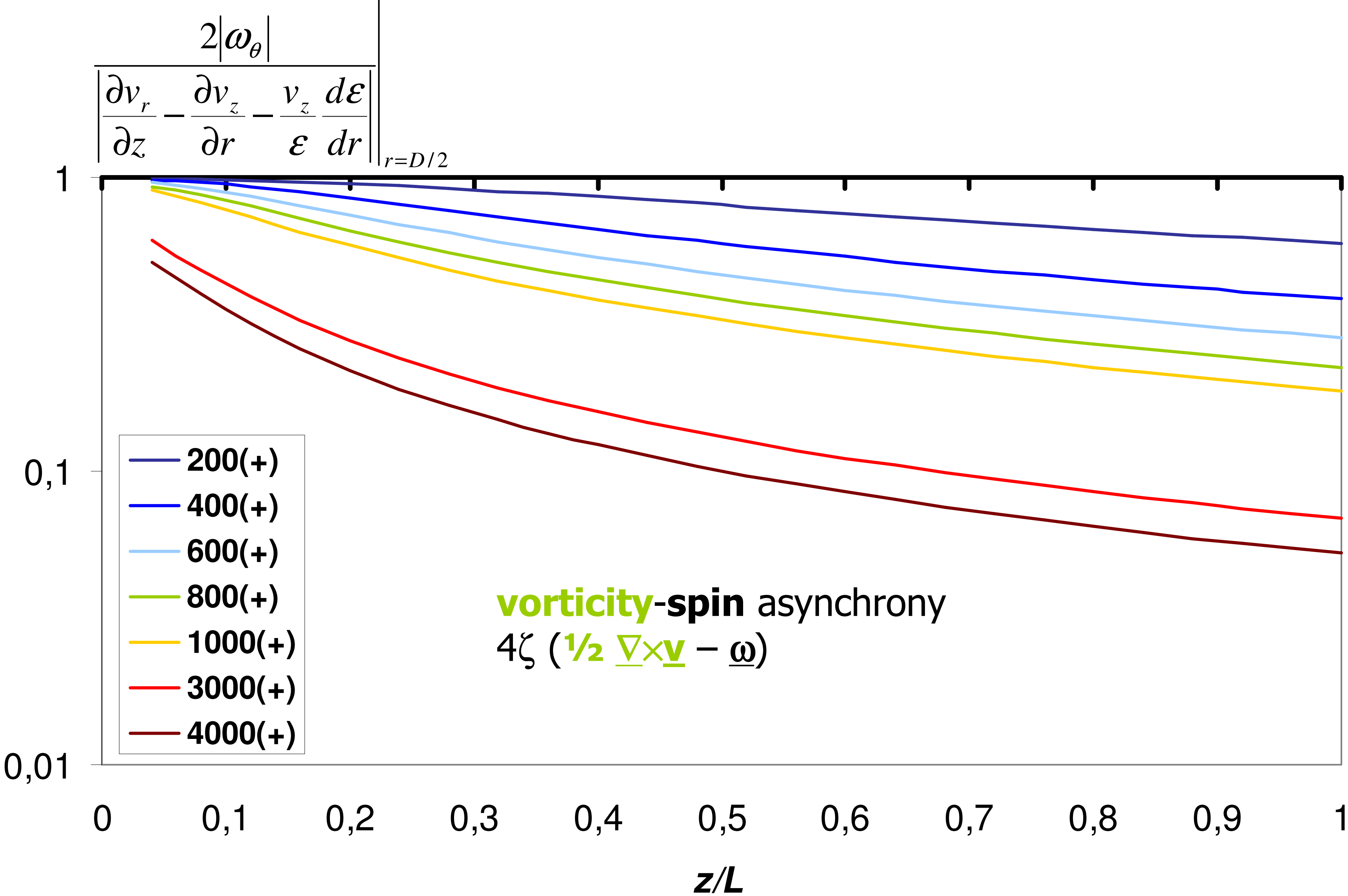
Magnetoviscothickening *versus* magnetoviscothinning?



Ratio of spin density to FF vorticity axial profile @ wall

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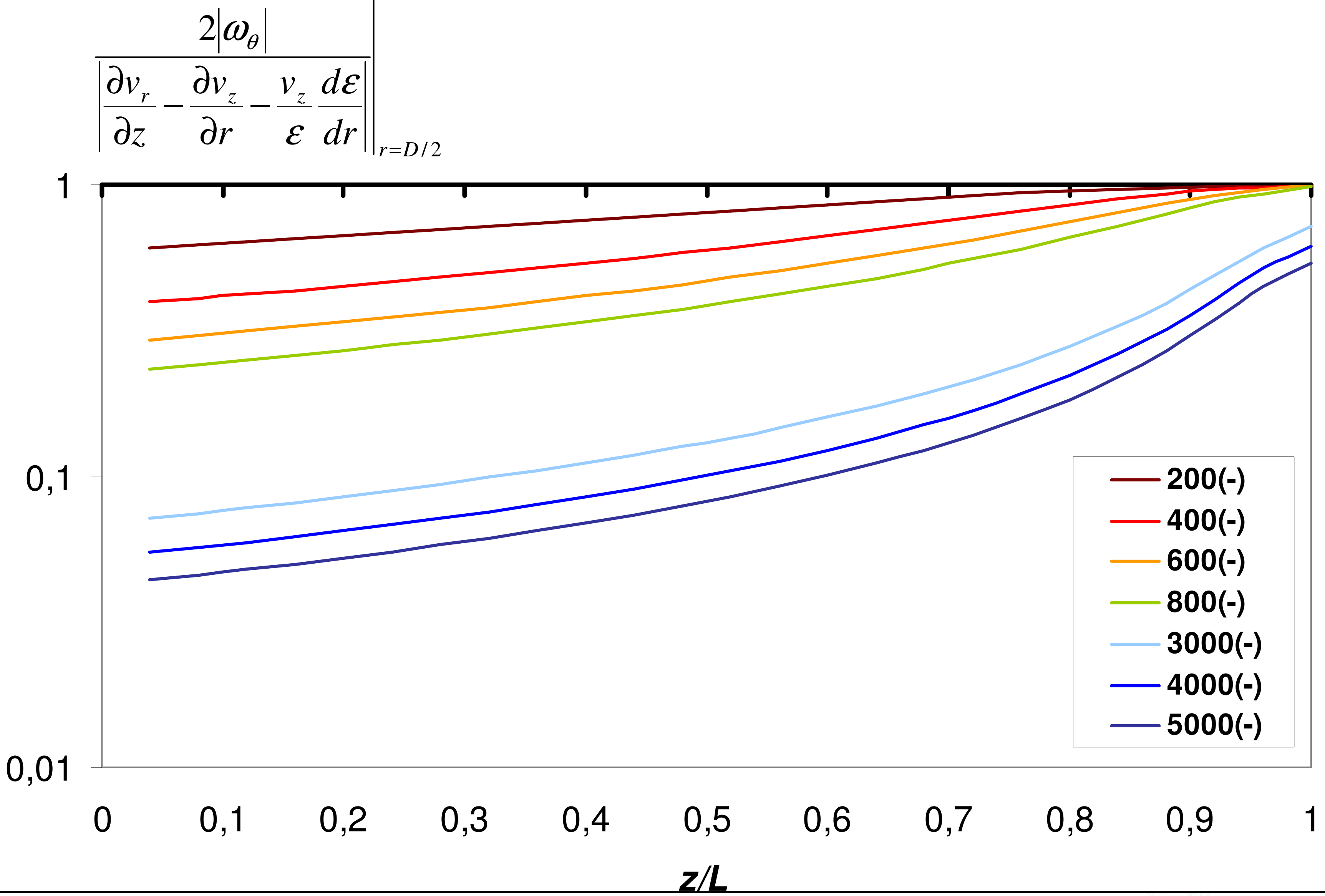
Magnetoviscothickening *versus* magnetoviscothinning?



Ratio of spin density to FF vorticity axial profile @ wall

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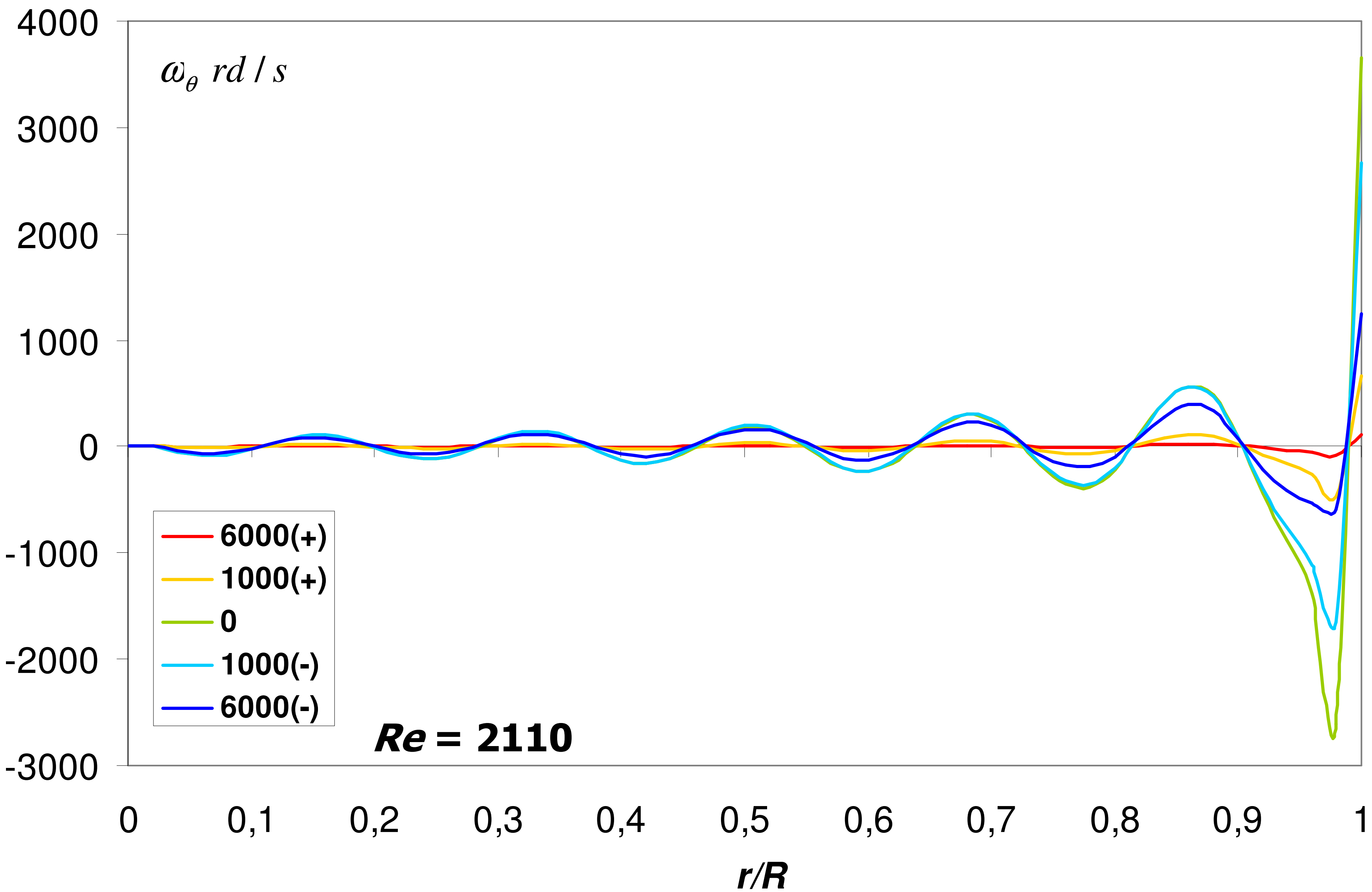
Other evidence of nanoparticle spin blockage



Spin density radial profile @ bed exit

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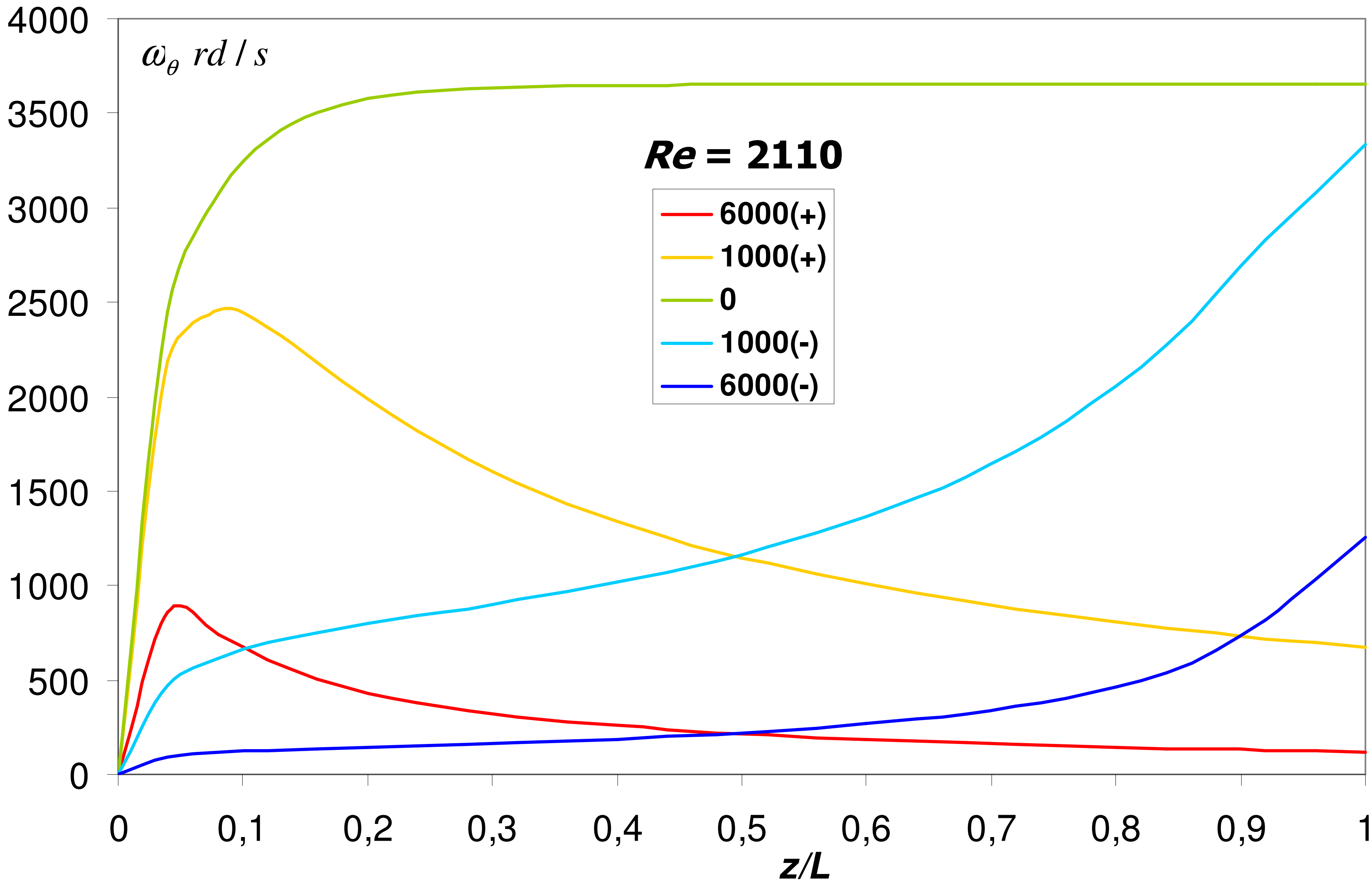
Other evidence of nanoparticle spin blockage



