# Simulations of solid-liquid suspensions

## *from dilute to dense*

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# Acknowledgement

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# Outline

- Introduction

    liquid-solid examples (why do we do this work?)

    ® industrial crystallization, fluidization, ...

- Dilute systems
    - a point-particle approach
    - stirred suspensions

**~1% solids vol.**

- DNS with solid-liquid interface resolution
    - methodology
    - particle-particle interactions in turbulent flow

**~10% solids vol.**

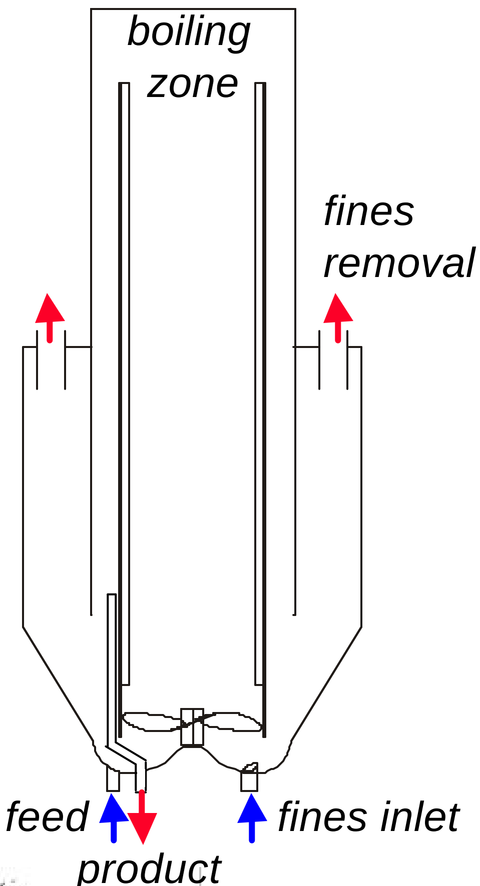
- Liquid-solid fluidization
    - inhomogeneities
    - bulk viscosity in Euler/Euler closure

**~50% solids vol.**

- Sheared suspensions

# (G)LS example: industrial crystallization

1100 L draft tube, baffled crystallizer

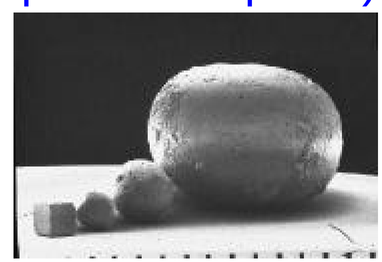



Transport physics
- turbulent flow
- multiphase (gas/liquid/solid) system
- heat and mass transfer

↕ particle-turbulence interaction

Micro-scale physics
- primary nucleation
- growth
- agglomeration
- attrition (secondary nucleation)

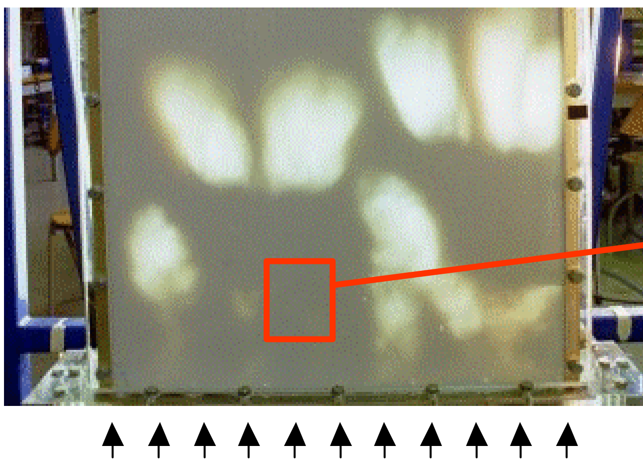
↕ collisions (particle-particle and particle-impeller)



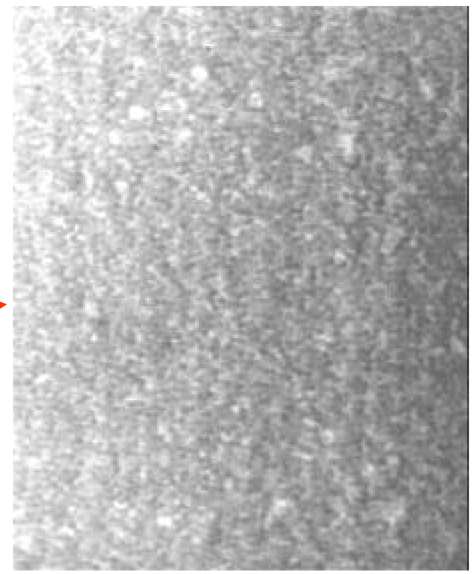
# GS/LS example: fluidization

## flat fluidized beds

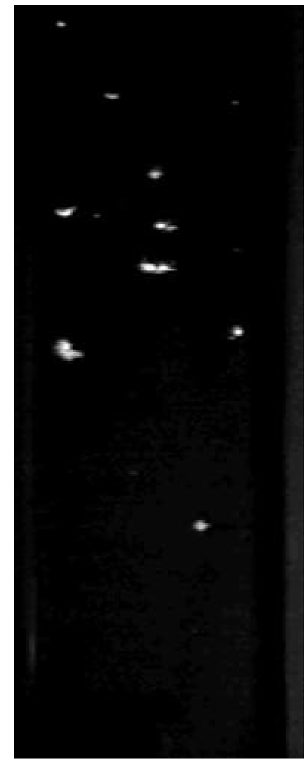
air-polystyrene system $\frac{\rho_s}{\rho_f} \approx 1000$



air



water-steel $\frac{\rho_s}{\rho_f} \approx 8$



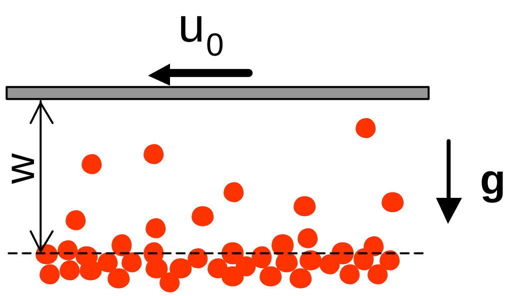
courtesy É. Guazelli
(IUSTI, CNRS-UMR)

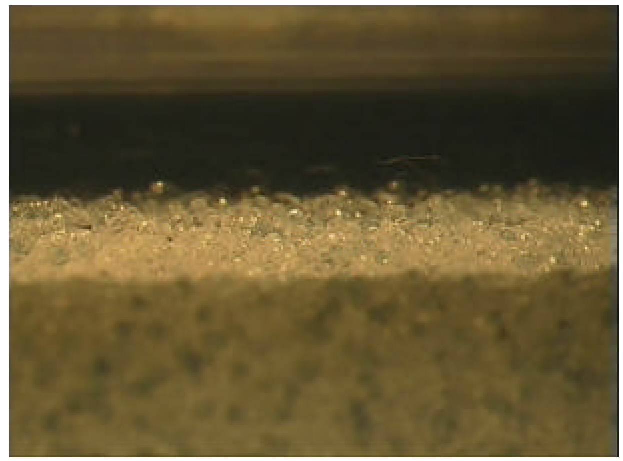
inhomogeneities

- mixing
- transfer processes

# LS example: sheared granular bed

glass beads in water $\frac{\rho_s}{\rho_f} \approx 2.5$



courtesy François Charru (IMFT)

sedimentation and resuspension

# Agitated liquid-solid flow

Issues:

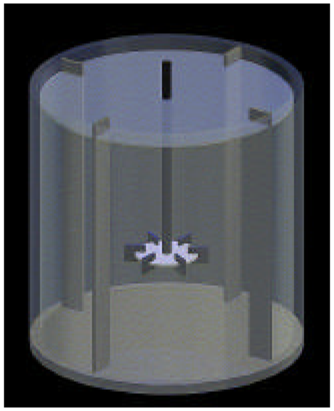
- attrition, formation of fines (role of collisions)
- power consumption
- scale-up
- ...

A starting point:

just-suspended experiments: Zwietering (1958); Baldi *et al*. (1978)

$$N_{js} = s\frac{d_p^{0.2}\,\mu_L^{0.1}\,(g\Delta\rho)^{0.45}\,\phi_m^{0.13}}{\rho_L^{0.55}\,D^{0.85}}$$

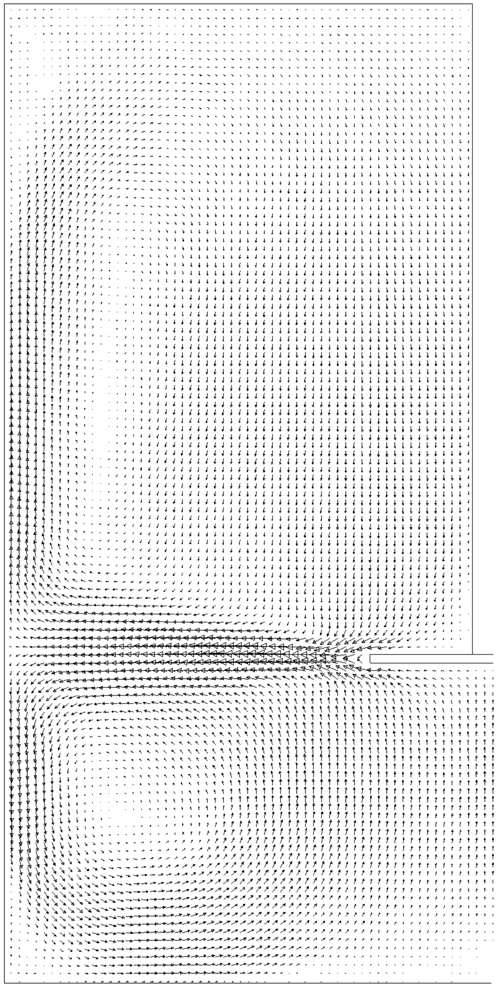
hydrodyn. interaction
collisions
2way coupling



with $s \approx 15$ for $\frac{T}{C} = \frac{T}{D} = 3$

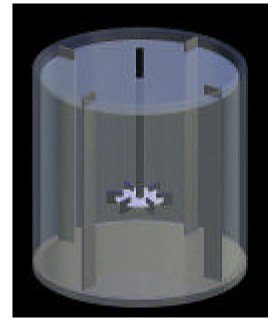
(Rushton turbine in baffled tank)

# Single-phase flow: LES



average flow

single realization

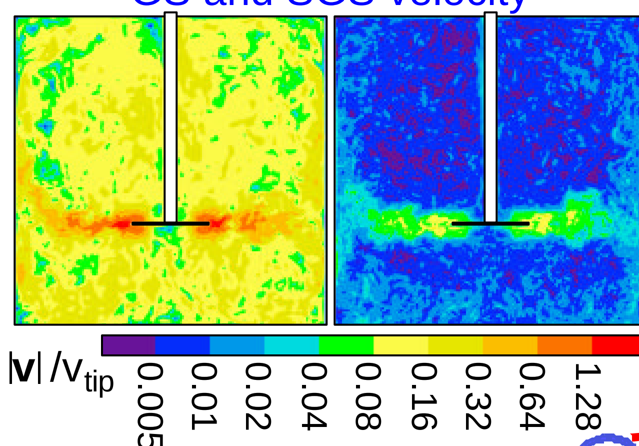



$$Re = \frac{ND^2}{\nu} = 10^5$$

Smagorinsky SGS model ($c_S$=0.1)