Spin density axial profile @ wall

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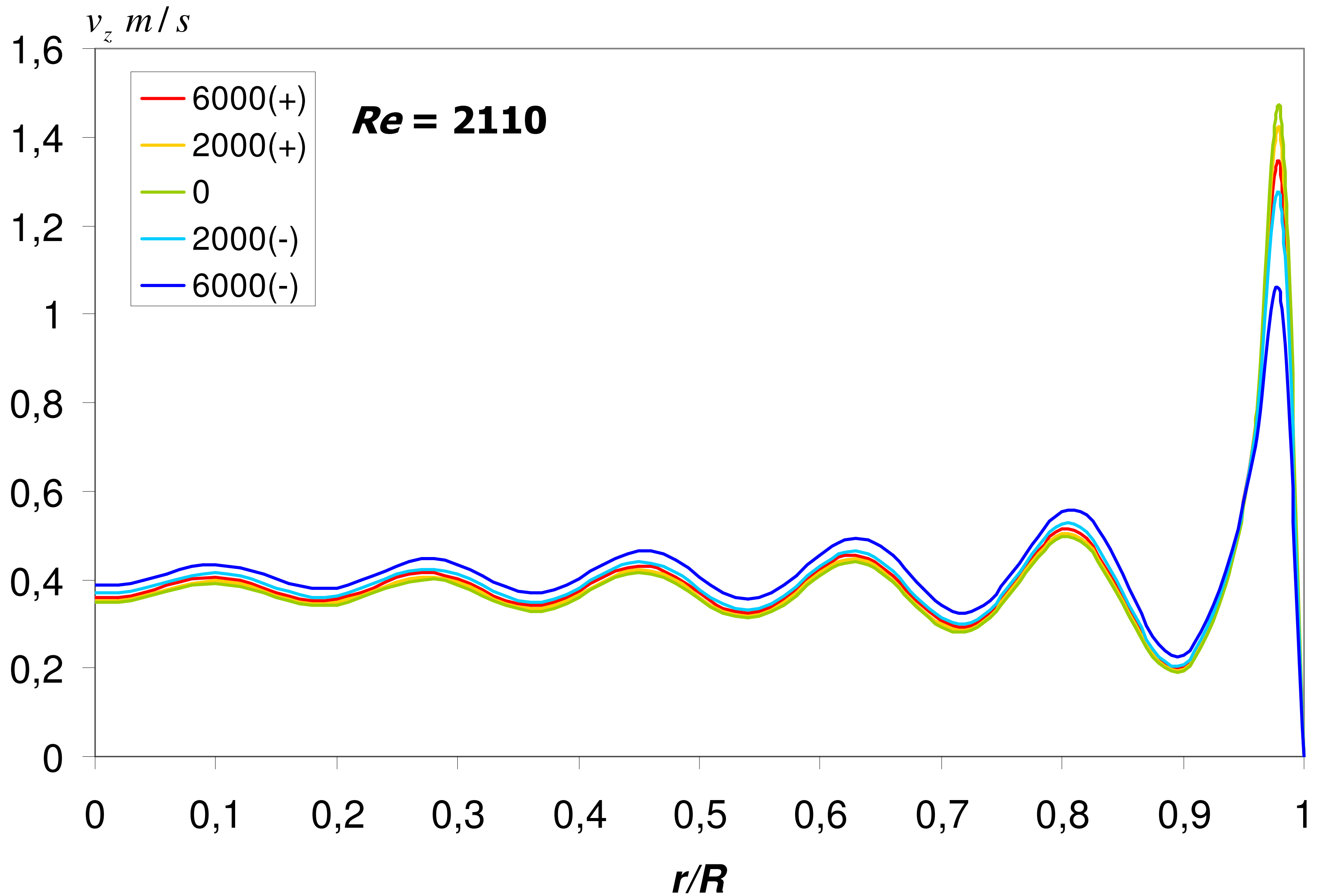
Evidence of mitigation of wall bypass fraction



Radial v_z profile of ferrofluid @ bed exit

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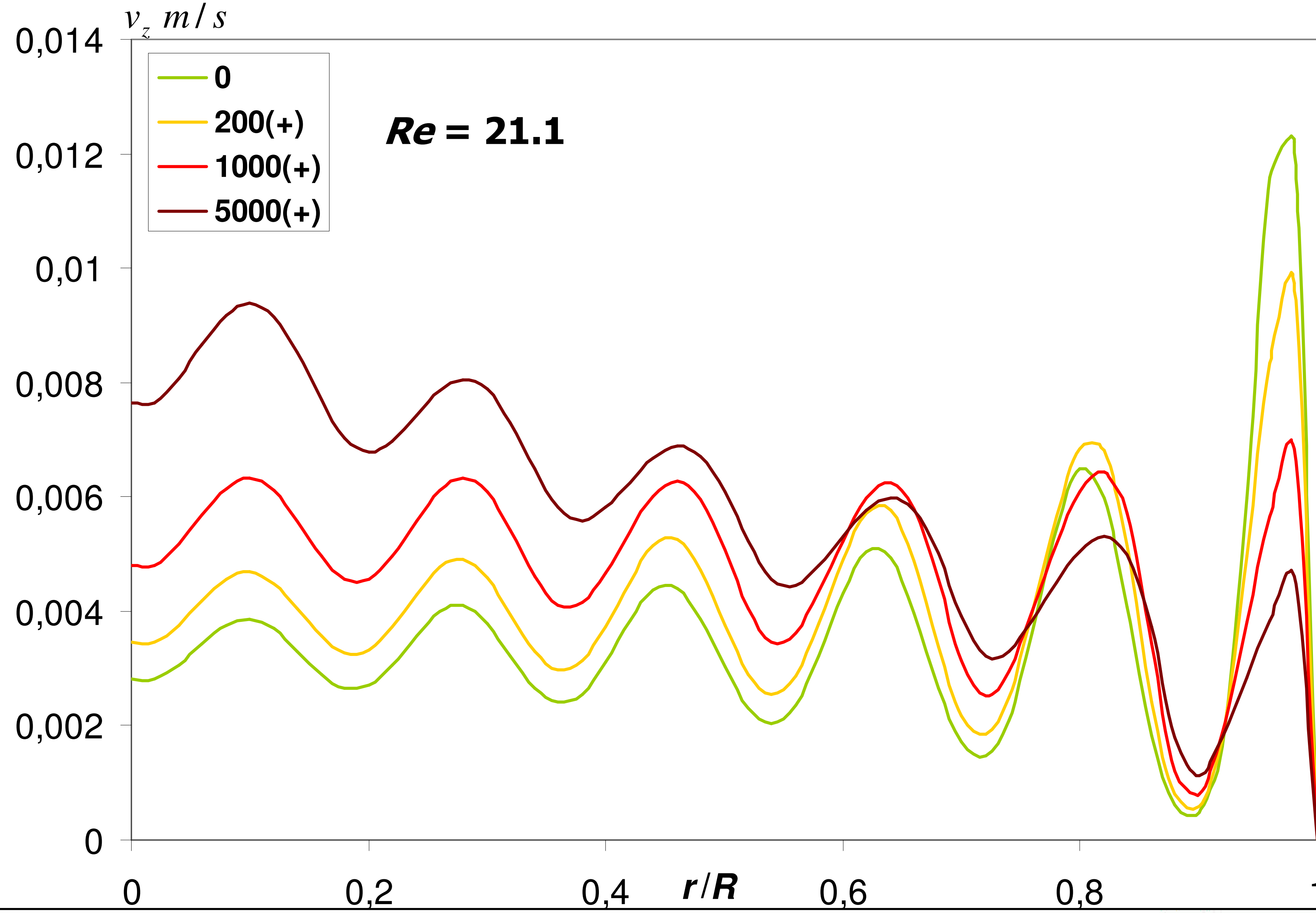
Evidence of mitigation of wall bypass fraction



Radial v_z profile of ferrofluid @ bed exit

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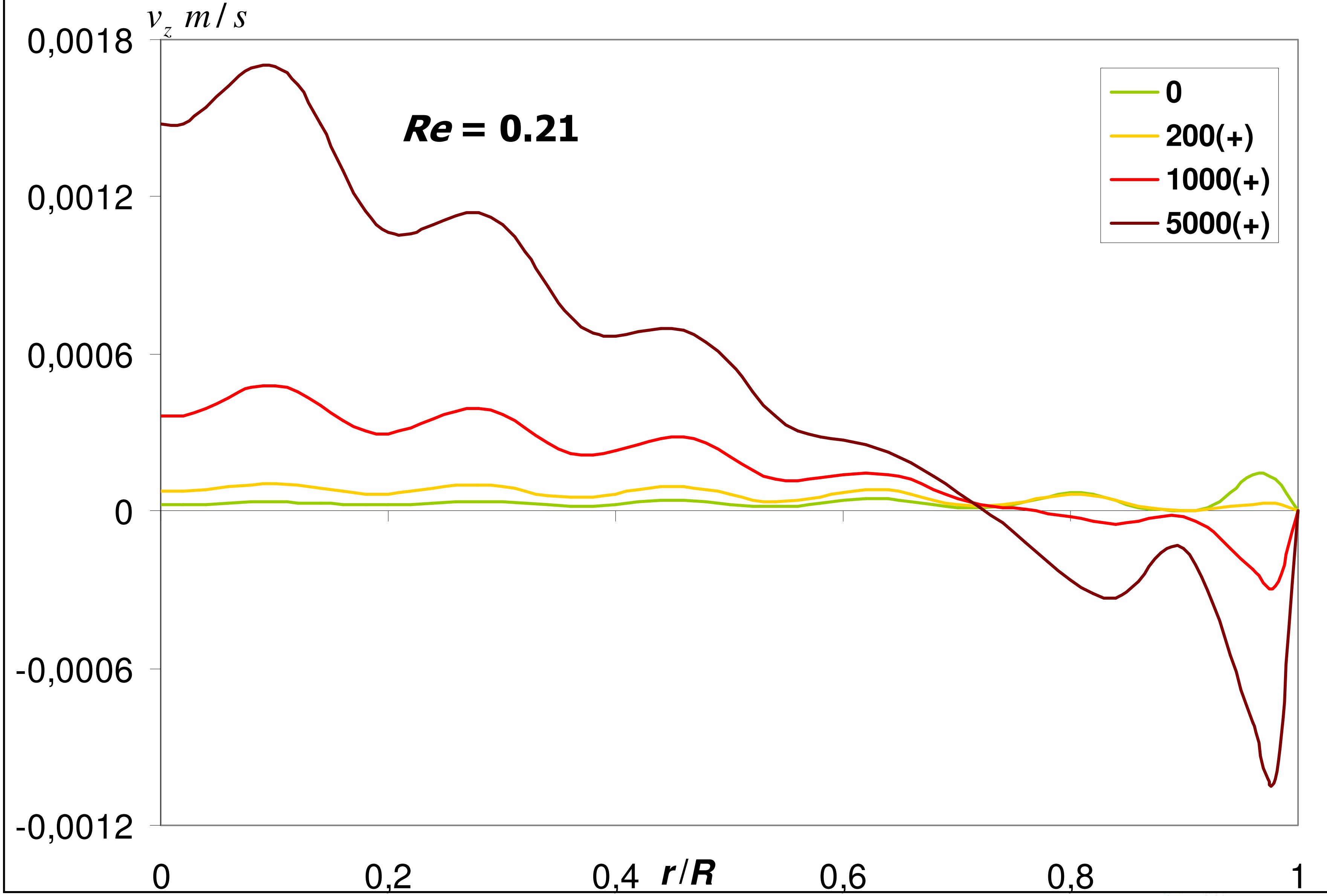
Evidence of mitigation of wall bypass fraction



Radial v_z profile of ferrofluid @ bed exit

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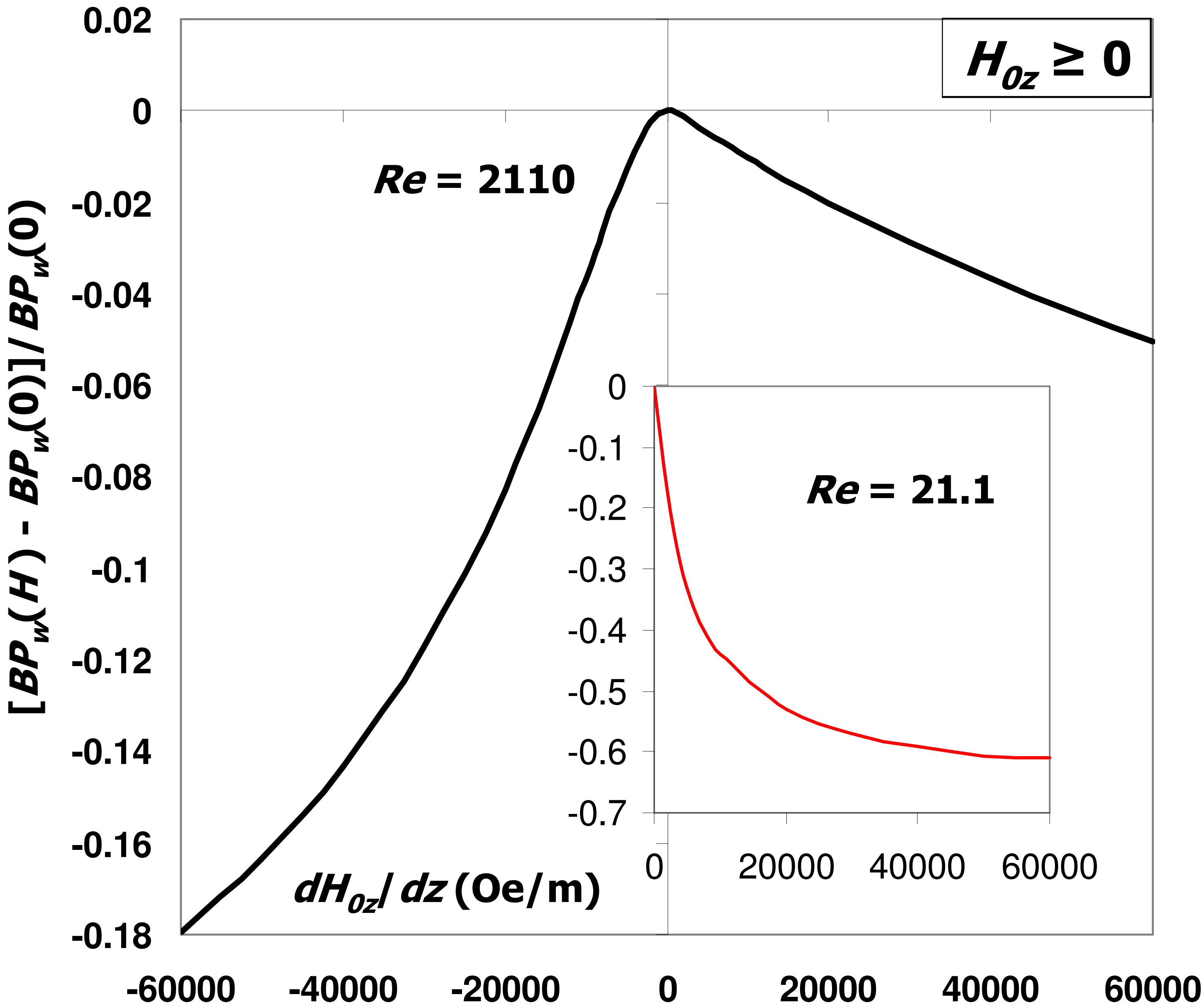
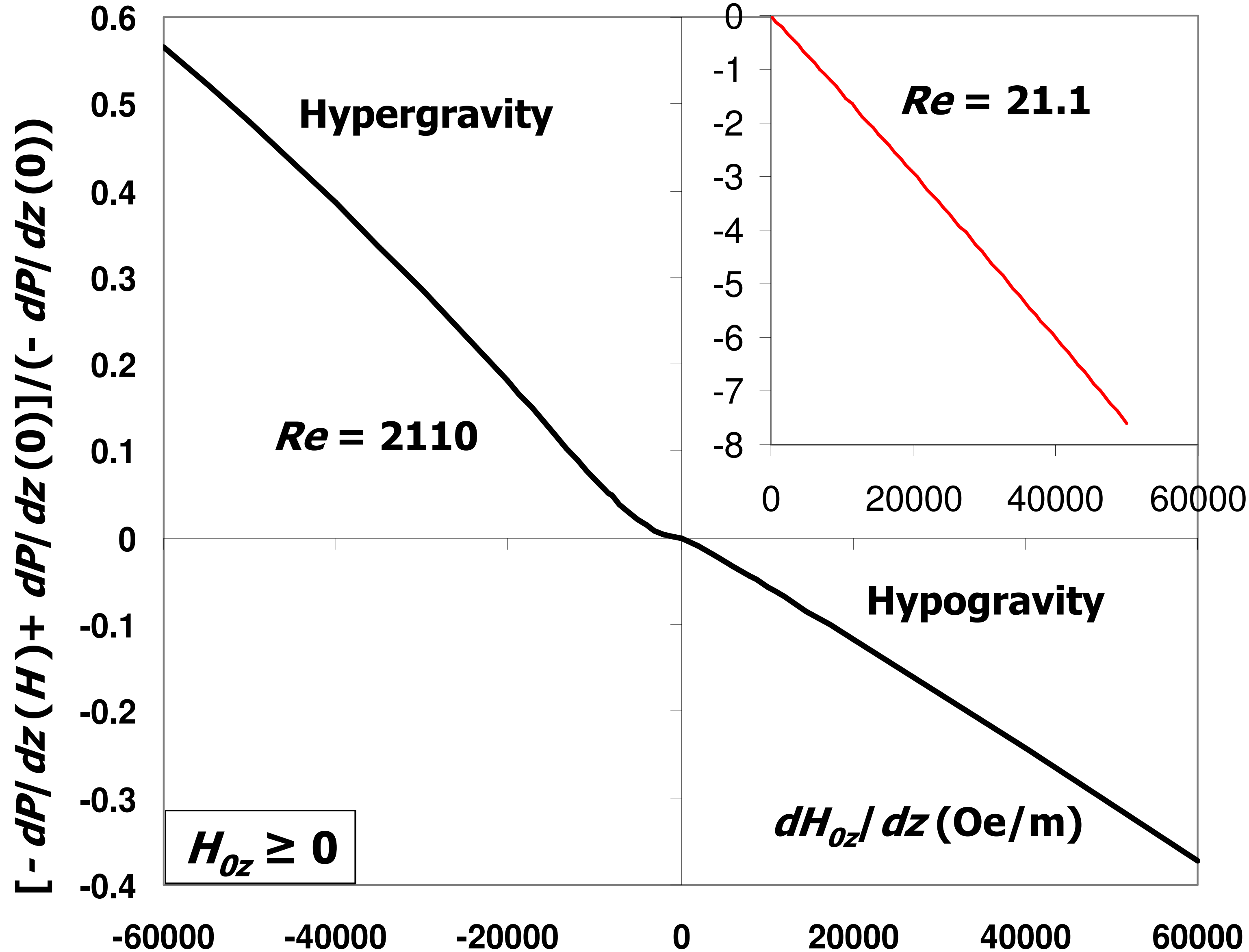
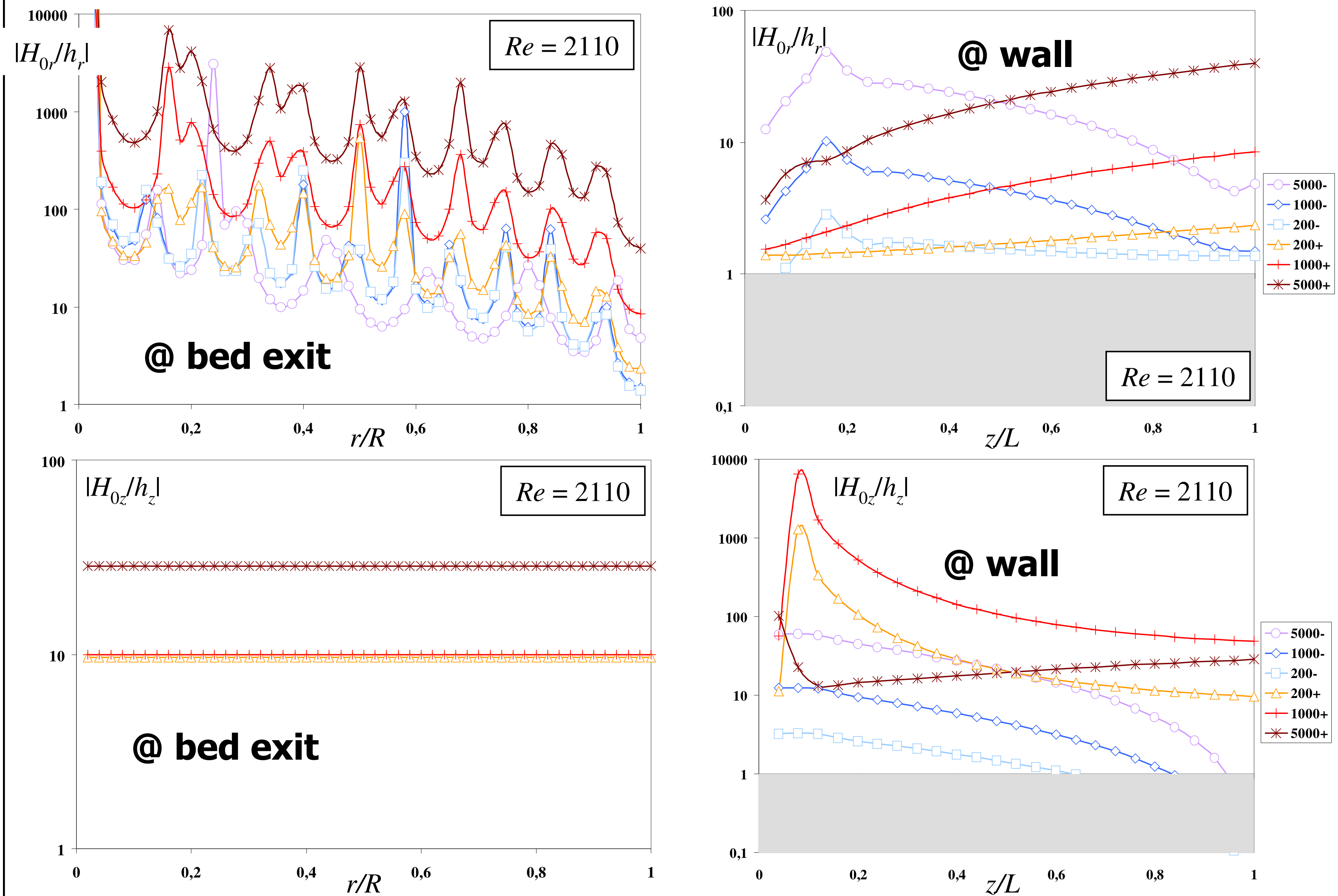
Evolution of wall bypass fraction vs. magnetic field gradient



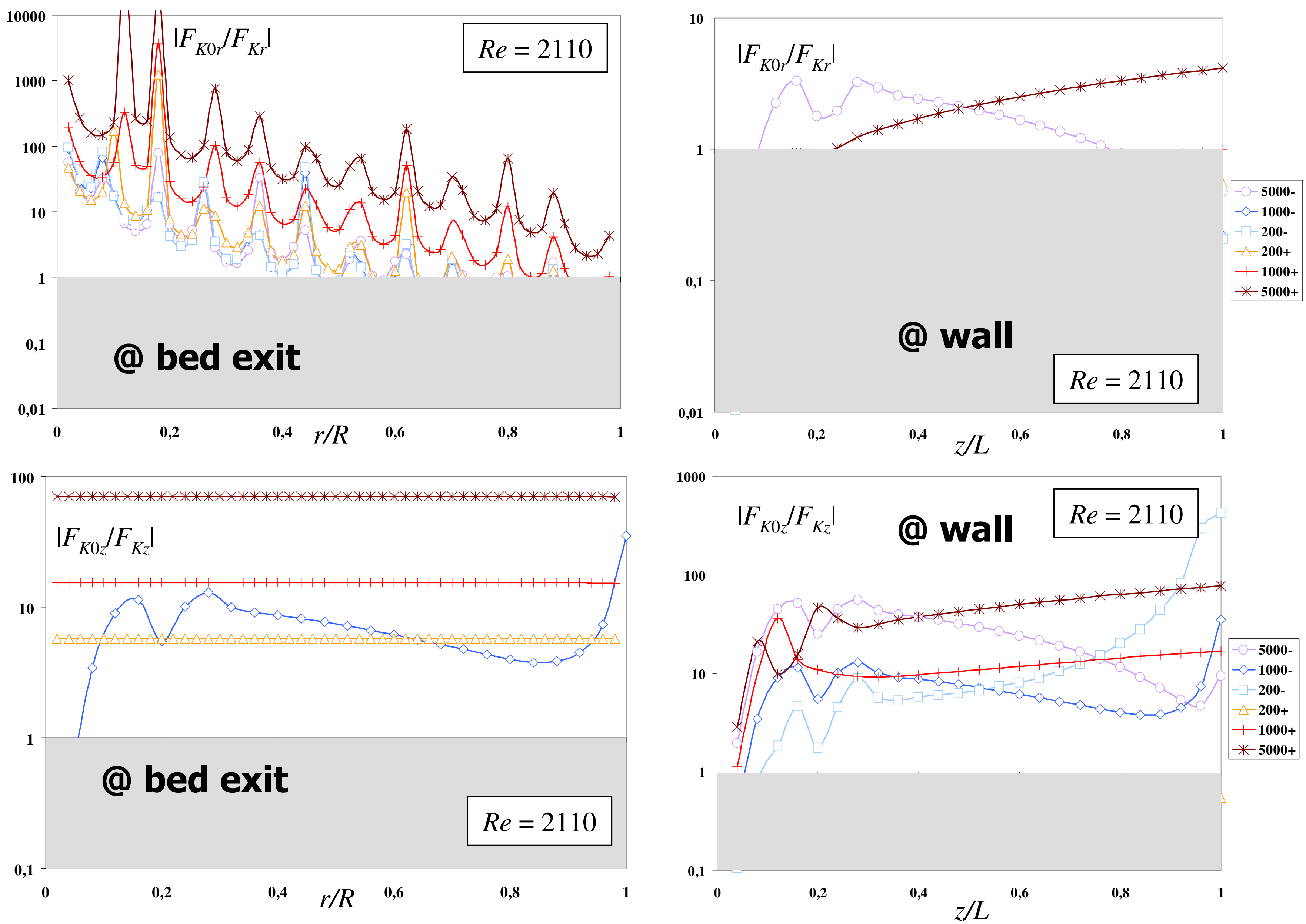
Evolution of pressure gradient vs. magnetic field gradient



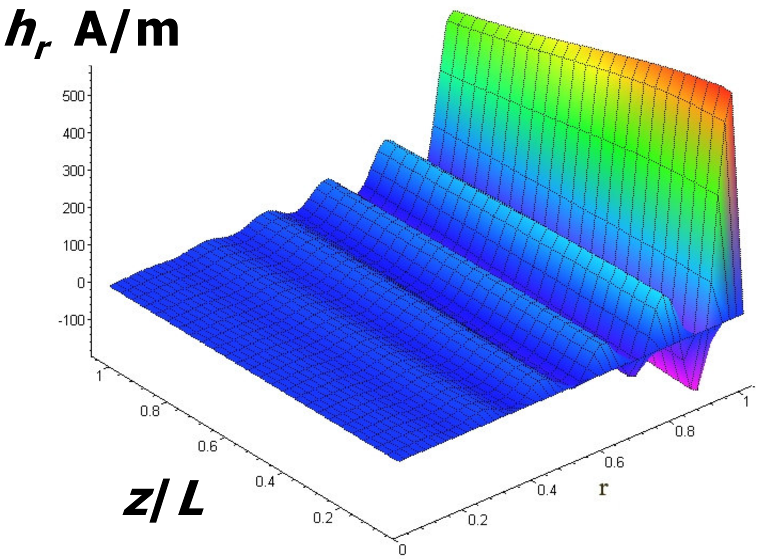
Induced vs. external magnetic fields: Hypothesis validation

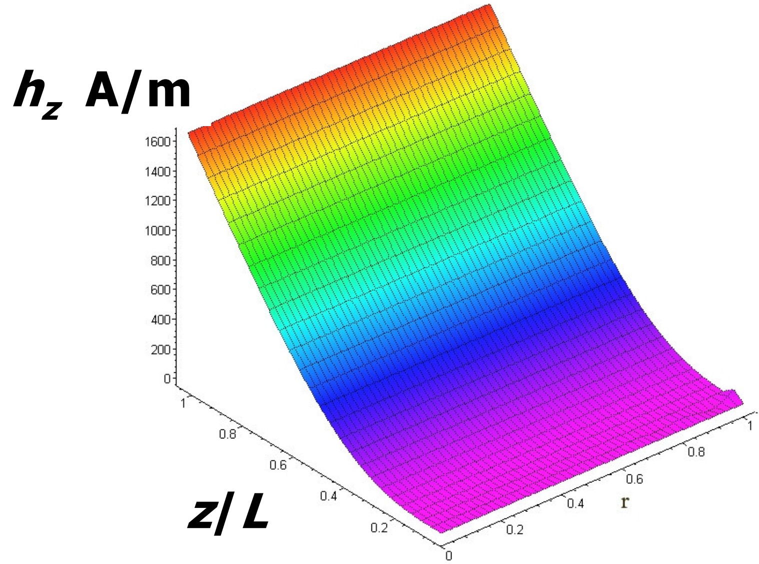


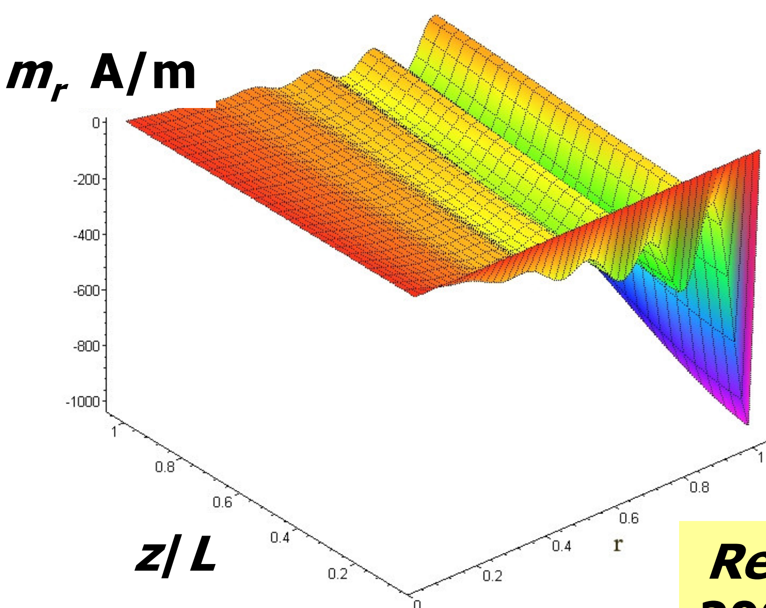
Induced vs. external Kelvin forces: Hypothesis validation