Lattice-Boltzmann discretization

single realization: magnitude of GS and SGS velocity



# Solid phase dynamics: Lagrangian

Equations of motion for the spherical particles

$$\frac{d\mathbf{x_p}}{dt} = \mathbf{v_p}$$

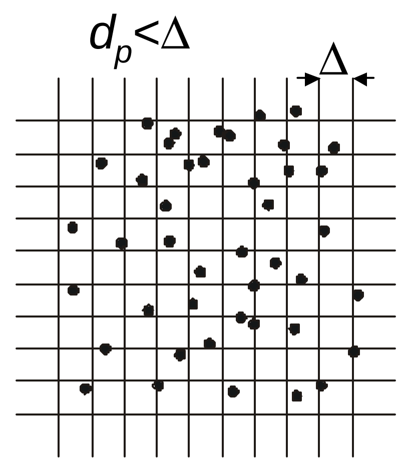
$$(m_p + m_a)\frac{d\mathbf{v_p}}{dt} = \sum \mathbf{F_p}$$

$$I\frac{d\mathbf{?}_\mathbf{p}}{dt} = \sum \mathbf{T_p}$$

added mass

*forces:*
gravity, drag, lift,
stress gradients } from *single*-particle correlations

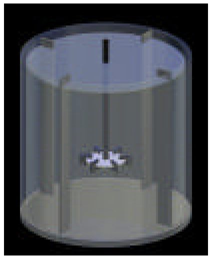
*particle rotation:*
Magnus force
rotational slip velocities




Collisions:

- hard-sphere particle-particle and particle-wall collisions
- 2 parameters: restitution coefficient *e* (=1 mostly)
  friction coefficient $\mu_f$ (=0 mostly)

Derksen, *AIChE J* **49** (2003)



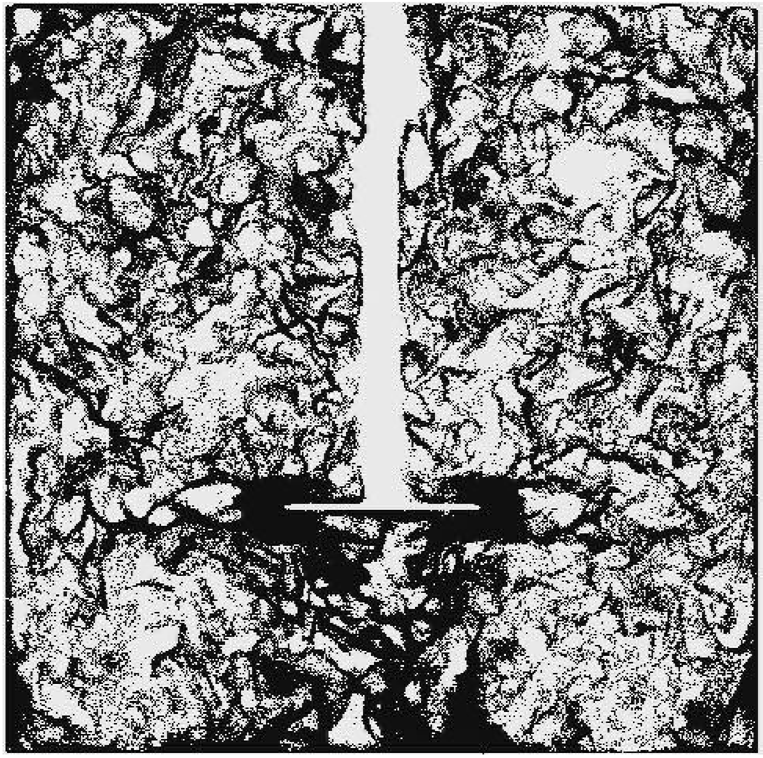
# Results: impressions

10 liter vessel, $d_p$=0.3 mm,

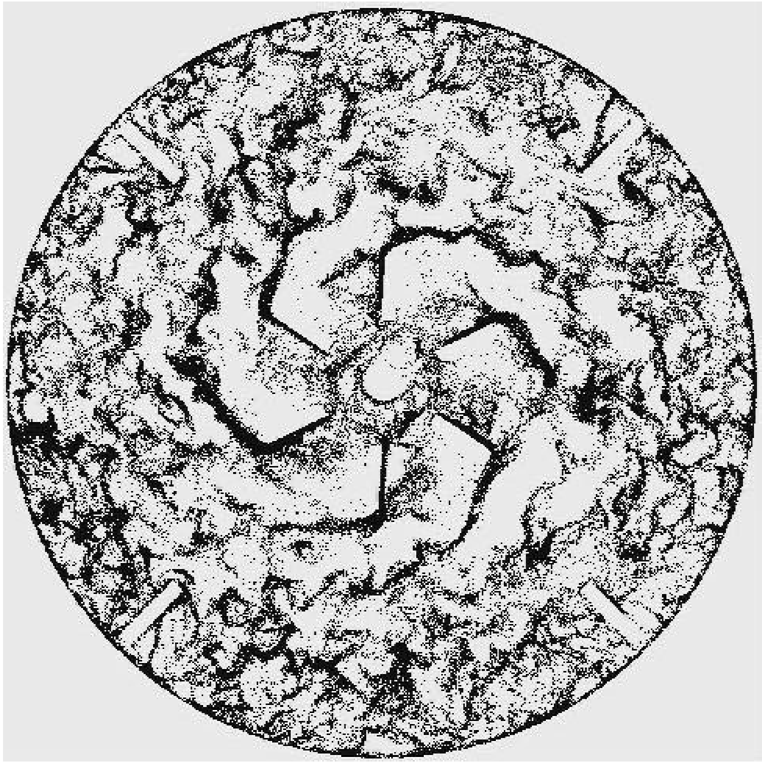
$\rho_{part}/\rho_{liq}$=2.5, $\phi_V$=3.6%, $n_p$=2.4 $10^7$

$$Re = \frac{ND^2}{\nu} = 10^5$$

$$St = \frac{\rho_{part}}{\rho_{liq}} \frac{d_p^2\, 6N}{18\nu} = O(1)$$

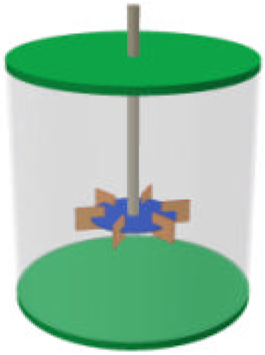


vertical cross section



horizontal cross section

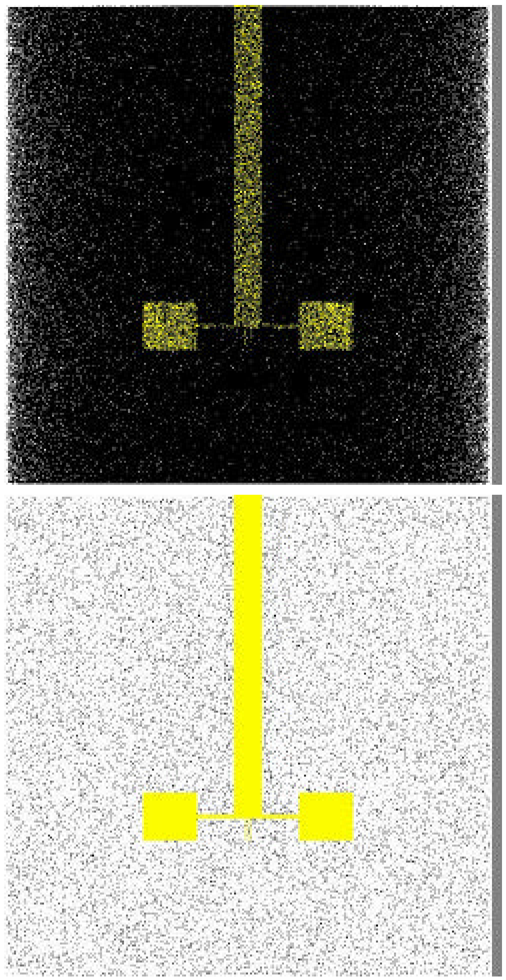
# Intermezzo: unbaffled tank



5.4 liter

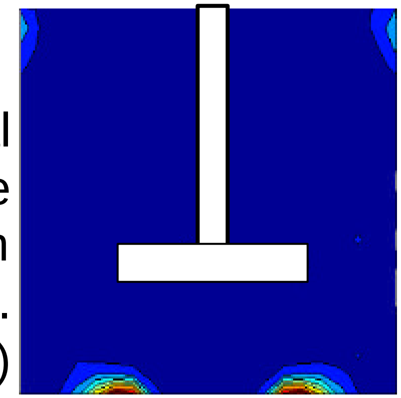
$d_p$=0.5 mm

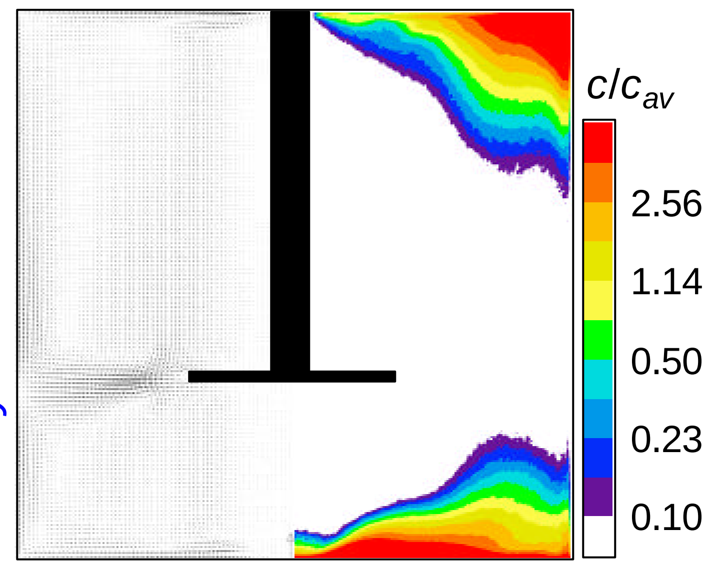
$\rho_{part}/\rho_{liq}$=2.5



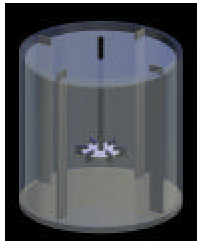

**time averages**

experimental particle concentration (thanks to A. Brucato)