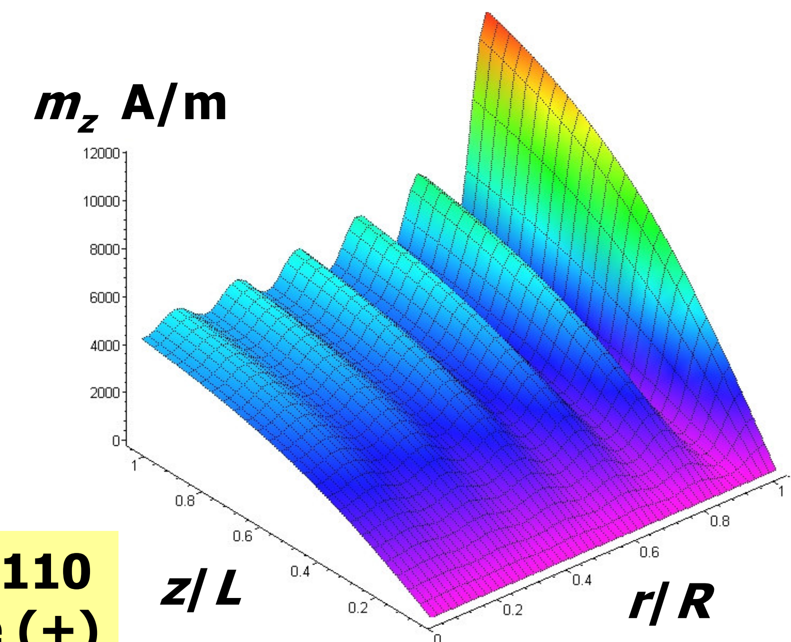


Induced magnetic field & magnetization field









***Re* = 2110**
200 Oe (+)

Ferrofluids, Conclusion

- Volume-average framework of FF flow in porous media using gradient magnetic fields

- Magnetostatic/hydrodynamic decoupling (negligible induced fields and their gradients) possible for positive high magnetic field gradient

- Magnetoviscothickening prevailing mechanism

- Substantial improvement of wall bypass achieved

- Pressure drop increases/decreases depending on whether hypergravity/hypogravity prevails in bed

- Possible chemical engineering applications in situations where low bed/particle diameter ratio unavoidable

- Experimental validation

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Non-magnetic fluid applications

-Evaluate ground-based artificial gravisensing of multiphase reactors using gradient magnetic fields

-Propose rational framework commuting magnetic effects into artificial gravity effects : hydrodynamics of non-magnetic para/diamagnetic fluids (weakly or nonelectrically conducting)

Case study - Magnet-Bore Fitted Miniature Trickle Bed

- **Measure two-phase pressure drop, liquid holdup & wetting efficiency; $B\nabla B$ ON & OFF: air/water; air/aqueous $MnCl_2$ (Mars-gravity: $g = 3.73\ m/s^2$)**
- **Propose 0-order volume-average formulation including $\vec{F}_m$**
- **Data analysis with slit model**

BACKGROUND

Magnetization Kelvin body force density

$$\underline{\mathbf{F}}_{\text{Kelvin}} = \mu_0 \underline{\underline{\nabla}} \left(\underline{\mathbf{H}} + \underline{\mathbf{h}}\right) \cdot \underline{\mathbf{M}}$$

Para ($\chi>0$)/diamagnetic ($\chi<0$) materials: $1+\chi \approx 1$

$$\underline{\mathbf{F}}_{\text{Kelvin}} = \frac{\chi}{\mu_0 (1+\chi)^2} \underline{\underline{\nabla}} \left(\underline{\mathbf{B}} + \underline{\mathbf{b}}\right) \cdot \left(\underline{\mathbf{B}} + \underline{\mathbf{b}}\right) \approx \frac{\chi}{\mu_0} \underline{\underline{\nabla}}\, \underline{\mathbf{B}} \cdot \underline{\mathbf{B}}$$

(STRONG Inhomogeneous magnetic induction : SUPERCONDUCTING MAGNET)

Net body force along gravitational field direction

$$\vec{F}_z^{\,m\oplus g} = \left(\rho g + \frac{\chi}{\mu_o} B_z \frac{dB_z}{dz} \right) \vec{e}_z$$

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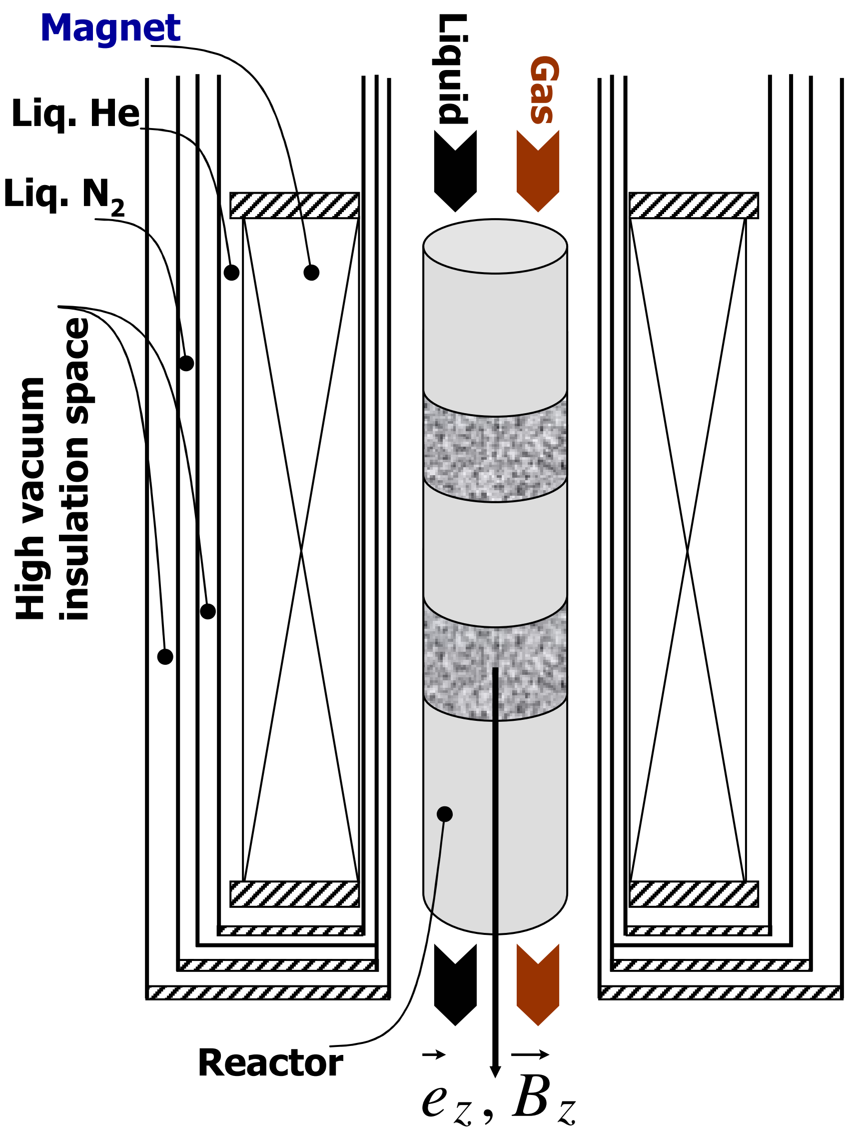
Artificial gravity sustained in a superconducting magnet



$$\vec{F}_{\alpha z}^{\,m\oplus g} = \rho_\alpha \tilde{g}_\alpha \vec{e}_z$$

Artificial gravity factor

$$\gamma_\alpha = \frac{\tilde{g}_\alpha}{g} = \left(1 + \frac{\chi_\alpha}{\rho_\alpha g \mu_o} B_z \frac{dB_z}{dz} \right)$$

Hypergravity

$$\left.\begin{matrix} \chi_\alpha > 0; \quad \nabla_z B_z^2 > 0 \\ \chi_\alpha < 0; \quad \nabla_z B_z^2 < 0 \end{matrix}\right\} \Rightarrow \quad \gamma_\alpha > 1$$

Hypogravity

$$\left.\begin{matrix} \chi_\alpha > 0; \quad \nabla_z B_z^2 < 0 \\ \chi_\alpha < 0; \quad \nabla_z B_z^2 > 0 \end{matrix}\right\} \Rightarrow \quad 0 < \gamma_\alpha < 1$$

Mars gravity $\gamma_\alpha = 0.38$

Levitation $\gamma_\alpha = 0$

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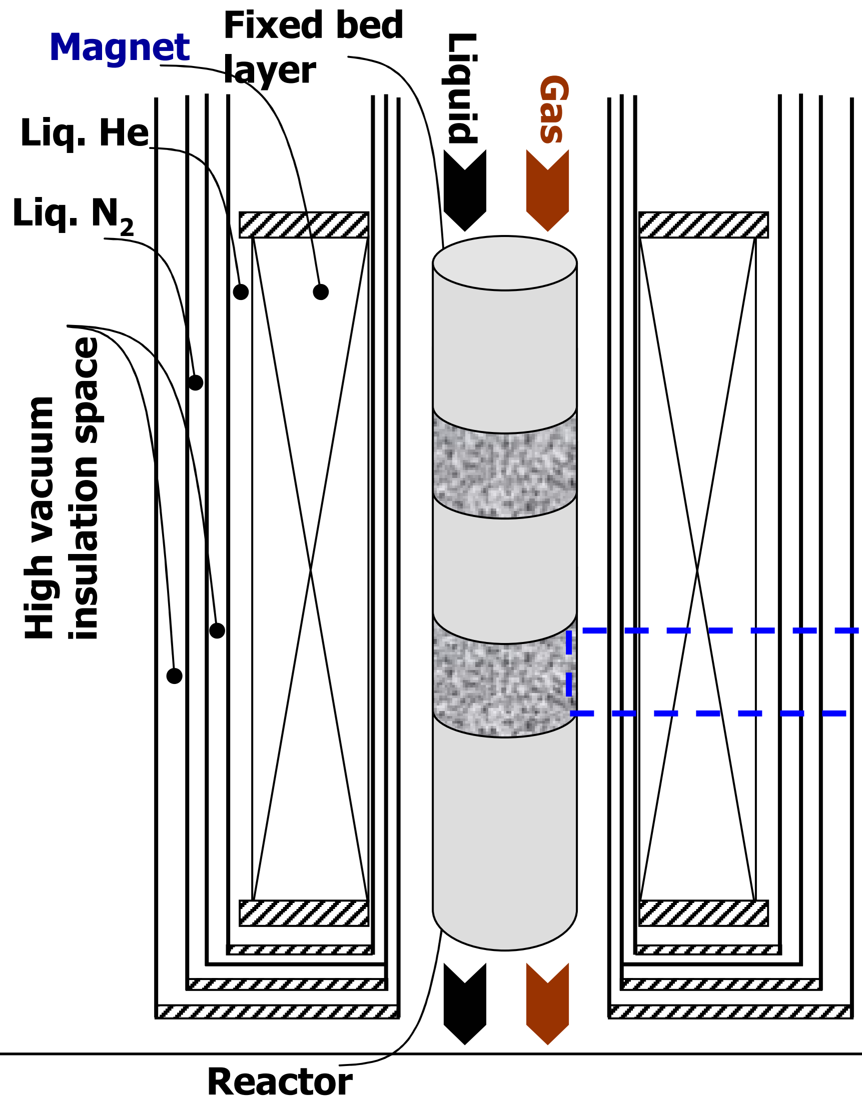
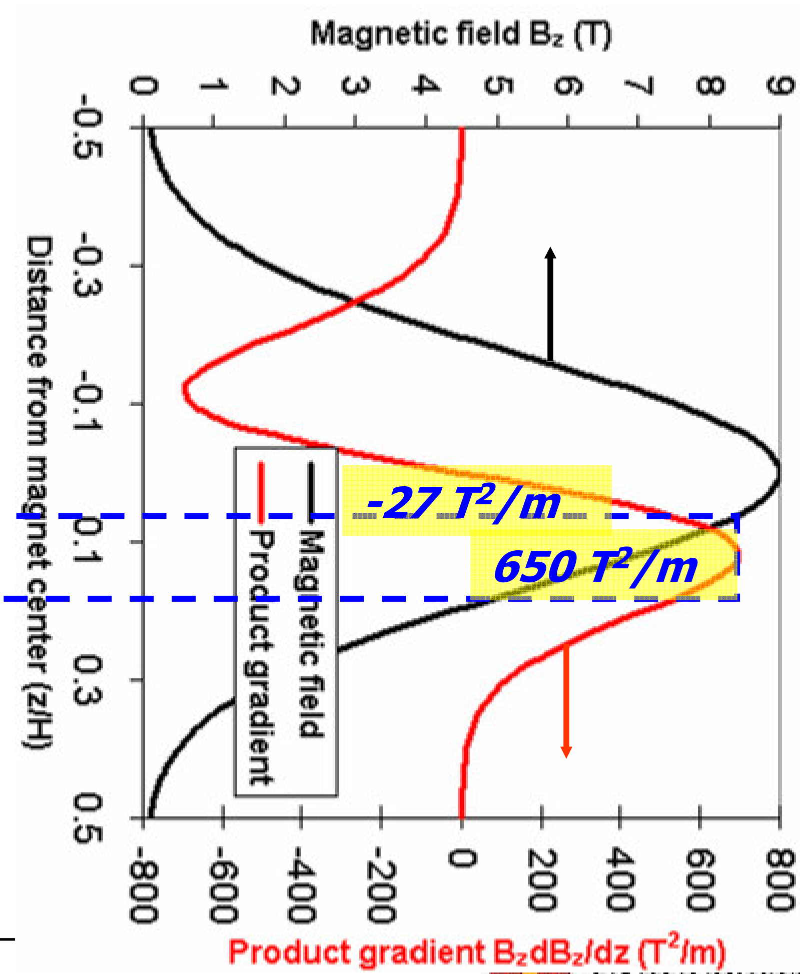
ULaval 9-T NbTi Superconducting Magnet Setup

Atmospheric magnet bore

Magnetic field & magnetic field gradient distributions in a 9 Tesla superconducting magnet (Trickle bed inside magnet bore)





ULaval 9-T NbTi Superconducting Magnet Setup

Peak magnetic flux density	9 T
Peak $B_z dB_z/dz$	±650 T^2/m
Length	30 cm
OD	15 cm
ID	5 cm
Free ID	2.5 cm

Artificial gravity

Water superf. velocity	0 – 9 mm/s
Water susceptibility	-9.0 10^{-6} (-)
$B_z dB_z/dz$=+650 T^2/m	$\Rightarrow \gamma_\ell \approx 0.52$
Air superf. velocity	0 – 50 mm/s
Air susceptibility	0.38 10^{-6} (-)
$B_z dB_z/dz$ =+650 T^2/m	$\Rightarrow \gamma_g \approx 17.0$
T & P	ambient
Glass beads	0.6 & 1 mm
Bed Length	40 mm
ID	19 mm