# The quest for (just) suspended



$N$=16.5 rev/s
$N_{js}$=13.5 rev/s

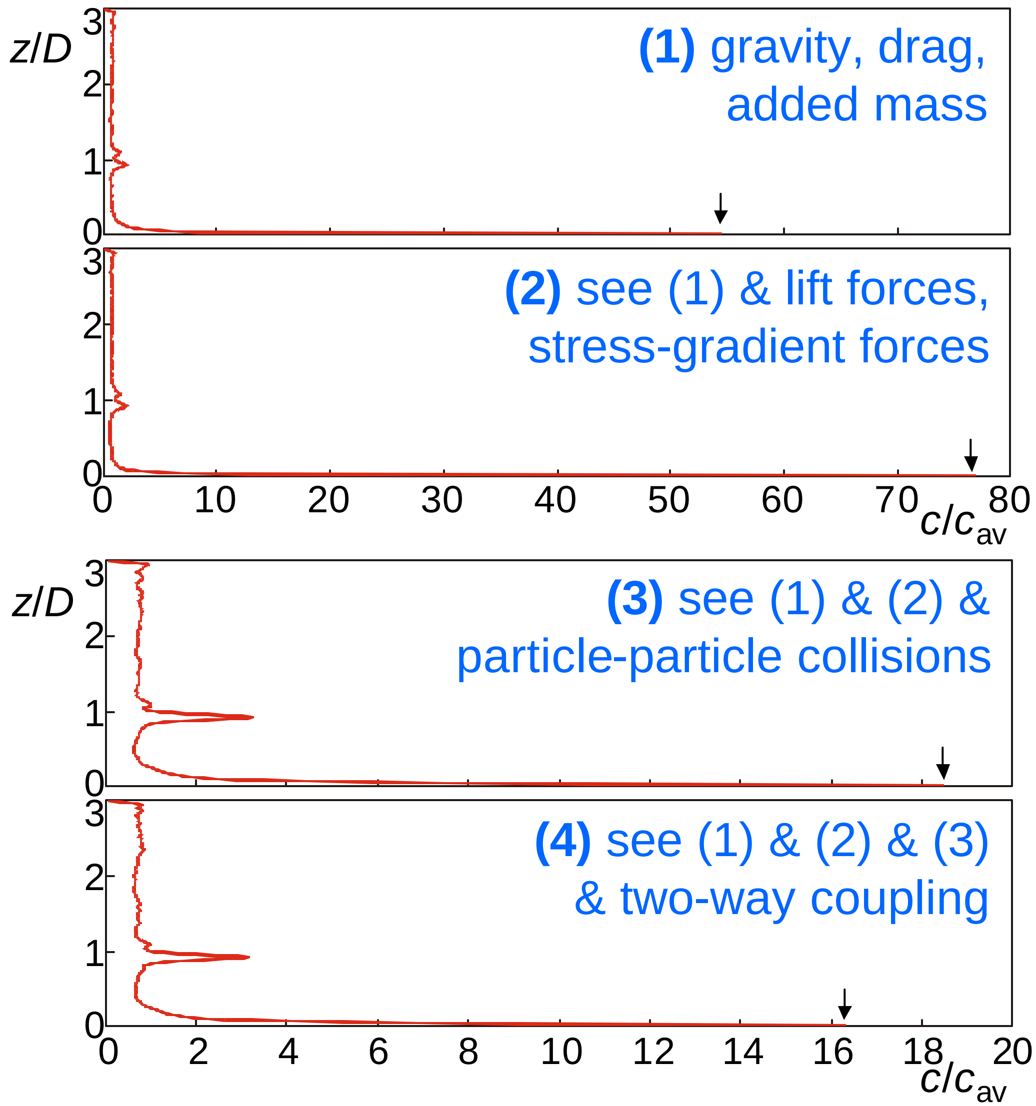
vertical concentration profiles at $2r/T$=0.45

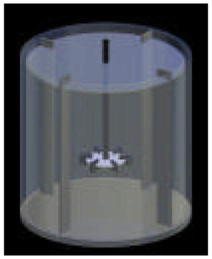
closest packing of spheres: $c_{cp}/c_{av}$=78

ratio drag and lift force

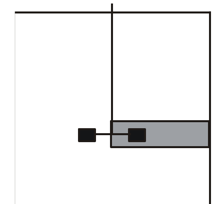
$$\frac{F_{lift}}{F_{drag}} \approx 0.1\sqrt{\frac{d_p^2\omega}{\nu}}$$

e.g. if $\omega$=10$N$: $F_{lift}$»0.5$F_{drag}$





# Particle-particle collisions



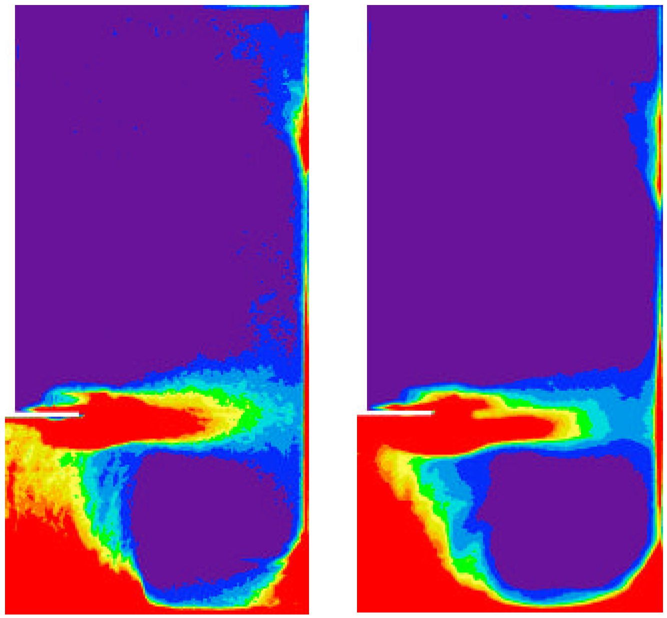
**collision frequencies** (color scale according to Von Smulochowski)

1 9

$r_{coll}/r_{coll,Sm}$

$d_p$=0.3 mm, $\phi_V$=0.95%

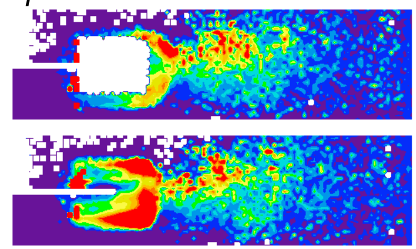
$d_p$=0.47 mm, $\phi_V$=3.6%



refer collision rates to Von Smulochowski: $r_{coll,Sm} = \frac{4}{3}\dot{\gamma} d_p^3 M^2 \qquad \dot{\gamma} = \sqrt{\frac{\varepsilon}{\nu}}$

**collision intensities**

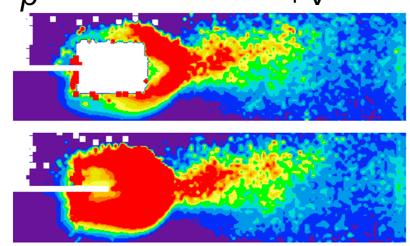
$d_p$=0.3 mm, $\phi_V$=0.95%



0°

55°

$d_p$=0.47 mm, $\phi_V$=3.6%

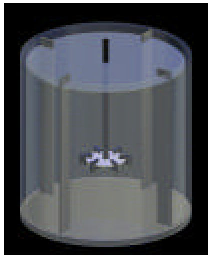


0°

55°

0.02 0.1 0.18

$v_{rel}^2/v_{tip}^2$



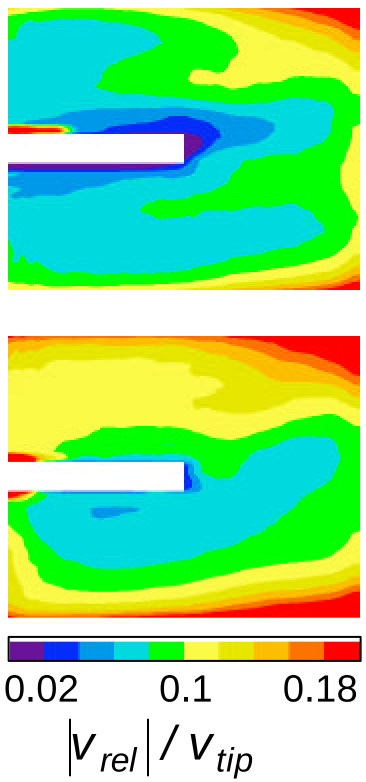
# Particle-impeller collisions

## collision intensities

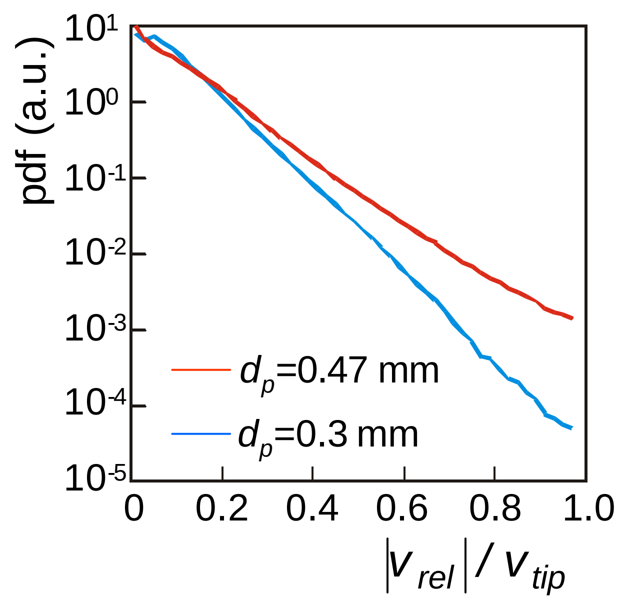
*front surface of impeller blade*

$d_p$=0.3 mm, $\phi_V$=0.95%

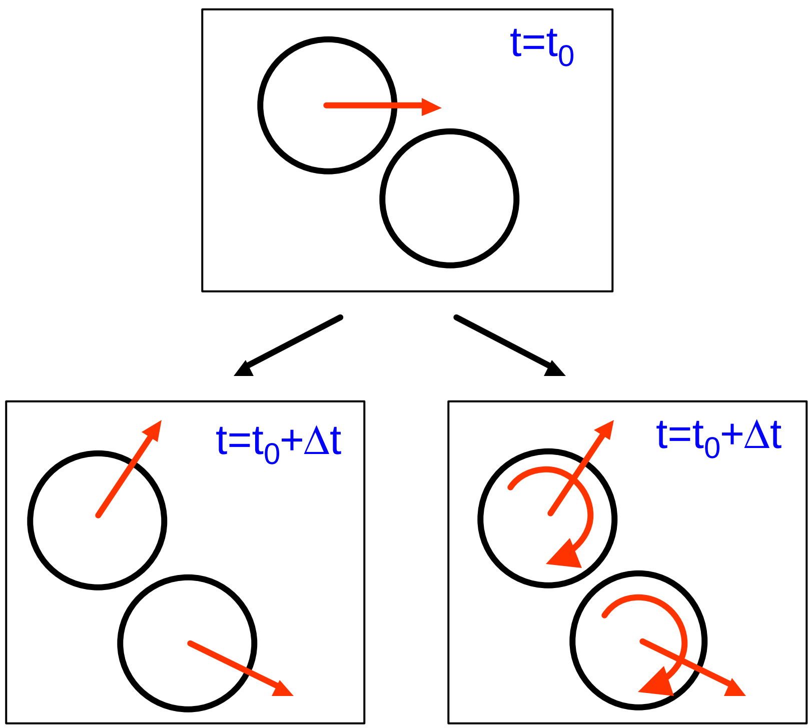
$d_p$=0.47 mm, $\phi_V$=3.6%




*probability density function (pdf) of particle-impeller collision velocities*



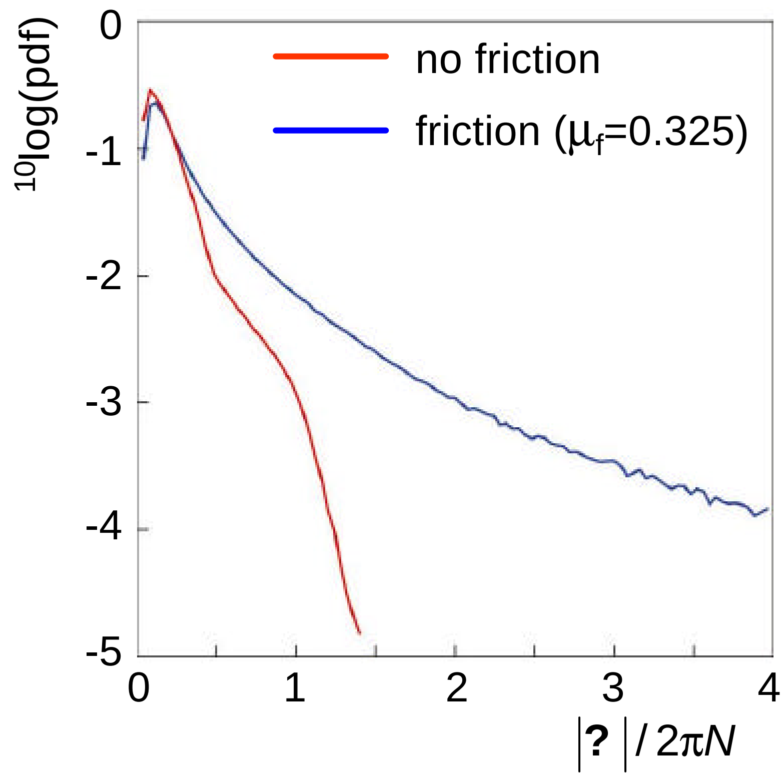
# Collision mechanics



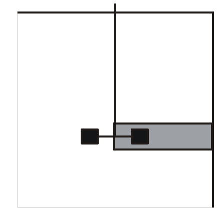

elastic, *frictionless* collision

elastic, *frictional* collision

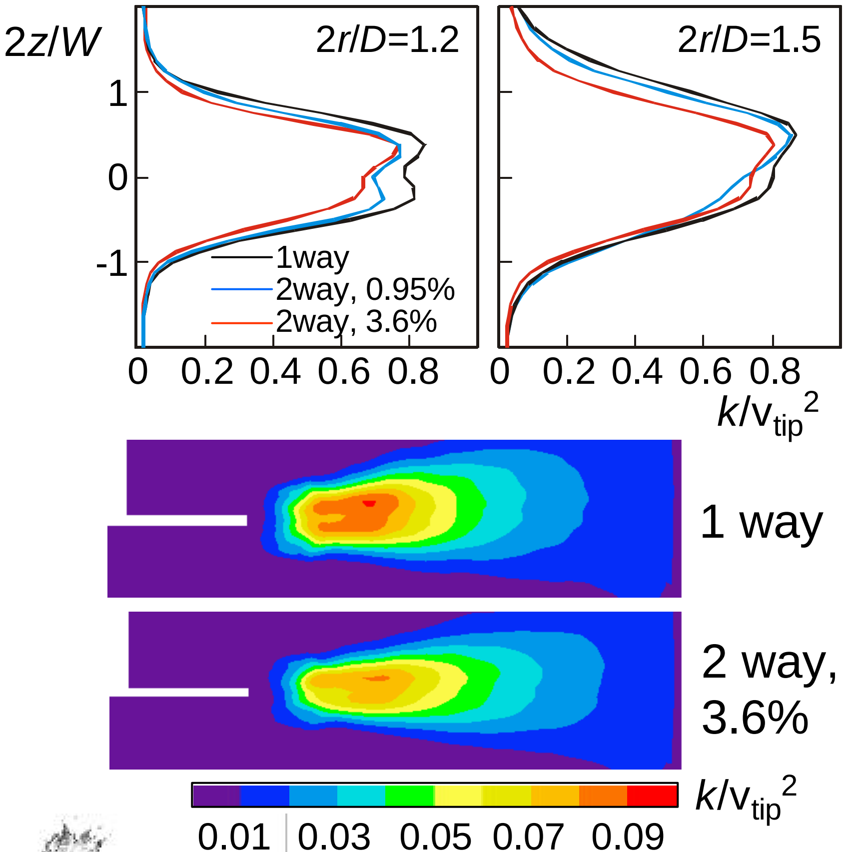
pdf of the particle's angular velocity
(single realization, entire vessel)



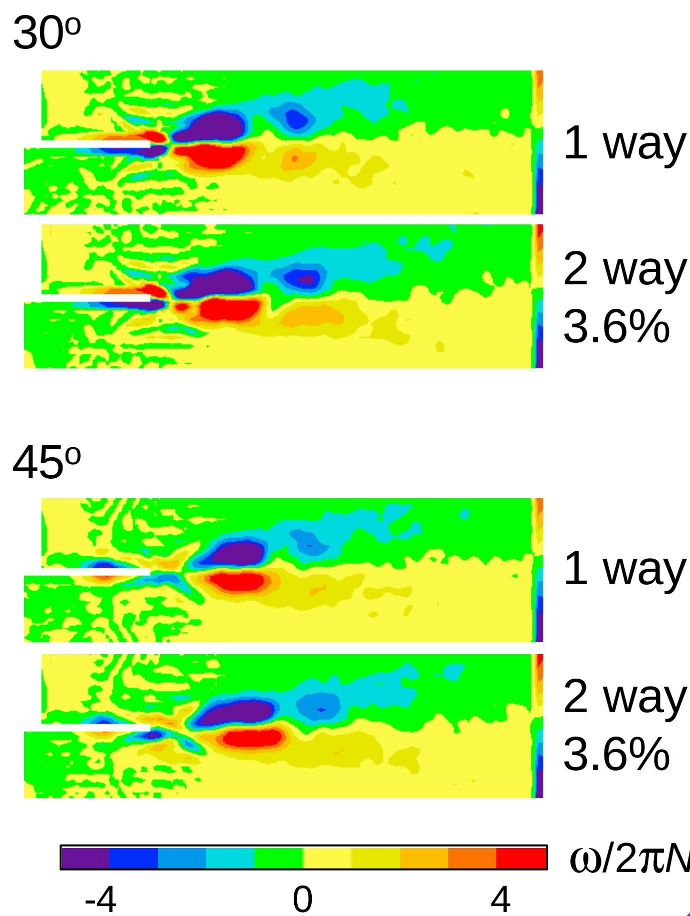
# One- and two-way coupling



## phase-averaged turbulent kinetic energy in the impeller outstream

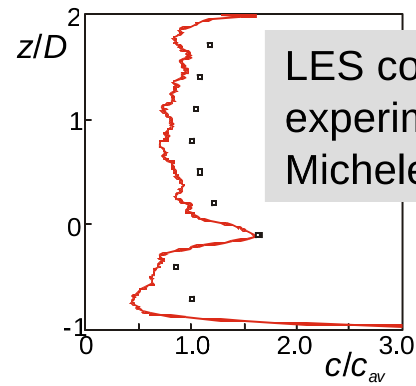



## trailing vortex system

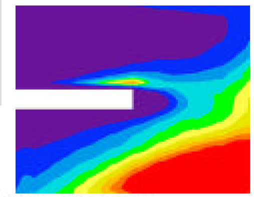


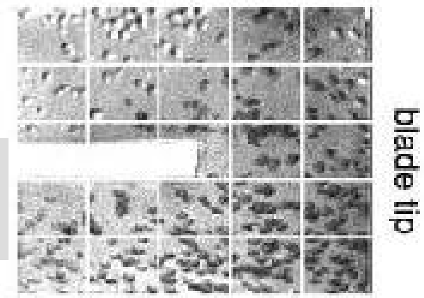
# How to proceed?

Experimental validation
- particle concentration profiles
- particle-impeller collisions
- ......

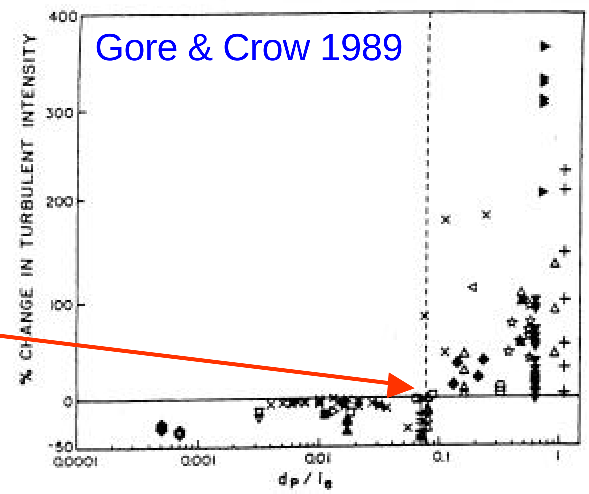



LES compared to experimental data of Michelettti et al. 2003





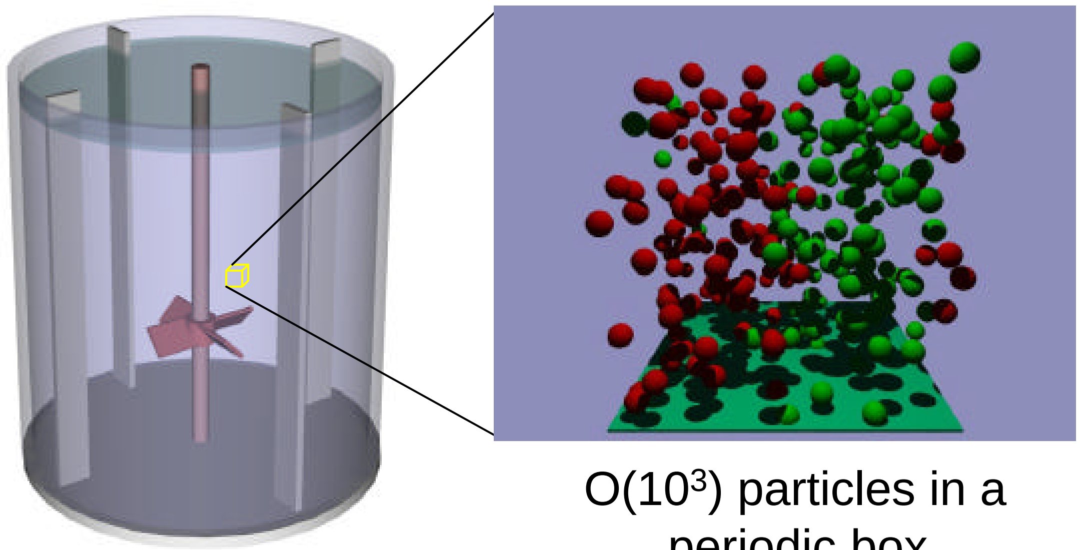

impact tests Kee & Rielly 2004




Solid-liquid system with density ratio≈2.5
- many concepts borrowed from solid-gas systems
  - point particles
  - neglect history force
  - neglect lubrication
  - simple two-way coupling