Mars gravity

$MnCl_2$ 34.5% sol. superf. velocity	0 – 1mm/s
$MnCl_2$ sol susceptibility	4.23 10^{-4} (-)
$B_z dB_z/dz$=-27 T^2/m	$\Rightarrow \gamma_\ell \approx 0.38$
Air superf. velocity	0 – 60 mm/s
Air susceptibility	0.38 10^{-6} (-)
$B_z dB_z/dz$=-27 T^2/m	$\Rightarrow \gamma_g \approx 0.38$
T & P	ambient
Glass beads	0.6 & 1 mm
Bed Length	37 mm
ID	17 mm
Viscosity/density	3 cp, 1347 kg/m^3

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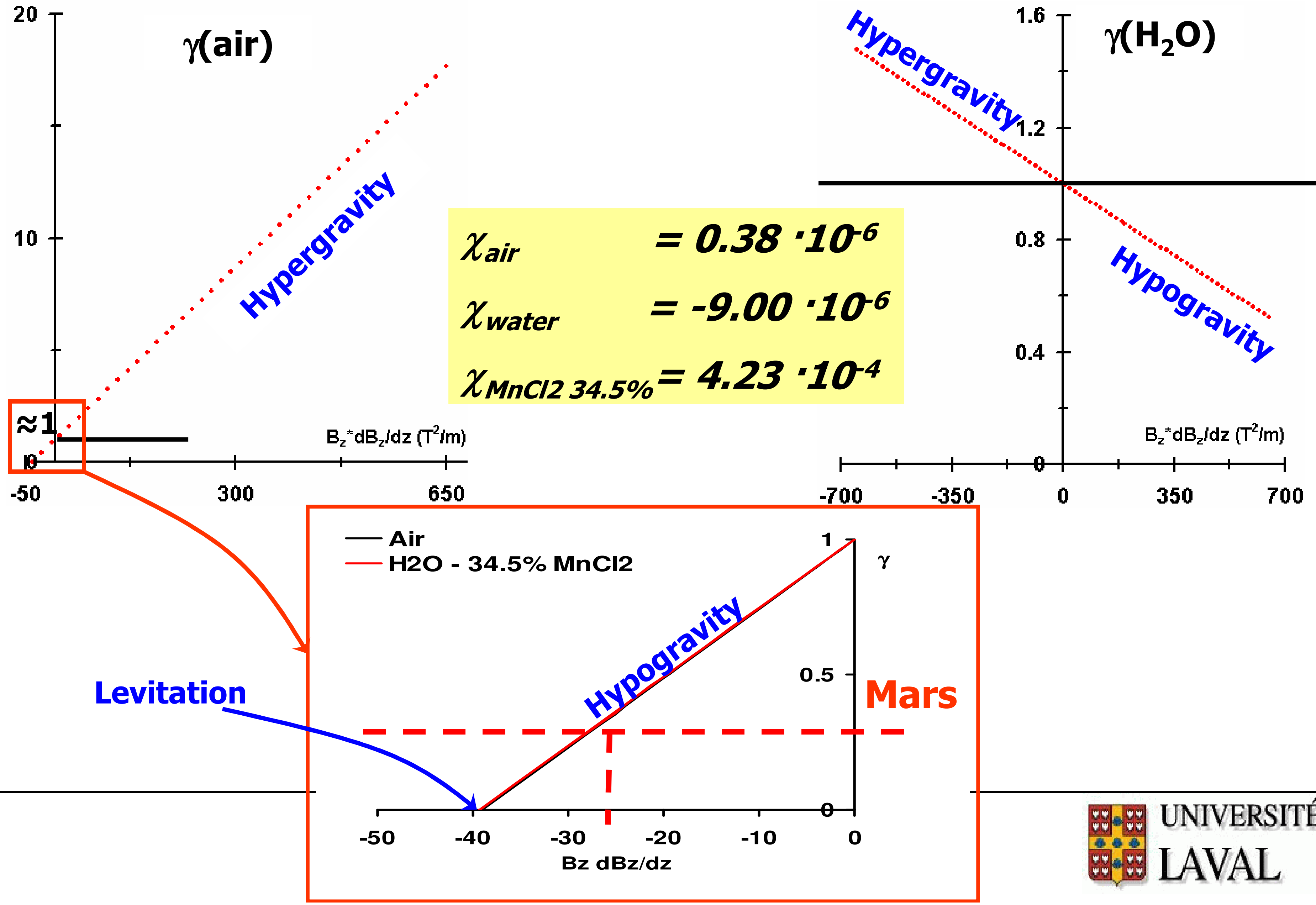
Artificial gravity factor (γ) vs. $B_z dB_z/dz$ for (paramag.) air, (diamag.) H_2O, $H_2O+MnCl_2$ (paramag.) -ambient-



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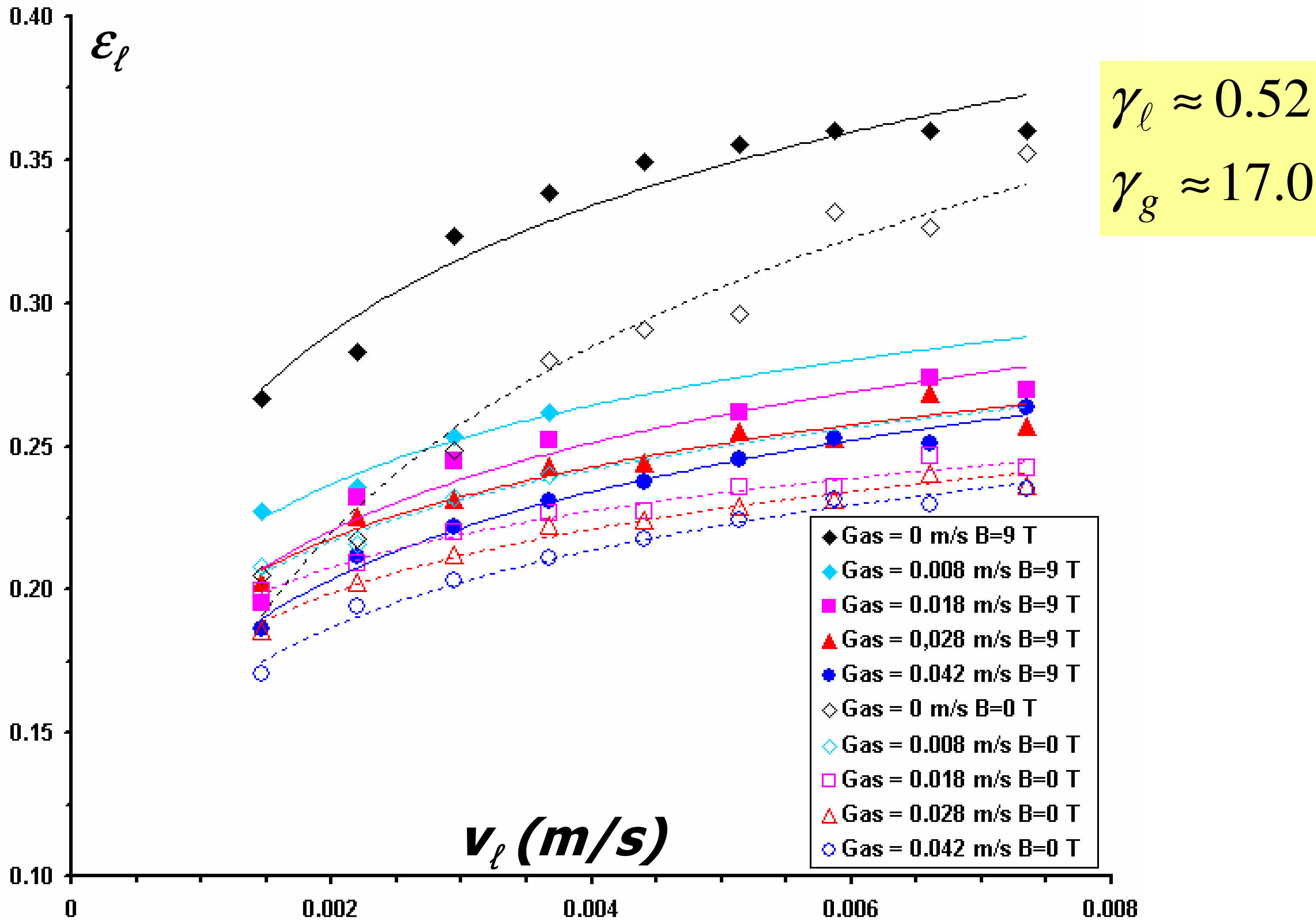
Effect of magnetic field on liquid holdup @ various liquid & gas superf. velocities – 1 mm glass beads – air/water



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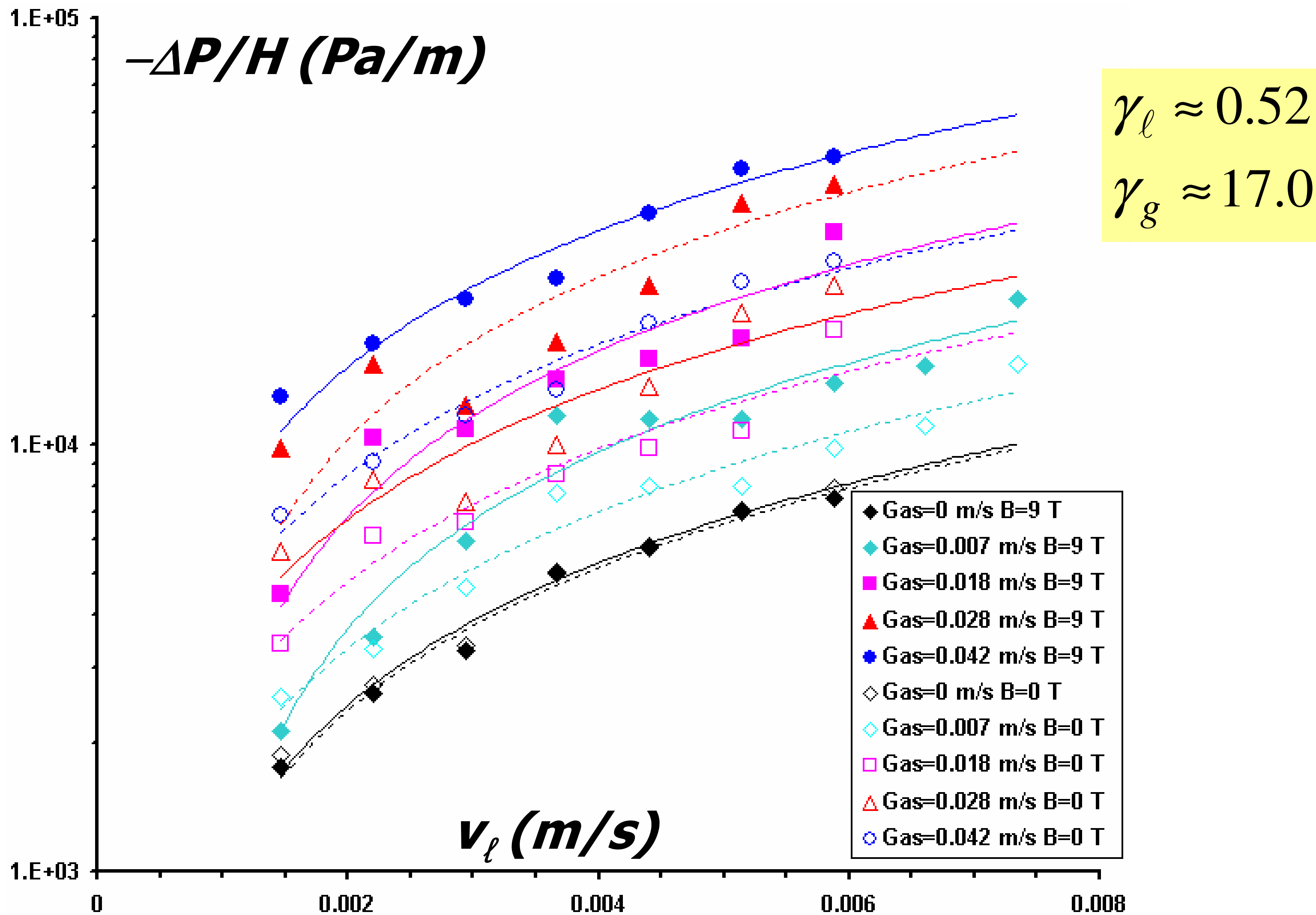
Effect of magnetic field on pressure drop @ various liquid & gas superf. velocities – 1 mm glass beads – air/water



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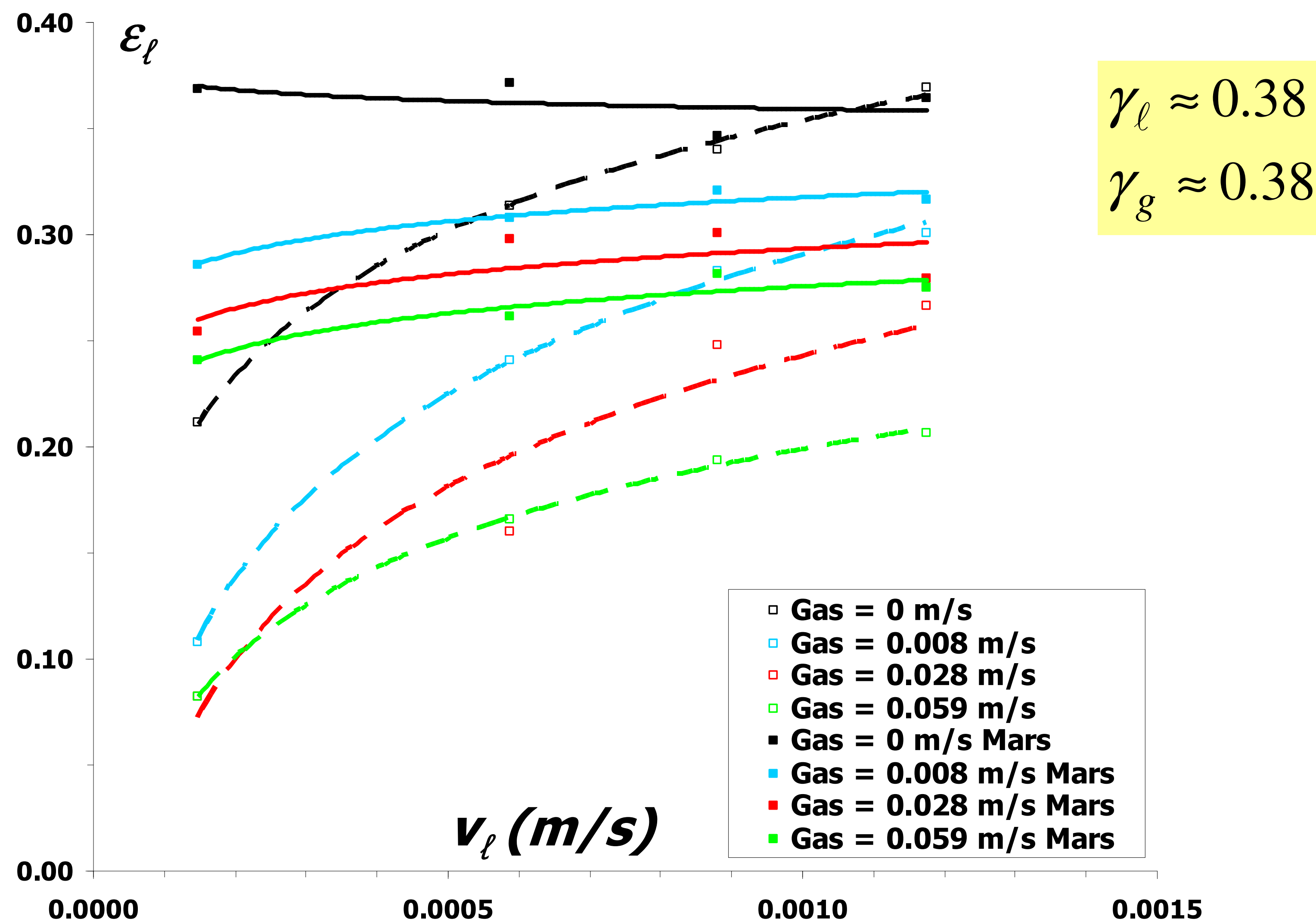
Liquid holdup @ various liquid & gas superf. velocities – 0.6 mm glass beads – air/water+34.5% $MnCl_2$ (Mars gravity)