**We need to investigate finite-particle-size effects**

# DNS with interface resolution[1]



O($10^3$) particles in a periodic box

*Particles:*

Lattice-Boltzmann simulation
Fully resolved particles

*Turbulence:*

Fluctuating bodyforce[2]

*Particle interactions:*

Through LB fluid
Lubrication forces
Hard-sphere p-p collisions

- Particle – turbulence interaction
- Collision statistics

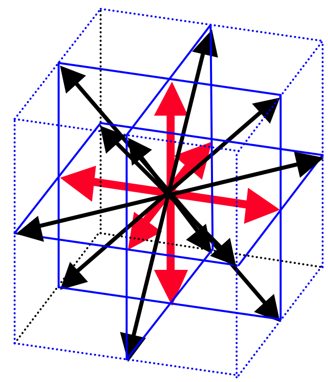
[1] Ten Cate et al. *JFM* 2004
[2] Alvelius *PoF* 1999

# Lattice-Boltzmann (LB) discretization

Particles move from one lattice site to the other and collide:

$$N_i(\mathbf{x}+\mathbf{c}_i, t+1) = N_i(\mathbf{x},t) + \Gamma_i(\mathbf{N})$$

$$\rho u_\alpha = \sum_i c_{i\alpha} N_i$$



Space, time, *and velocity* are discretized:

***local operations: good parallel efficiency***

***uniform, cubic lattice***

2nd order (space and time) representation of a Navier-Stokes-*like* equation, *e.g.*:

$$\frac{\partial \rho u_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta}\rho u_\alpha u_\beta = -\frac{1}{3}\frac{\partial \rho}{\partial x_\alpha} + \nu \frac{\partial}{\partial x_\beta}\left(\frac{\partial \rho u_\beta}{\partial x_\alpha} + \frac{\partial \rho u_\alpha}{\partial x_\beta}\right) + f_\alpha$$

velocity/ physical time-step constraint

**this is incompressible Navier-Stokes if** $\left|\mathbf{u}^2\right| << c_{sound}^{\;2}$

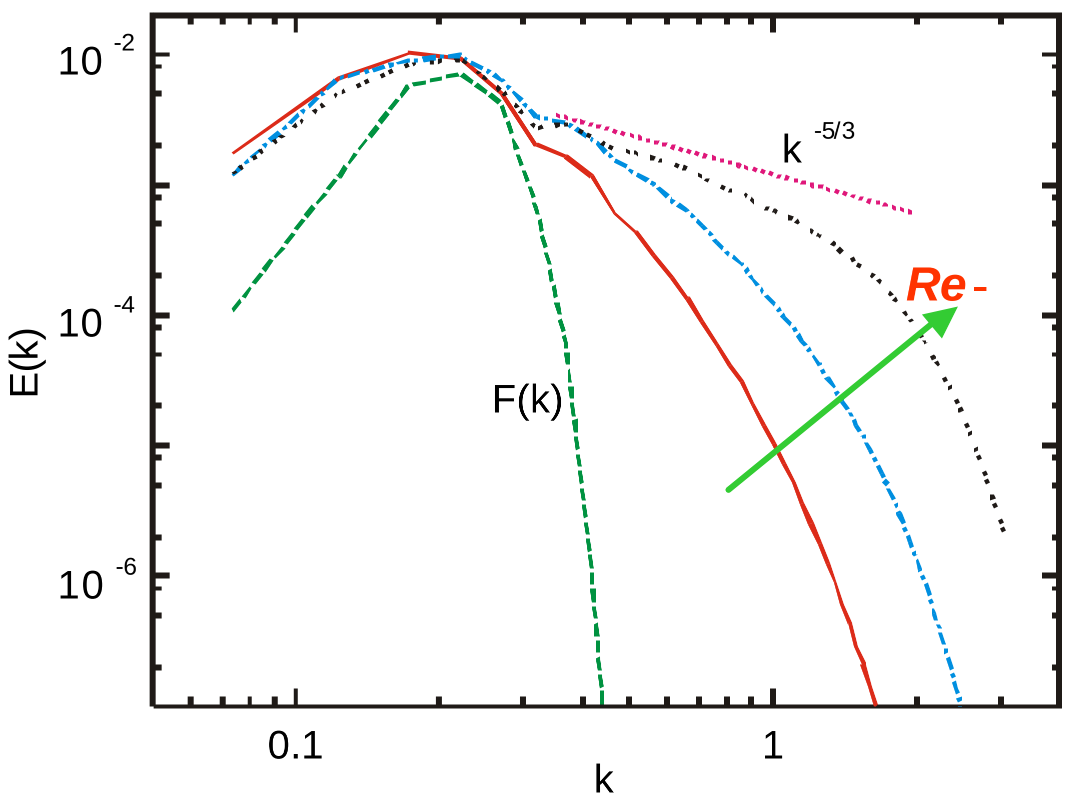
$$p = \frac{\rho}{3} \quad \rightarrow \quad c_{sound} = \sqrt{\frac{1}{3}}$$

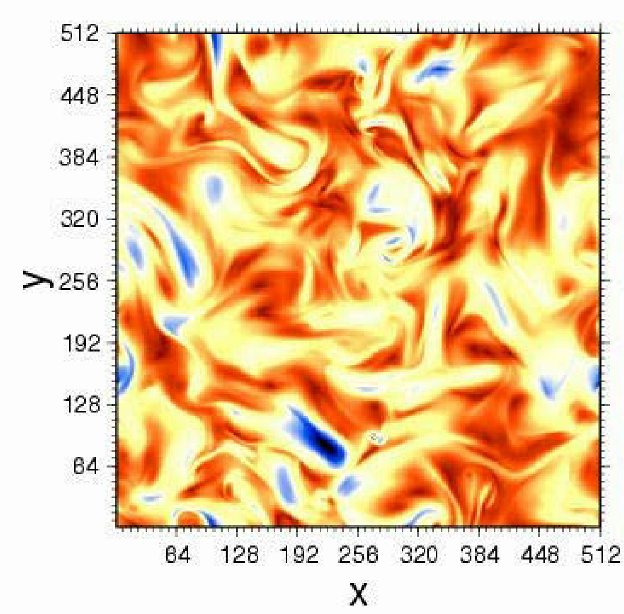
# Forced turbulence within LB

single-phase flow

$$Re_{forcing} = \frac{u_* l_*}{\nu} = 18{,}000$$

$\eta_K = 0.5\Delta$ $\quad 512^3$ cubic grid

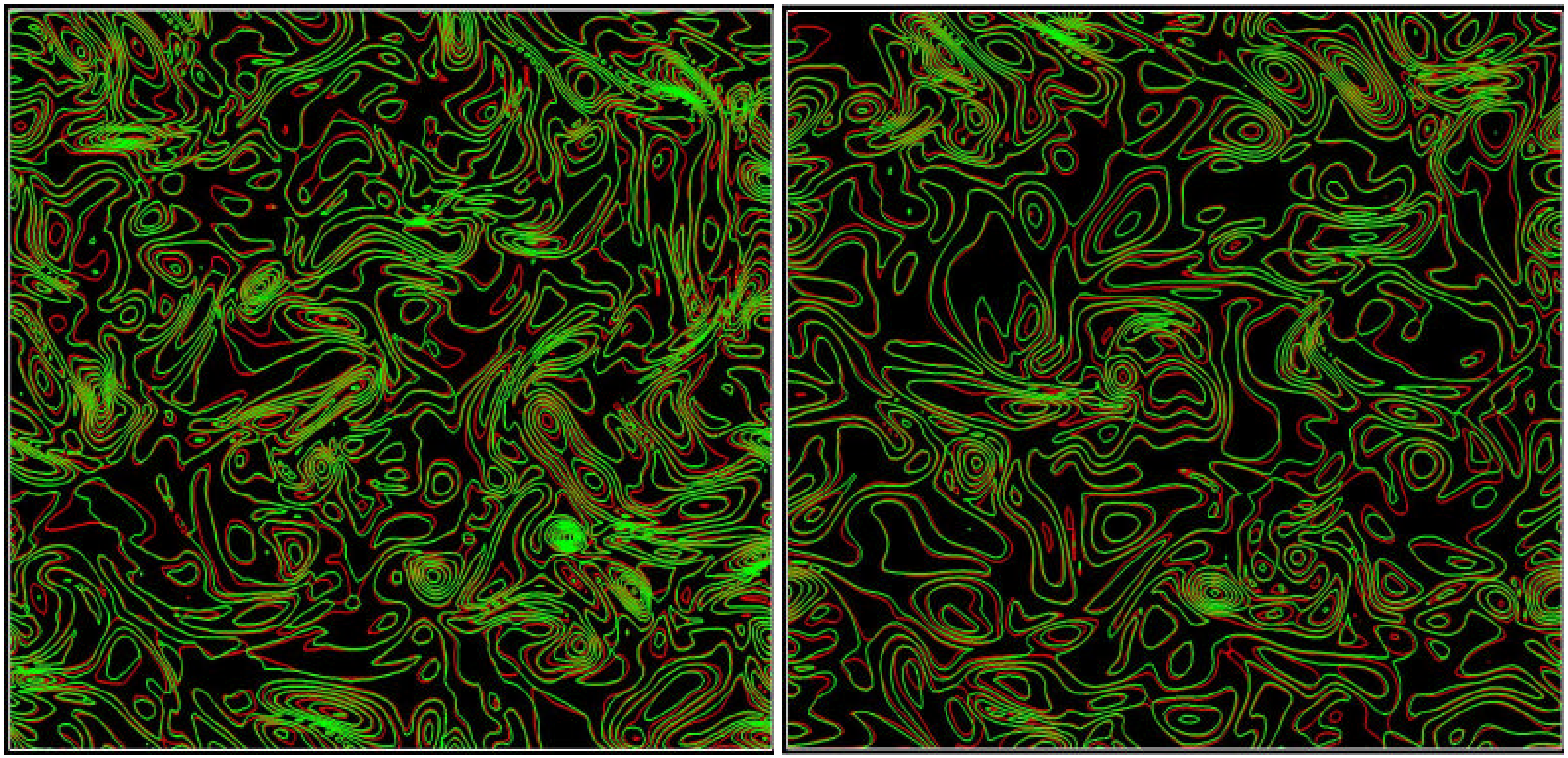



colors: velocity magnitude

# DNS of decaying homogeneous, isotropic turbulence

$$Re_\lambda = \sqrt{\frac{3K(t=0)}{2}}\frac{\lambda}{\nu} = 35$$



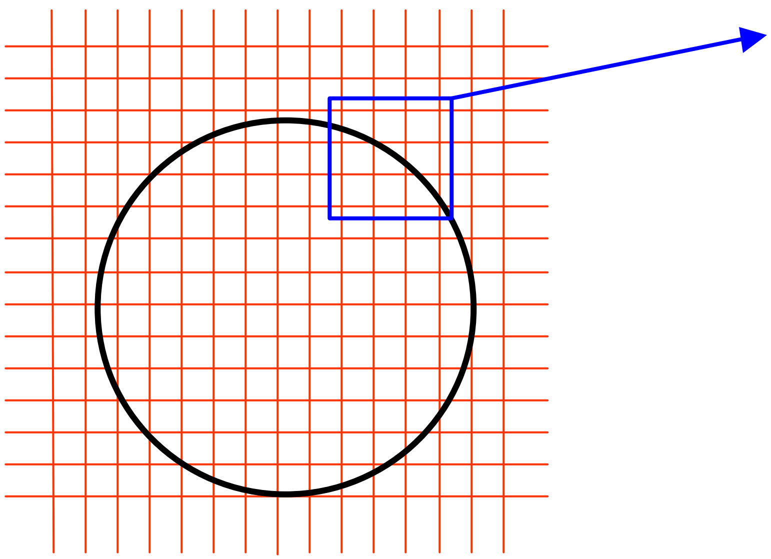

LB is a competitive tool for (single-phase) turbulence simulations

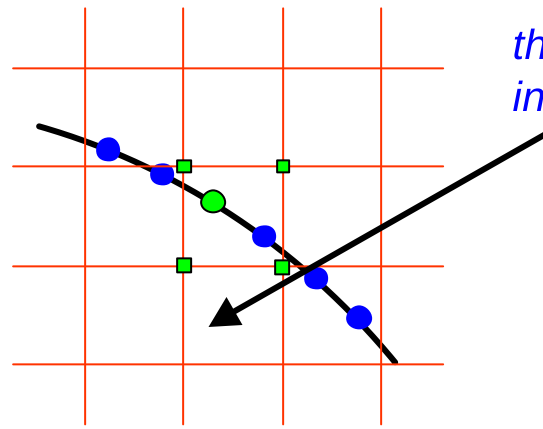
$|\nabla \times \mathbf{u}|$ contours at 2 moments in time started from same initial condition

courtesy Li-Shi Luo

# Boundary conditions: forcing method[1,2]

no-slip at the surface of the moving, spherical particles






at determine the fluid local velocity by interpolation from

$$? \mathbf{u} = \mathbf{u}_{\mathbf{fluid}} - (\mathbf{u}_{\mathbf{particle}} + ?_{\mathbf{particle}} \times \mathbf{r})$$

$$\mathbf{F}^{i+1} = \alpha \mathbf{F}^{i} - \beta ? \mathbf{u} \qquad \alpha=0.95;\ \beta=1.8$$

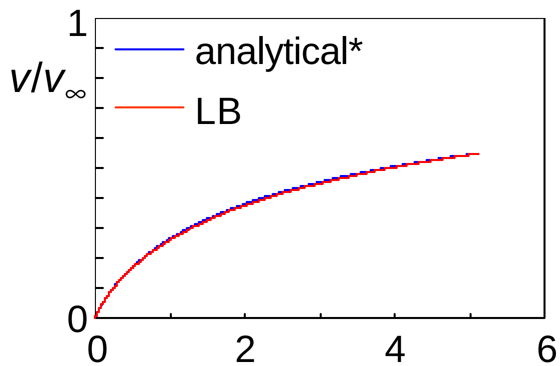
distribute $\mathbf{F}^{i+1}$ at to the lattice-nodes

$-\Sigma\mathbf{F}$ is the fluid to particle force

(1) Goldstein et al., *J. Comp. Phys*. **105** (1993)
*applied within spectral method*
(2) Ten Cate et al., *Phys. Fluids* **14** (2002)
*applied within LB method*

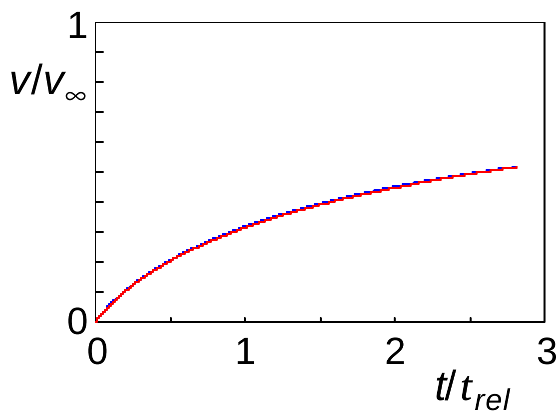
# Single-particle tests

Start-up of a sphere falling under gravity ($Re_p$<1)




$$\frac{\rho_s}{\rho_f} = 1.1$$

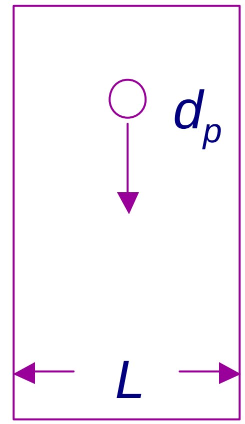
$$\tau_{rel} = \frac{\rho_s}{\rho_f}\frac{d_p^2}{18\nu}$$




$$\frac{\rho_s}{\rho_f} = 2.0$$

* Maxey & Riley, *Phys. Fl.*, **26** (1983)

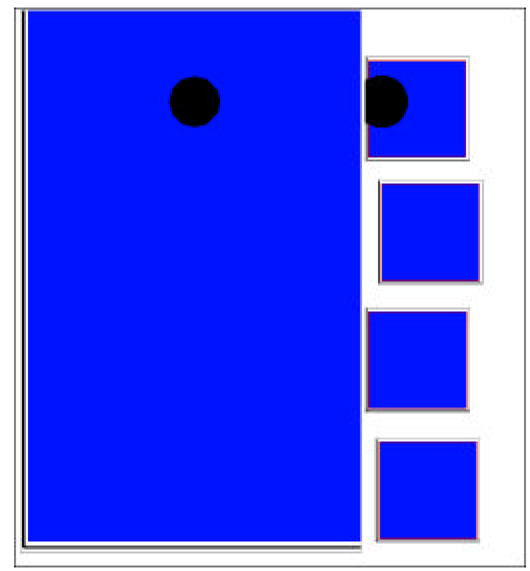
PIV experiment of a falling sphere in a closed box*




$$Re_p = \frac{U_{p,\infty} d_p}{\nu} = 1.5....32$$

$$\frac{d_p}{L} = 0.15$$

$Re_p = 32$



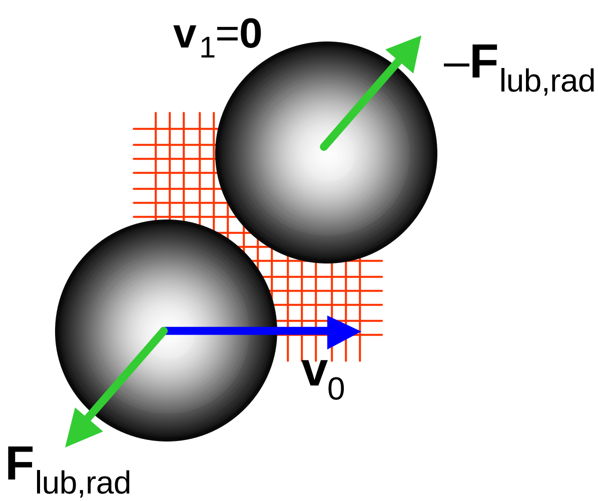
LB simulation PIV exp

*Ten Cate et al., *Phys. Fluids* **14** (2002)

# Lubrication forces (and torques)*

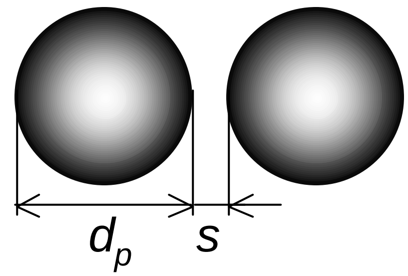
For particles in close proximity to another particle or a wall

Example:
radial lubrication force




radial lubrication force $\propto \dfrac{d}{s}$

tangential lubrication force $\propto \ln\left(\dfrac{d}{s}\right)$




lubrication switched on once $s \le \delta$
with $\delta = 0.1 d_p$ ($\approx \Delta$)

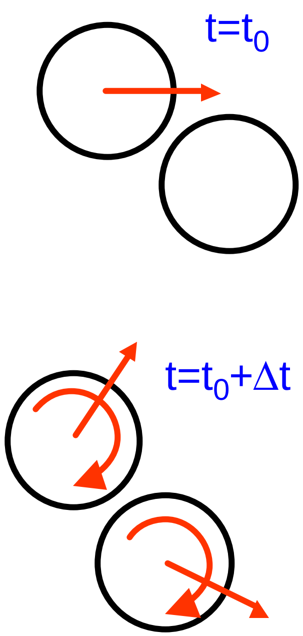
$$F_{\text{lub,rad}} \propto \frac{d_p}{s} - \frac{d_p}{\delta}$$

$$F_{\text{lub,tang}} \propto \ln\left(\frac{d_p}{s}\right) - \ln\left(\frac{d_p}{\delta}\right)$$

limit the lubrication force if $s \le \varepsilon$
with $\varepsilon = 10^{-4} d_p$
*(surface roughness)*

* Kim & Karrila: *Microhydrodynamics* (1991)
Nguyen & Ladd, *Phys. Rev. E*, **66** (2002)

# Hard-sphere collisions




A two parameter model*

- restitution coefficient $e$
- Coulomb friction coefficient $\mu_f$

*default settings*: $e=1$, $\mu_f=0$

no overlap between particles is allowed:

*event-driven* simulation of particle motion

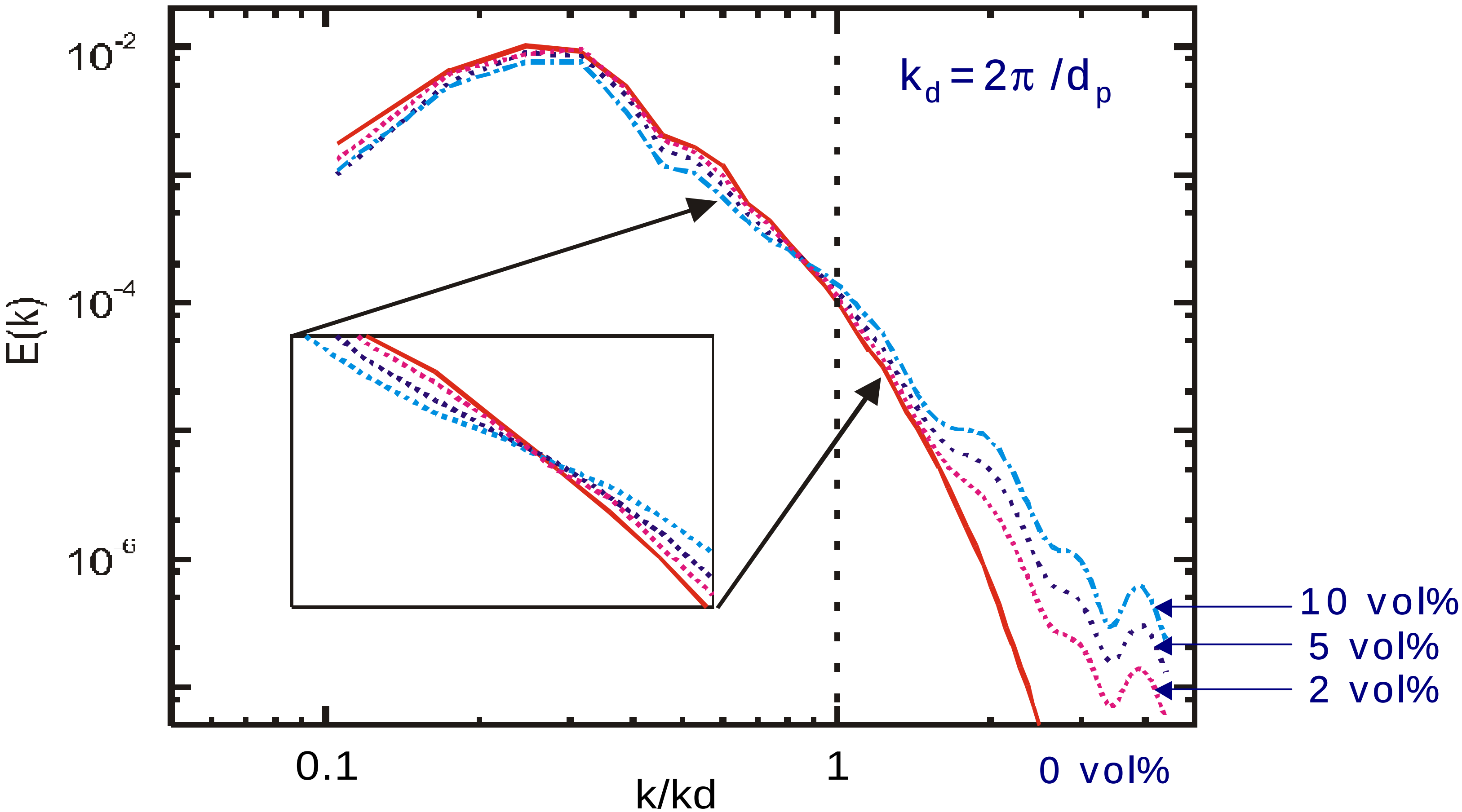
* Yamamoto et al., *JFM*, **442** (2001)