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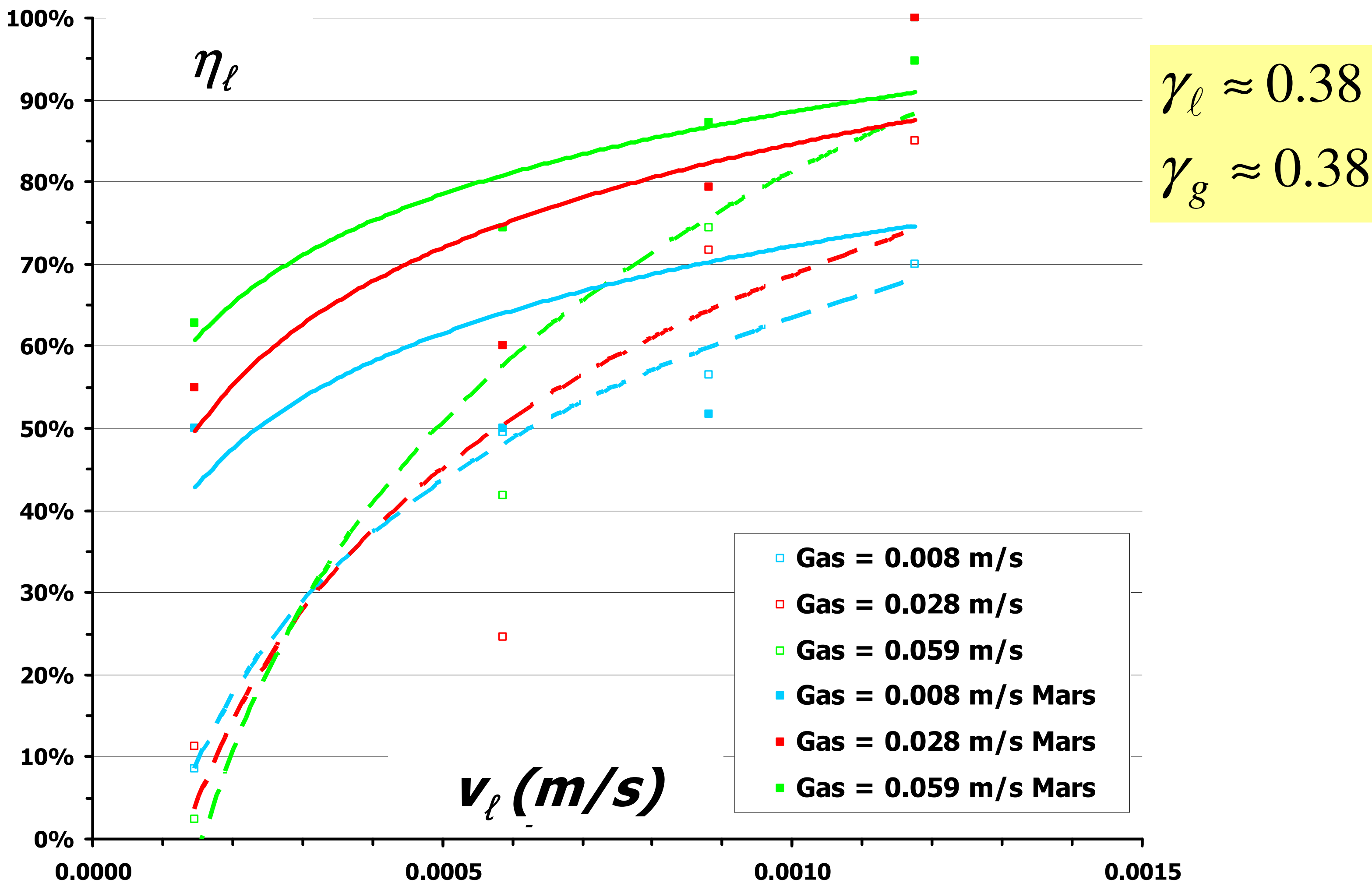
Wetting efficiency @ various liquid & gas superf. velocities – 0.6 mm glass beads – air/water+34.5% $MnCl_2$ (Mars gravity)



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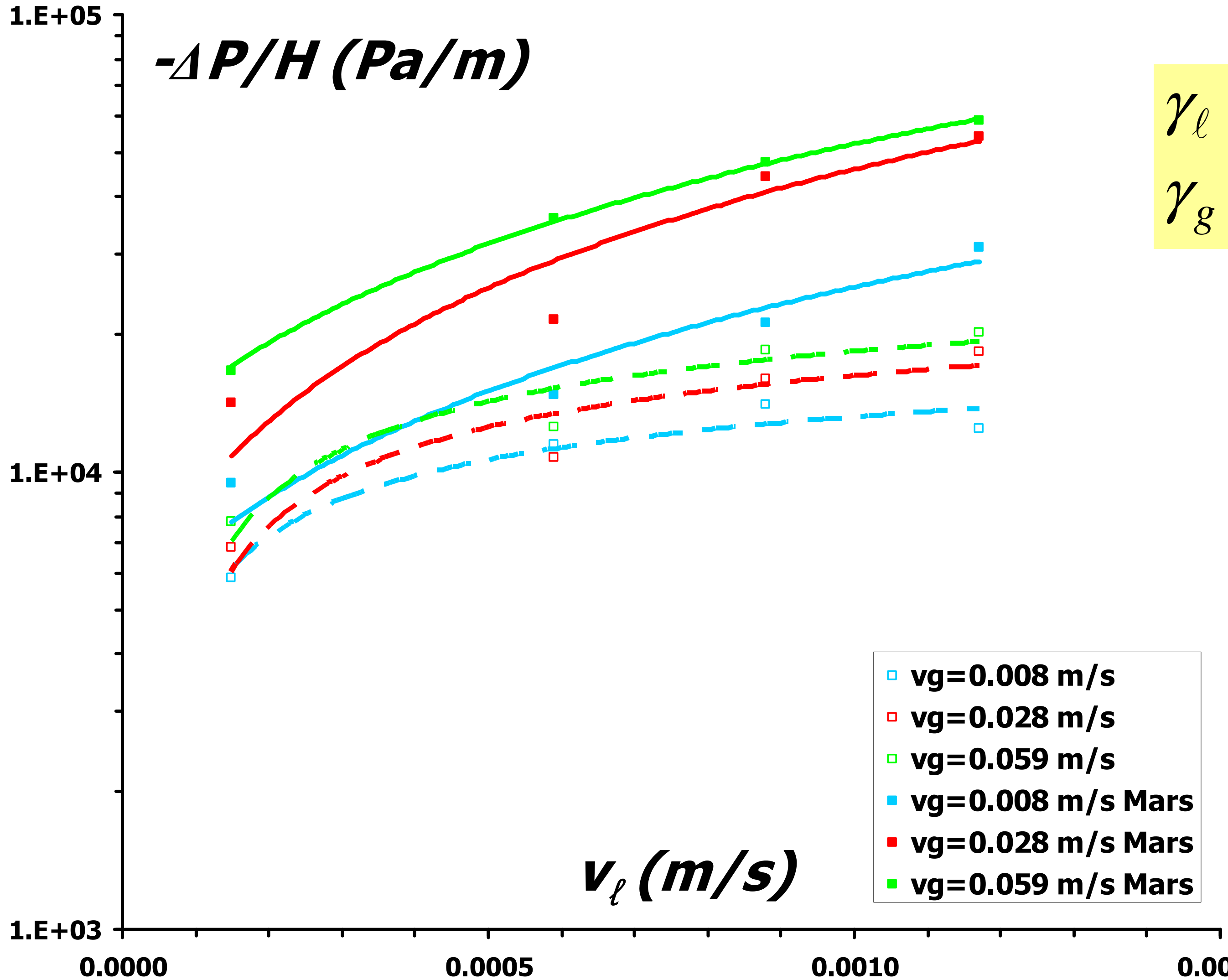
Pressure drop @ various liquid & gas superf. velocities – 0.6 mm glass beads – air/water+34.5% $MnCl_2$ (Mars gravity)



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LOCAL FLOW DESCRIPTION

Continuity $\nabla \cdot \underline{\mathbf{v}}_k = 0$ **In Ω_k (k=g,ℓ)**

Linear momentum $\frac{\partial}{\partial t} \rho_k \underline{\mathbf{v}}_k + \rho_k \nabla \cdot \underline{\mathbf{v}}_k \otimes \underline{\mathbf{v}}_k = \underline{\nabla} \cdot \underline{\underline{\mathbf{T}}}_k + \rho_k \underline{\mathbf{g}} + \mu_o \underline{\underline{\nabla \mathbf{H}}}_k \cdot \underline{\mathbf{M}}_k$ **In Ω_k (k=g,ℓ)**

$\underline{\mathbf{B}}_k = \underline{\mathbf{B}} + \underline{\mathbf{b}}_k \quad \underline{\mathbf{H}}_k = \underline{\mathbf{H}} + \underline{\mathbf{h}}_k$ **In Ω_k (k=g,ℓ,s)**

$\underline{\mathbf{B}}_k = \mu_o \left(\underline{\mathbf{H}}_k + \underline{\mathbf{M}}_k \right)$ **In Ω_k (k=g,ℓ,s)**

$\underline{\mathbf{M}}_k = \chi_k \underline{\mathbf{H}}_k$ **In Ω_k (k=g,ℓ,s)**

Ampère-Maxwell relation $\nabla \times \underline{\mathbf{H}}_k = \nabla \times \underline{\mathbf{h}}_k = \nabla \times \underline{\mathbf{H}} = \underline{\mathbf{0}}$ **In Ω_k (k=g,ℓ,s)**

Maxwell flux / Gauss law $\nabla \cdot \underline{\mathbf{B}}_k = \nabla \cdot \underline{\mathbf{b}}_k = \nabla \cdot \underline{\mathbf{B}} = 0 \quad \Rightarrow \quad \nabla \cdot \underline{\mathbf{h}}_k + \nabla \cdot \underline{\mathbf{M}}_k = 0$ **In Ω_k (k=g,ℓ,s)**

Adherence $\underline{\mathbf{v}}_g = \underline{\mathbf{0}}$ **on Γ_{gs}** $\underline{\mathbf{v}}_\ell = \underline{\mathbf{0}}$ **on $\Gamma_{\ell s}$** $\underline{\mathbf{v}}_\ell = \underline{\mathbf{v}}_g = \underline{\mathbf{v}}_i$ **on $\Gamma_{\ell g}$**

Continuity of tangential B component @ interfaces $\left(\underline{\mathbf{B}}_k - \underline{\mathbf{B}}_j \right) \cdot \underline{\mathbf{n}}_{kj} = 0$ **on Γ_{kj}**

Continuity of normal H component @ interfaces $\left(\underline{\mathbf{H}}_k - \underline{\mathbf{H}}_j \right) \times \underline{\mathbf{n}}_{kj} = \underline{\mathbf{0}}$ **on Γ_{kj}**

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Zero-order 1-D Volume-average Formulation

Main assumptions

- Simple upscaling-homogenization
- z-unidirectional, steady-state
- Fully developed flow, fully wetted bed

- Magnetization self-induced magnetic fields ignored
- Moses effect ignored
- Lorentz force ignored (electrolyte solution)
- STABLE LIQUID FILMS

$$-\varepsilon_g \frac{dP}{dz} + \varepsilon_g \rho_g g + \frac{\varepsilon_g \chi_g}{\mu_o} B_z \frac{dB_z}{dz} - F_{g\ell} = 0$$

$$-\varepsilon_\ell \frac{dP}{dz} + \varepsilon_\ell \rho_\ell g + \frac{\varepsilon_\ell \chi_\ell}{\mu_o} B_z \frac{dB_z}{dz} + F_{g\ell} - F_{\ell s} = 0$$

$$\gamma_g = \left(1 + \frac{\chi_g}{\rho_g g \mu_o} B_z \frac{dB_z}{dz}\right) \Rightarrow \quad -\varepsilon_g \frac{dP}{dz} + \varepsilon_g \rho_g \gamma_g g = F_{g\ell}$$

$$\gamma_\ell = \left(1 + \frac{\chi_\ell}{\rho_\ell g \mu_o} B_z \frac{dB_z}{dz}\right) \Rightarrow \quad -\varepsilon_\ell \frac{dP}{dz} + \varepsilon_\ell \rho_\ell \gamma_\ell g = -F_{g\ell} + F_{\ell s}$$

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Zero-order 1-D Volume-average Formulation

- Slit model drag closures **EXTENSION TO ARTIFICIAL GRAVITY CONDITIONS**

Holub, R. A., M. P. Duduković, P. A. Ramachandran, *Chem. Eng. Sci.*, 47, 2343 (1992)

$$\Psi_g = -\frac{dP}{dz}\frac{1}{\rho_g \gamma_g g} + 1 = \left(\frac{\varepsilon}{\varepsilon_g}\right)^3 \left(E_1 \frac{Re_g}{Ga_g} + E_2 \frac{Re_g^2}{Ga_g}\right)$$

$$\Psi_\ell = -\frac{dP}{dz}\frac{1}{\rho_\ell \gamma_\ell g} + 1 = \left(\frac{\varepsilon}{\varepsilon_\ell}\right)^3 \left(E_1 \frac{Re_\ell}{Ga_\ell} + E_2 \frac{Re_\ell^2}{Ga_\ell}\right)$$

$$Ga_g = \frac{\rho_g^2 \gamma_g g d_p^3 \varepsilon^3}{\eta_g^2 (1-\varepsilon)^3}$$

$$Ga_\ell = \frac{\rho_\ell^2 \gamma_\ell g d_p^3 \varepsilon^3}{\eta_\ell^2 (1-\varepsilon)^3}$$