# Settings for solid-liquid simulations

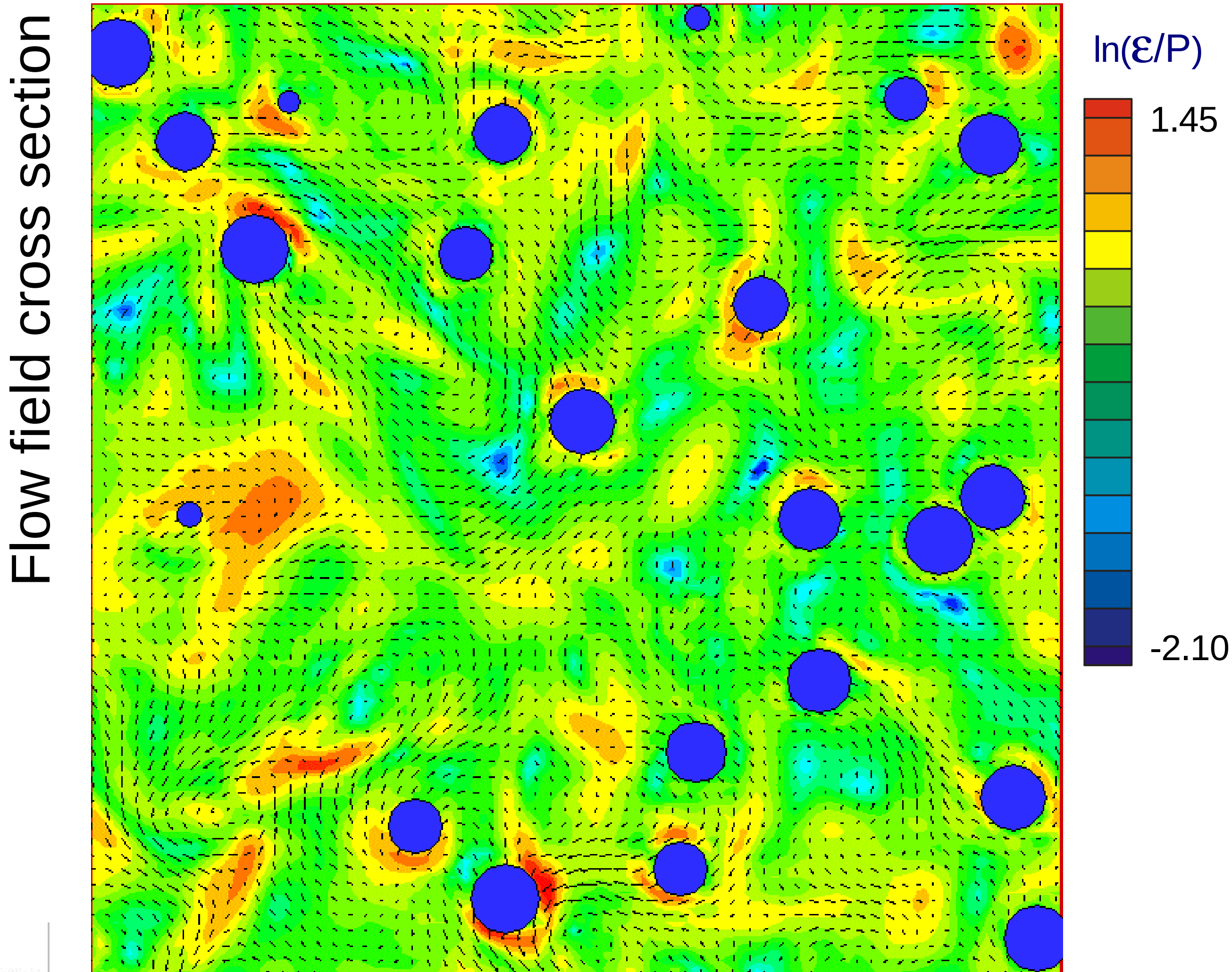
Particle diameter in grid units: 8
Kolmogorov scale in grid units: 1.2

| Vol % | $\rho_p / \rho_f$ | $N_p$ |
|---|---|---|
| 2 | 1.414 | 773 |
| 5 | 1.414 | 2200 |
| 10 | 1.414 | 3868 |
| 5 | 1.146 | 2200 |
| 5 | 1.728 | 2200 |

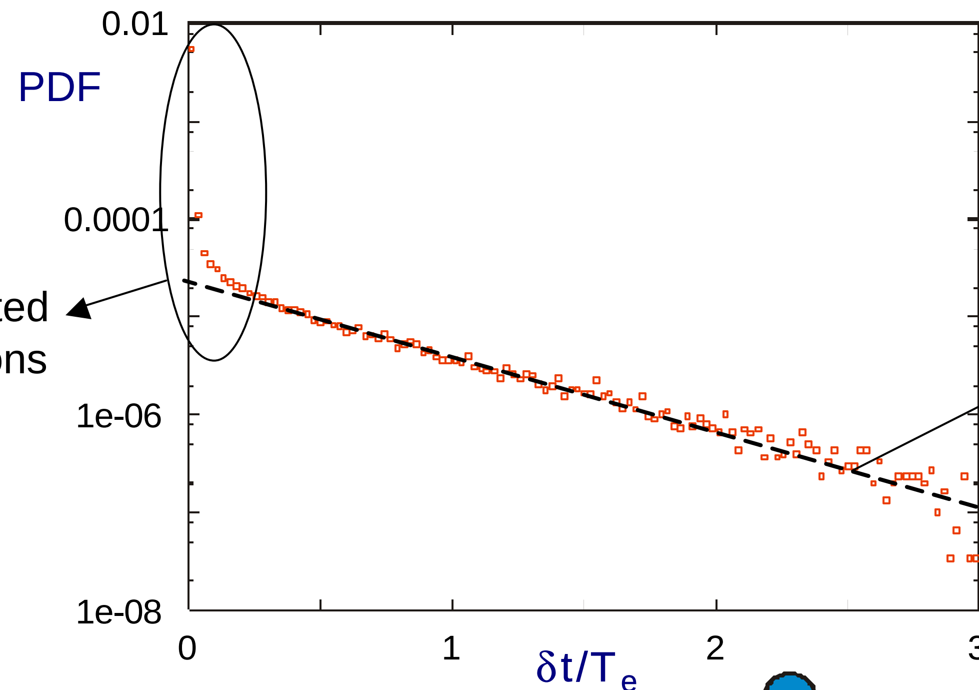
# Particle-turbulence interaction



# Energy dissipation rate



# Short range interactions

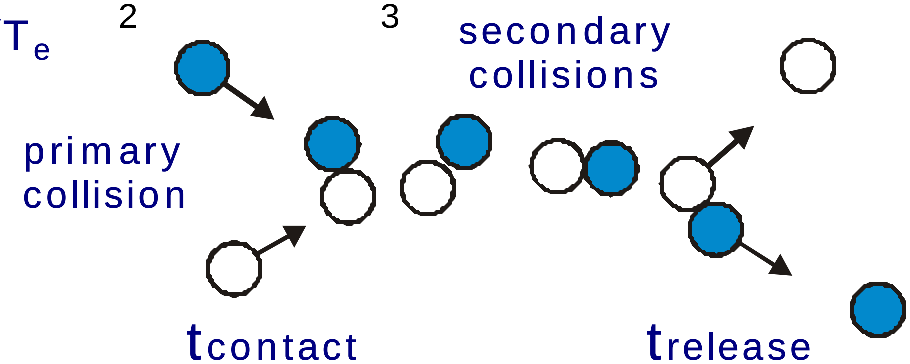



PDF of time between particle-particle contacts

correlated collisions

uncorrelated collisions (Poisson stats)

Multiple collisions

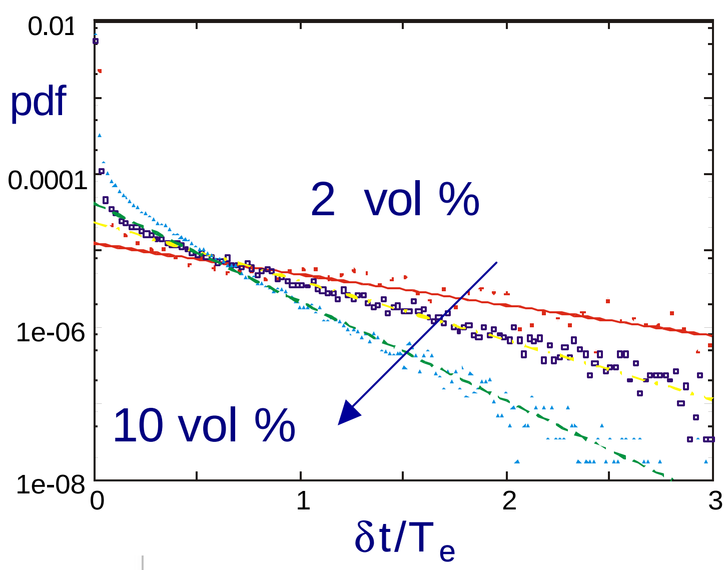


# Short range interactions (2)

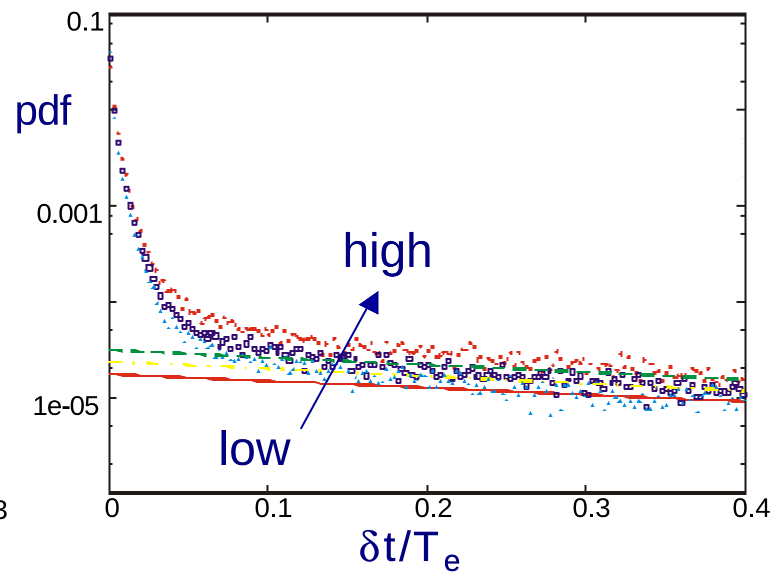
primary collisions:

exponential behavior of the PDF (Poisson-process)

effect of number density




effect of solids density



# What did we learn?

- Turbulence modulation by particles
- Primary and secondary collision mechanism demonstrated
- Collision time depends on volume fraction
  Primary/Secondary collision ratio depends on particle inertia