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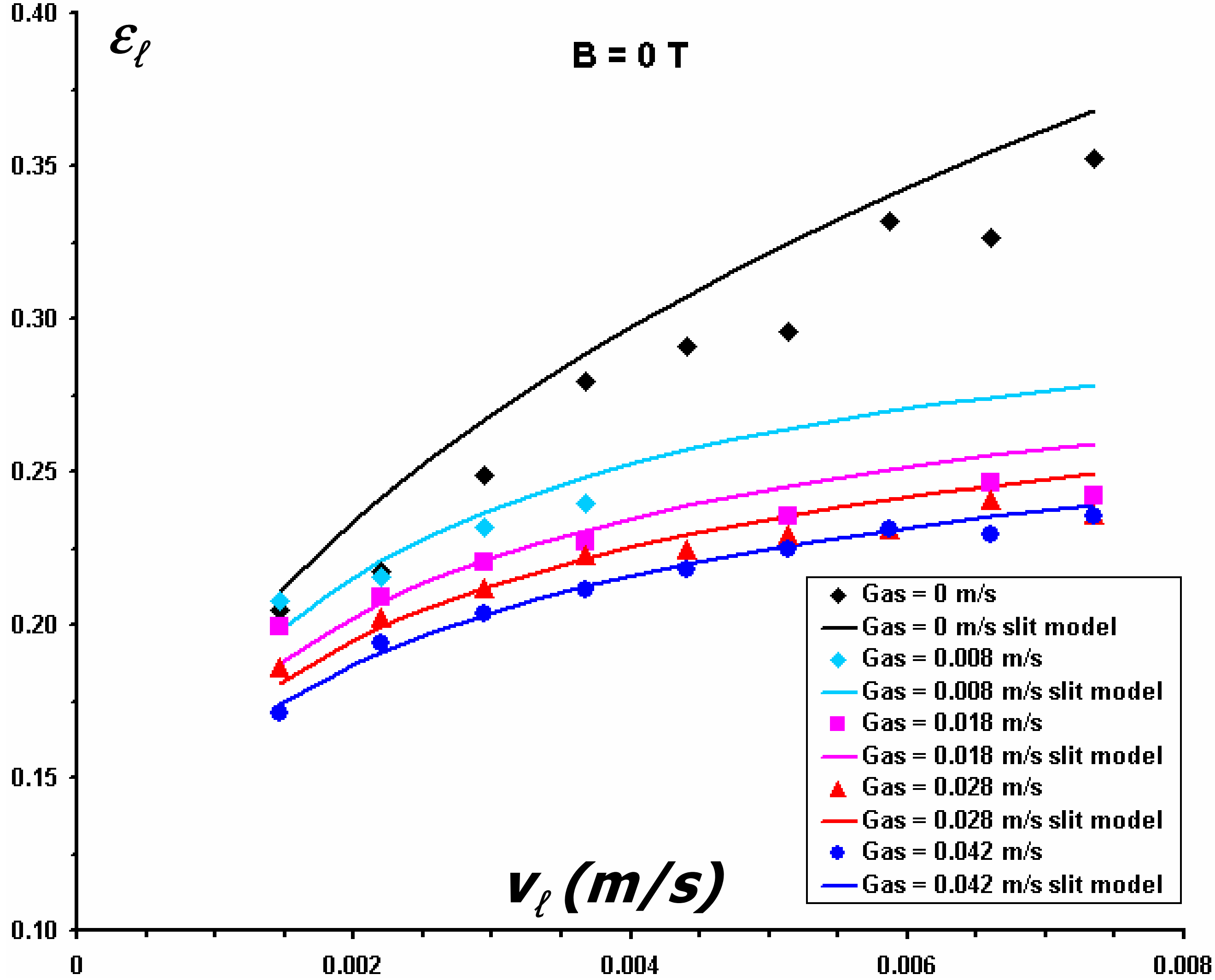
Comparison between Slit model & experimental holdup data – Magnetic field OFF – 1 mm glass beads



Holub, R. A., M. P. Duduković, P. A. Ramachandran, *Chem. Eng. Sci.*, 47, 2343 (1992)

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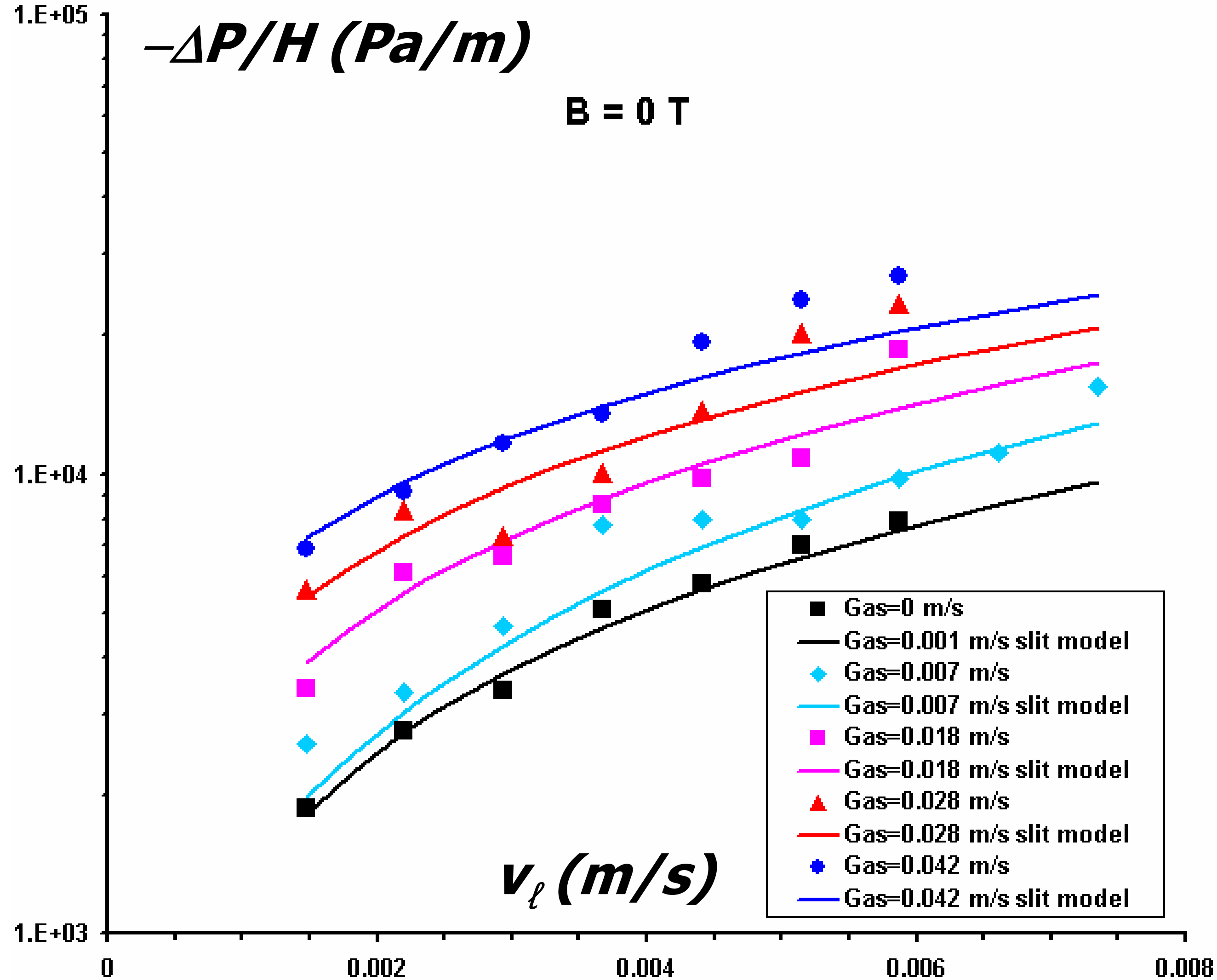
Comparison between Slit model & experimental pressure drop data – Magnetic field OFF – 1 mm glass beads



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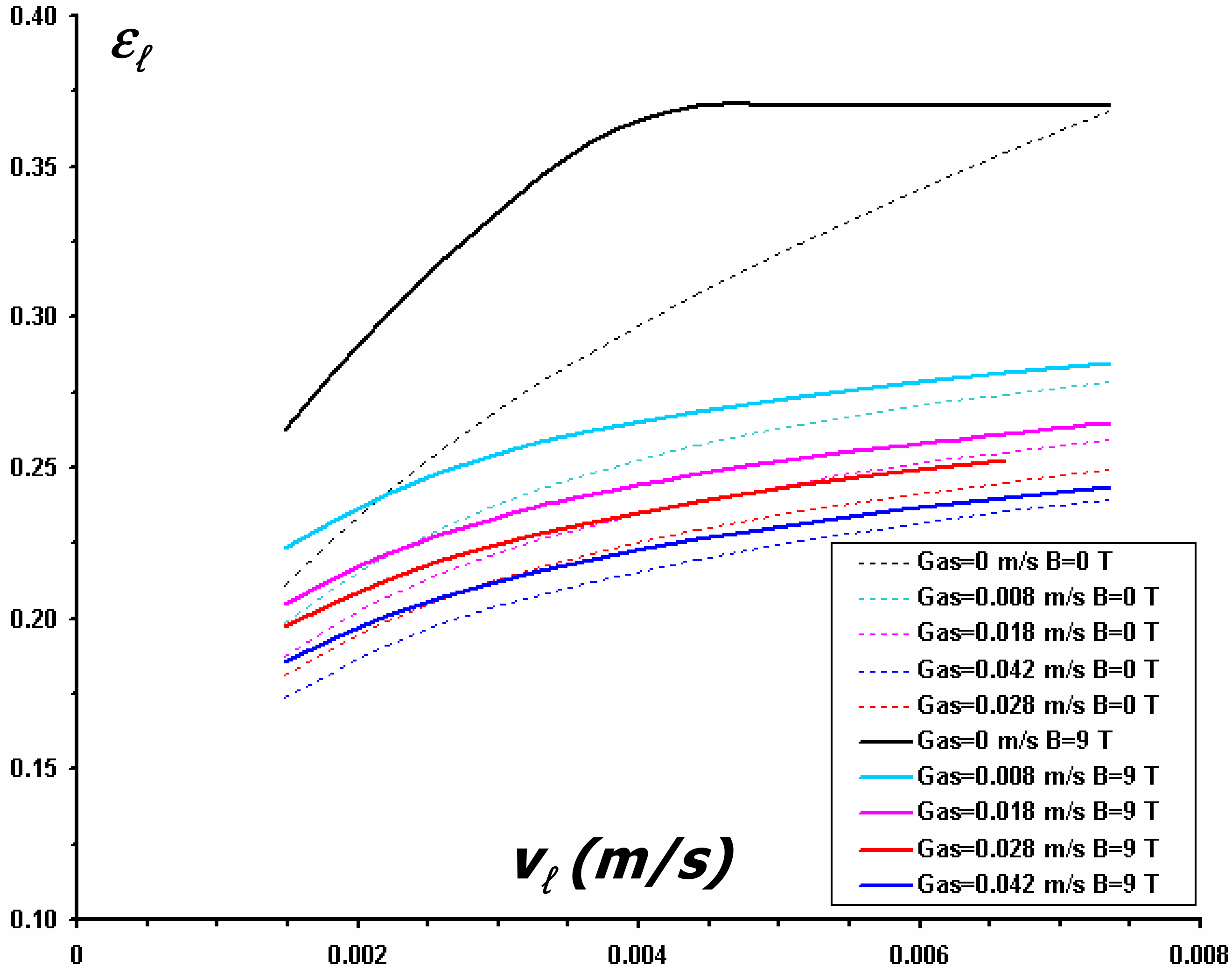
Trends predicted from slit model if magnetic field effect is commuted into artificial gravity effect. Liquid holdup @ various liquid & gas superf. velocities – 1 mm glass beads



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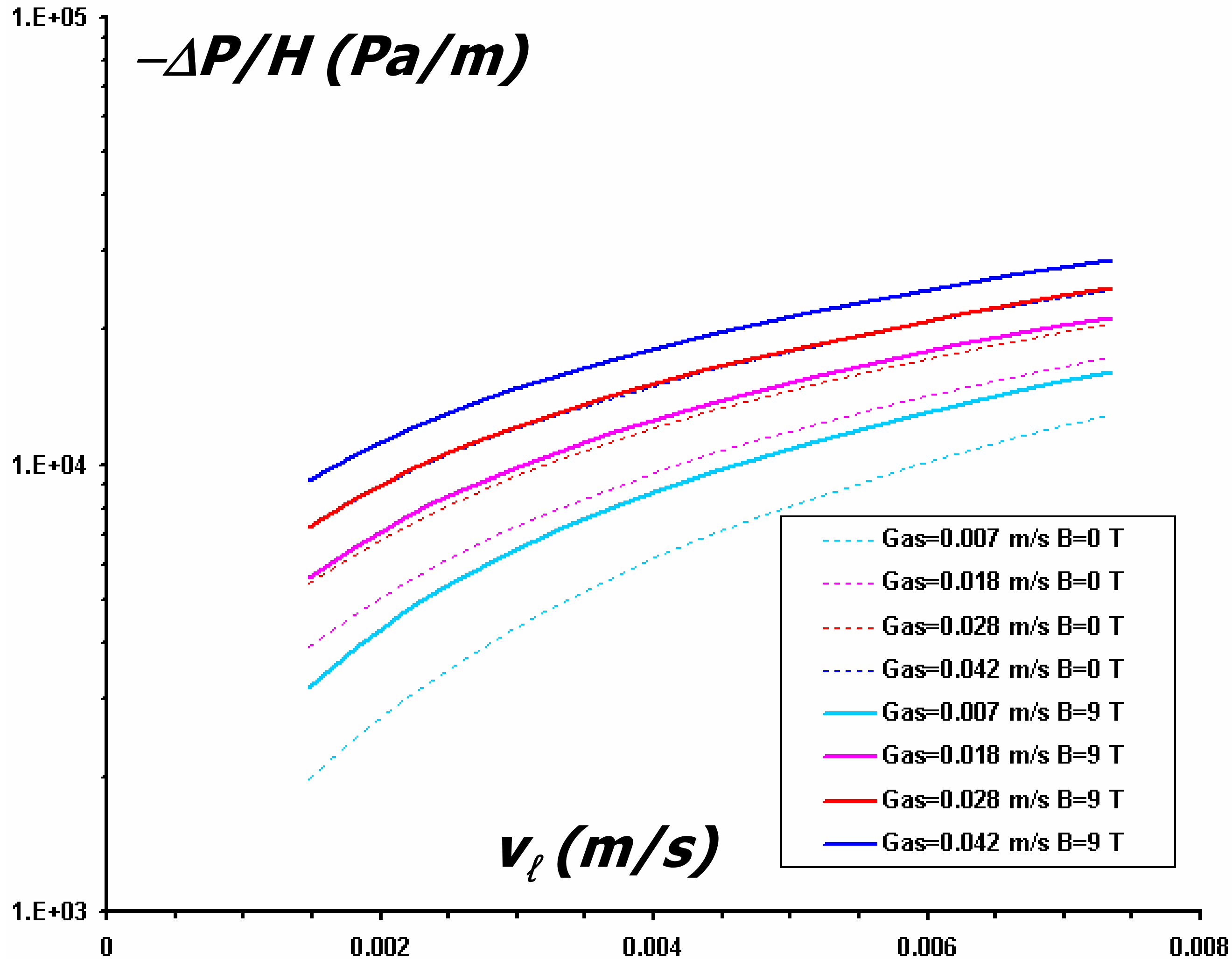
Trends predicted from slit model if magnetic field effect is commuted into artificial gravity effect. Pressure drop @ various liquid & gas superf. velocities – 1 mm glass beads



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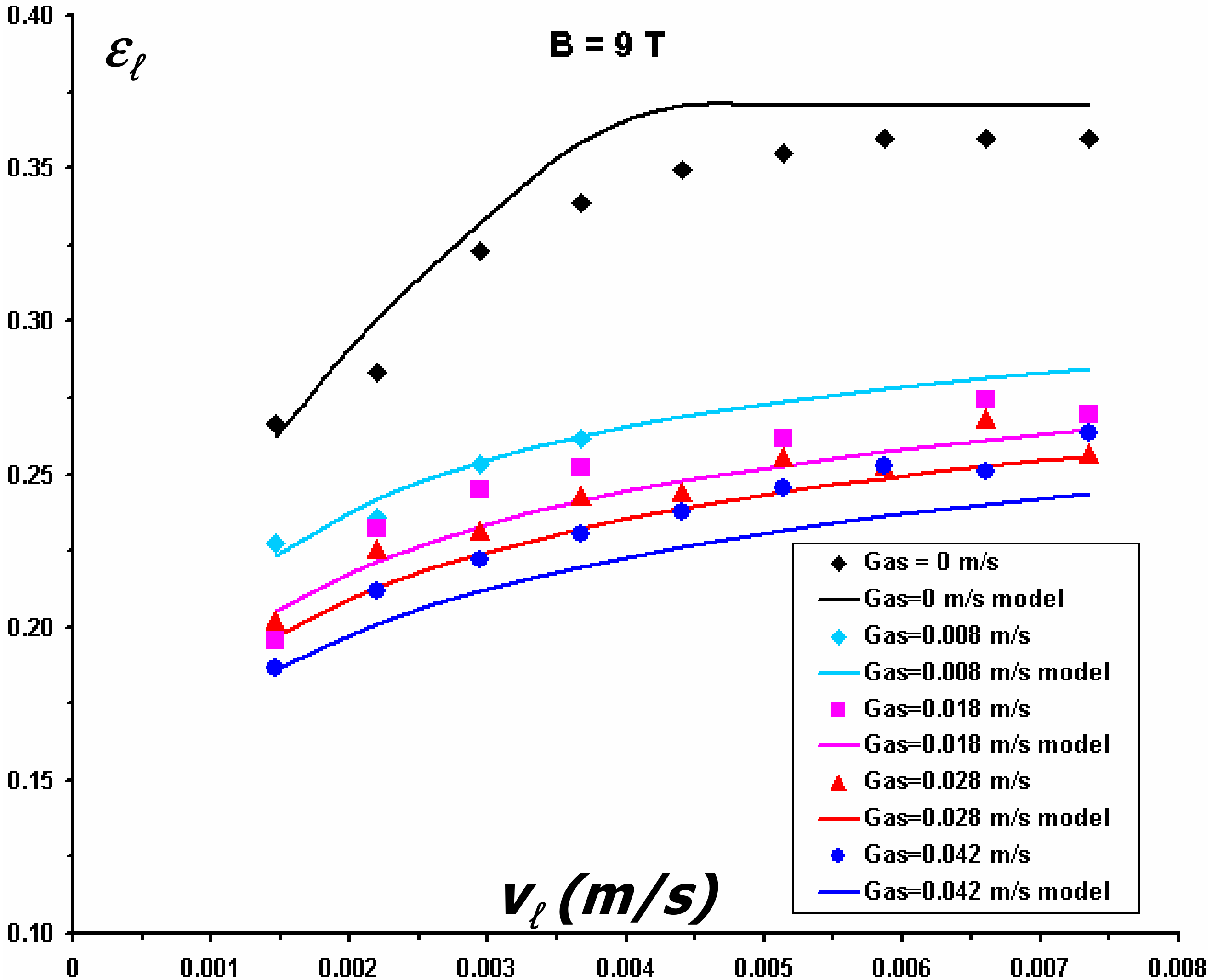
Comparison between Slit model & experimental holdup data – Magnetic field ON – 1 mm glass beads, air/water



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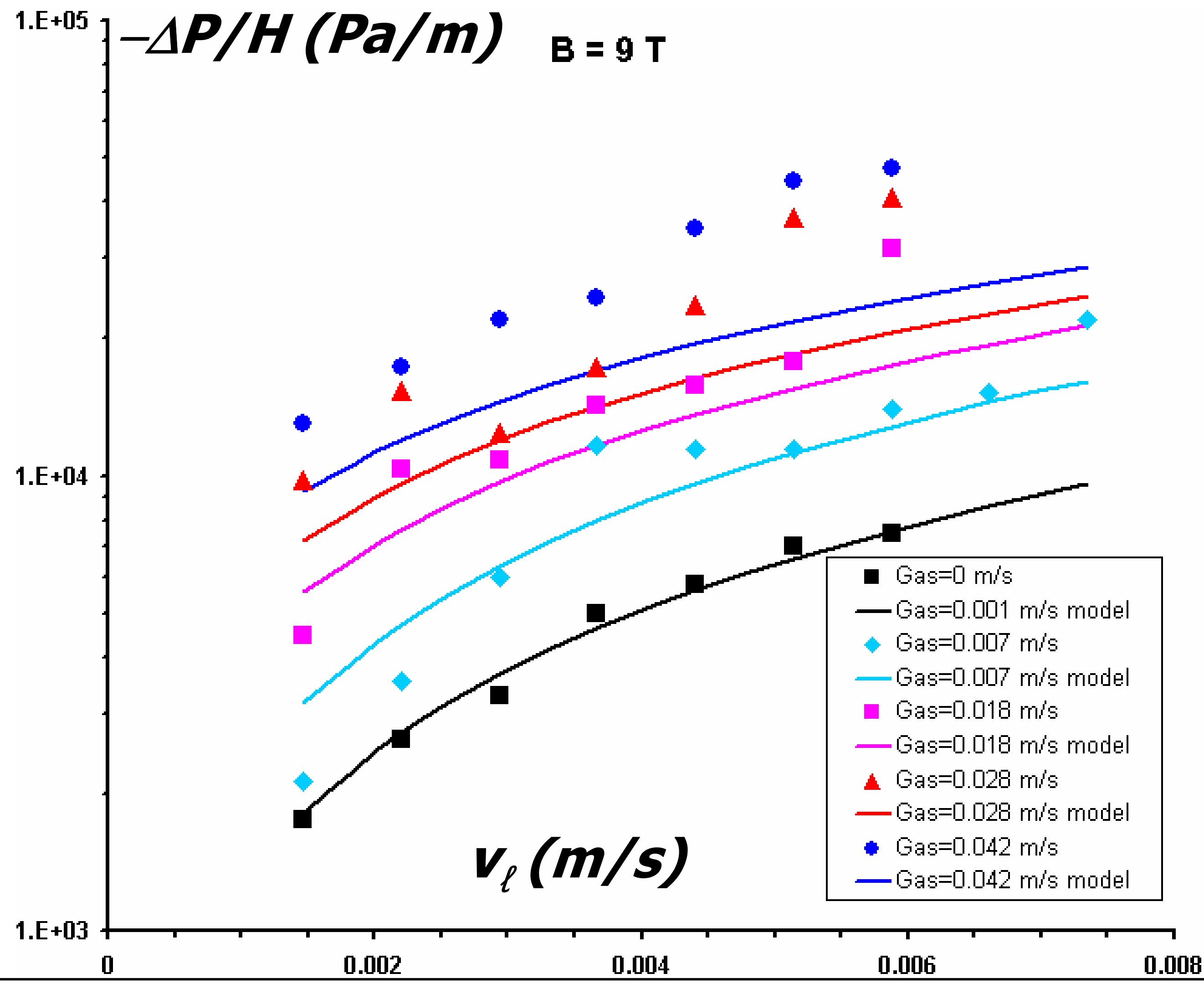
Comparison between Slit model & experimental pressure drop data – Magnetic field ON – 1 mm glass beads, air/water



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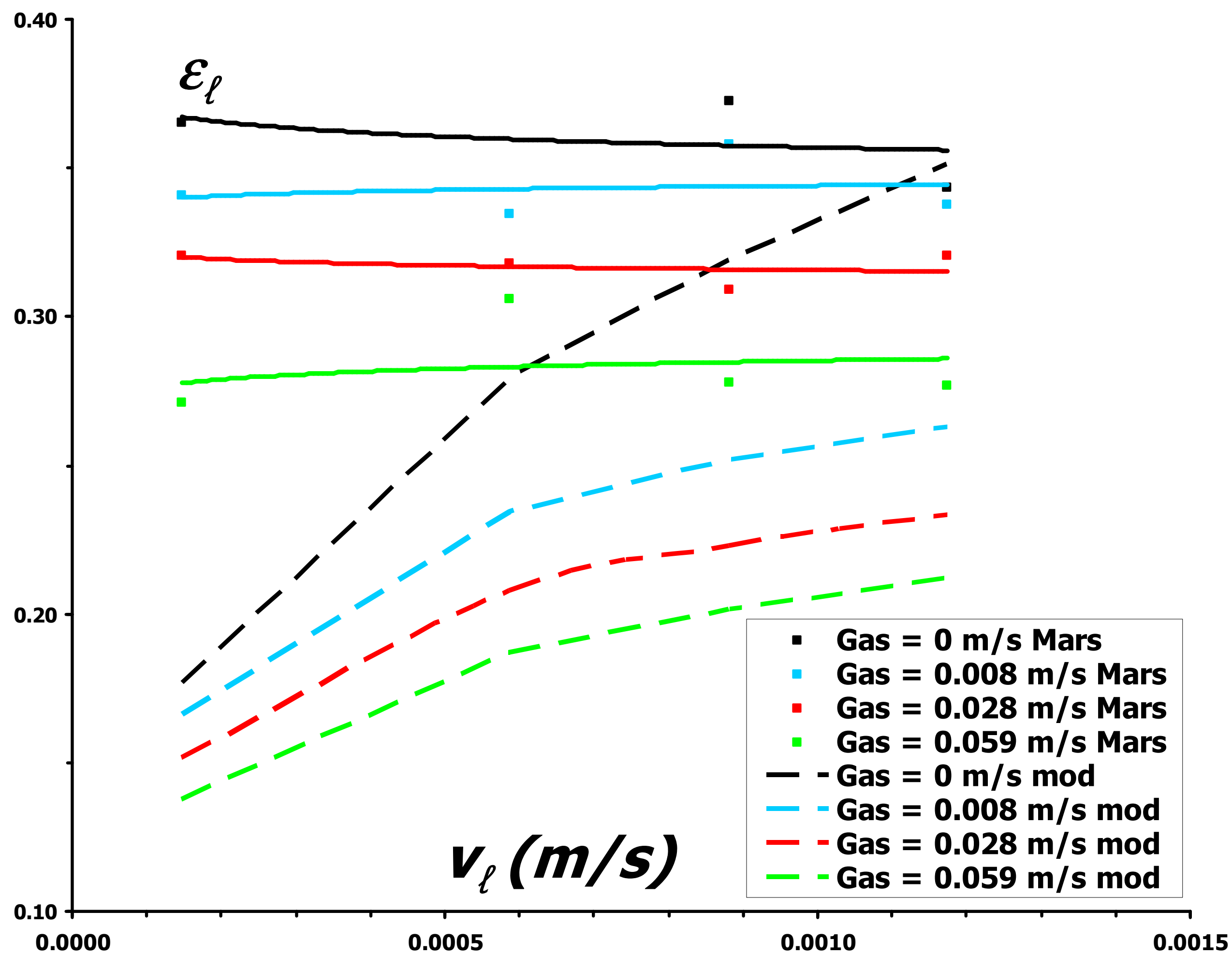
Comparison between Slit model & experimental holdup data – Magnetic field ON – 1 mm glass beads (Mars gravity)



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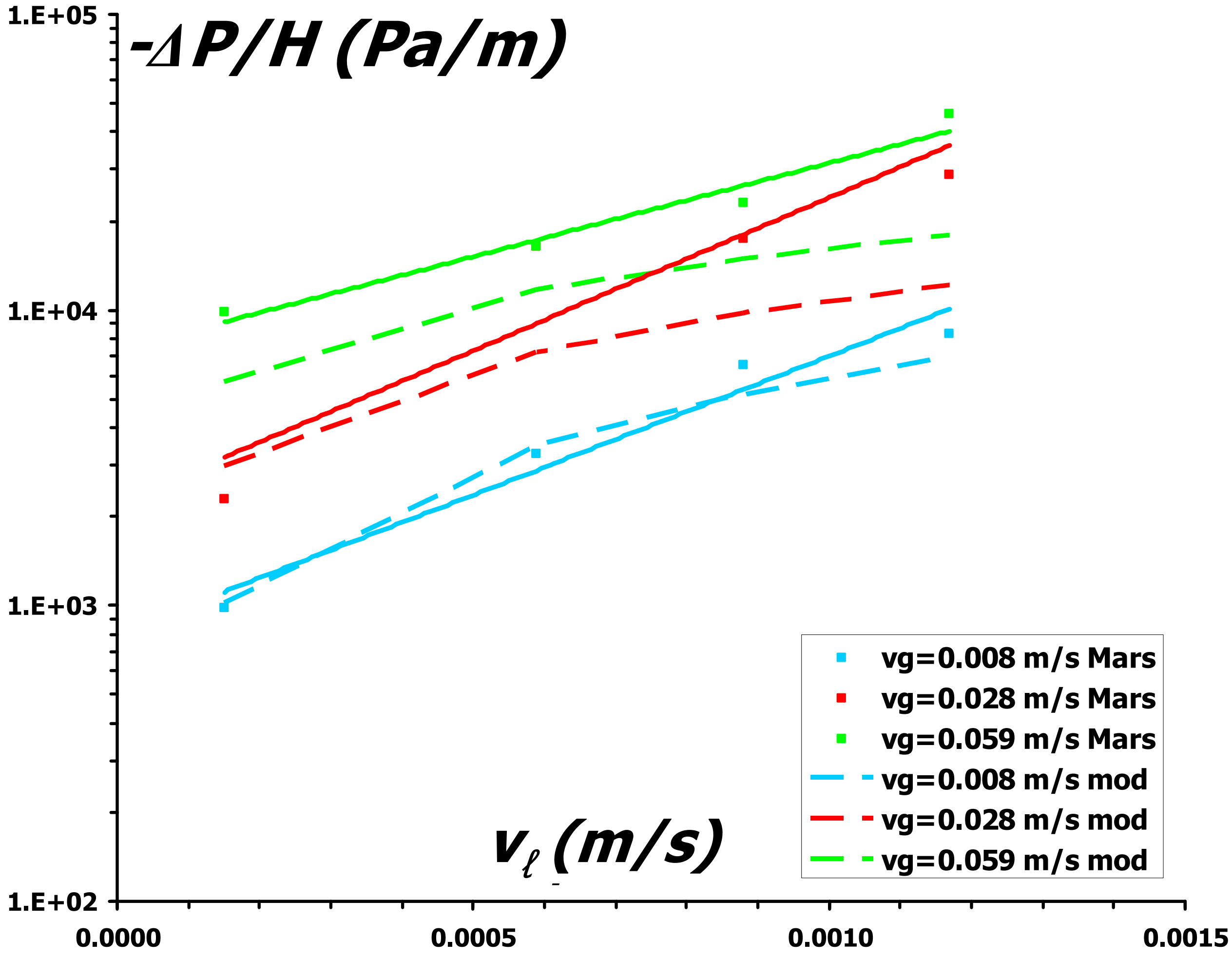
Comparison between Slit model & experimental pressure drop data – Magnetic field ON – 1 mm glass beads (Mars gravity)



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Concluding remarks

- Ground-based artificial gravisensing of multiphase reactors using gradient magnetic fields seems reasonable

- Magnetic Kelvin body force commutable into artificial gravity body force (fluid mechanically speaking)

- Liquid hypogravity increases liquid holdup & afflicts film stability. Gas hypergravity promotes interfacial interactions/drags, pressure drop

- Mars gravity: pressure drop, liquid holdup, wetting efficiency increase w/r to *g*-Earth case

- Slit model describes satisfactorily artificial gravity INASMUCH as film flow assumption holds, interactions accounted for, or low gas superficial velocity

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