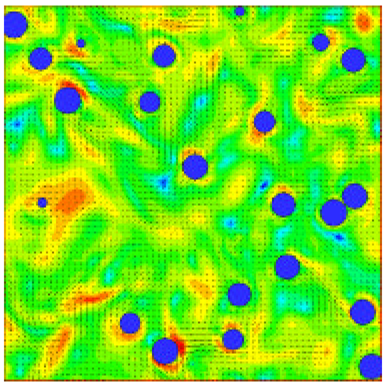
***To do***

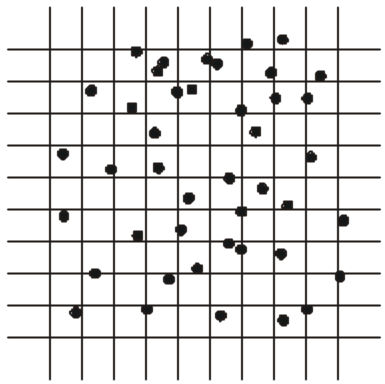
compare point-particle LES/DNS and full DNS on periodic domains

relative particle velocities

collisions statistics

....





# Much denser systems: liquid-solid fluidization

$\phi_s \approx 0.5$
no turbulence

Same methodology as for turbulent suspension

Lattice-Boltzmann method for the fluid flow

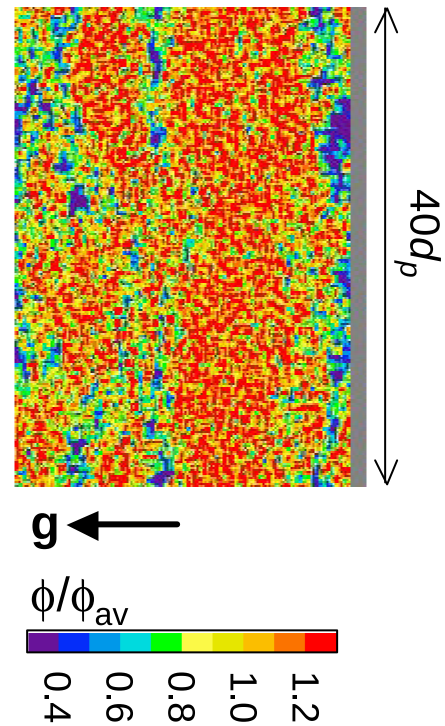
  with immersed boundary technique for no-slip at solid-liquid interface

Hard-sphere collision algorithm

Lubrication forces

water-steel $\frac{\rho_s}{\rho_f} \approx 8$


$\frac{\rho_s}{\rho_f} = 8$ $\phi_{av}$=0.49

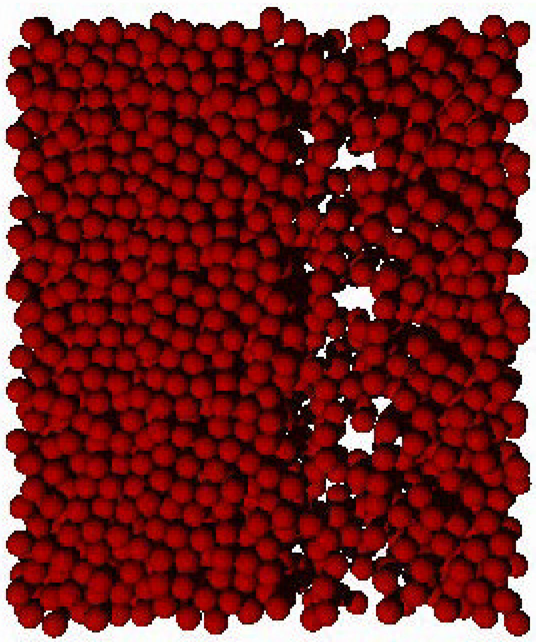



**3D flat beds (depth=12$d_p$)**

# Detailed view of void formation

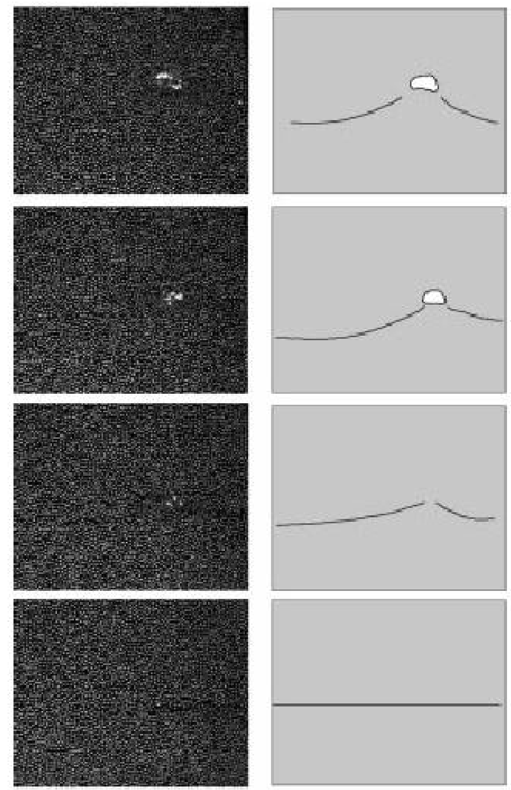
Onset of 2D instabilities in a flat bed

3D domain: $20d_p$x$24d_p$x$6d_p$ $\phi_{av}$=0.505 $\frac{\rho_s}{\rho_f} = 16$



$g$ ← $e$=1, $\mu_f$=0

Compare to the scenario as measured by Duru & Guazzelli*



1 cm ($d_p$=1 mm)

$g$ ↓

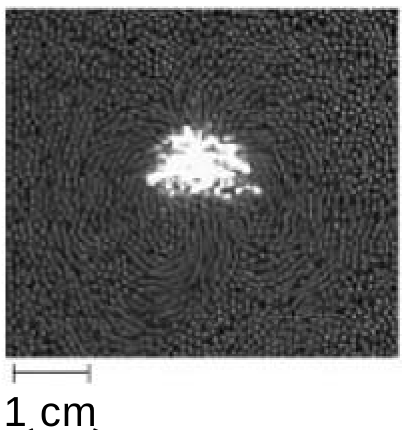
* *JFM* **470** (2002)

# Voids in flat beds (ctd)

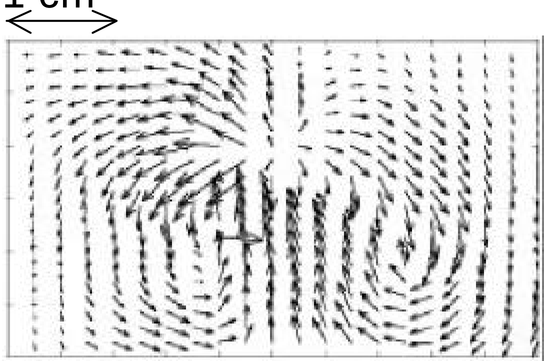
## Experiments*

1 mm steel beads in water $\frac{\rho_s}{\rho_f} = 8$

g




bubble formation

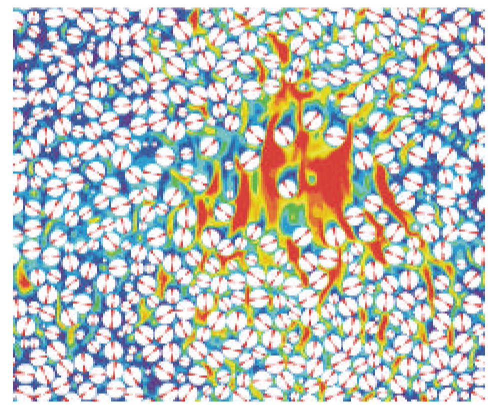


particle velocities (PIV)

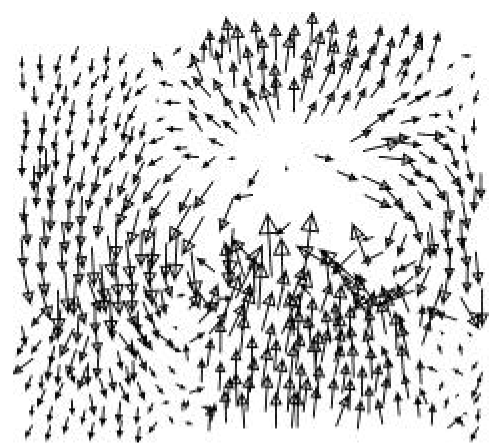
* Duru & Guazzelli *JFM* **470** (2002)

## Simulations

0.7 mm beads in water $\frac{\rho_s}{\rho_f} = 16$



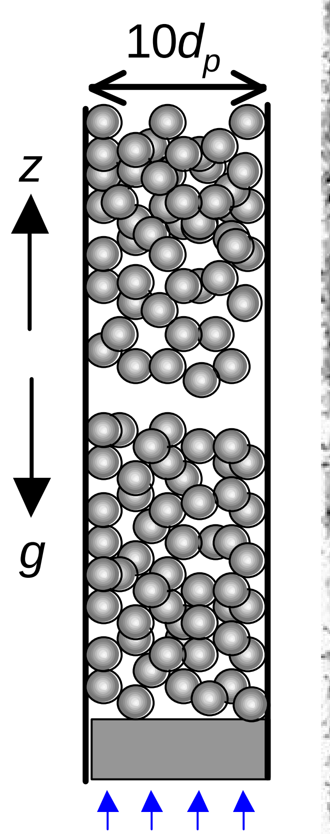
g

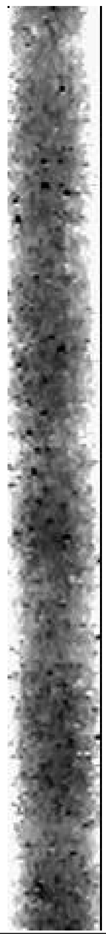


particle velocities

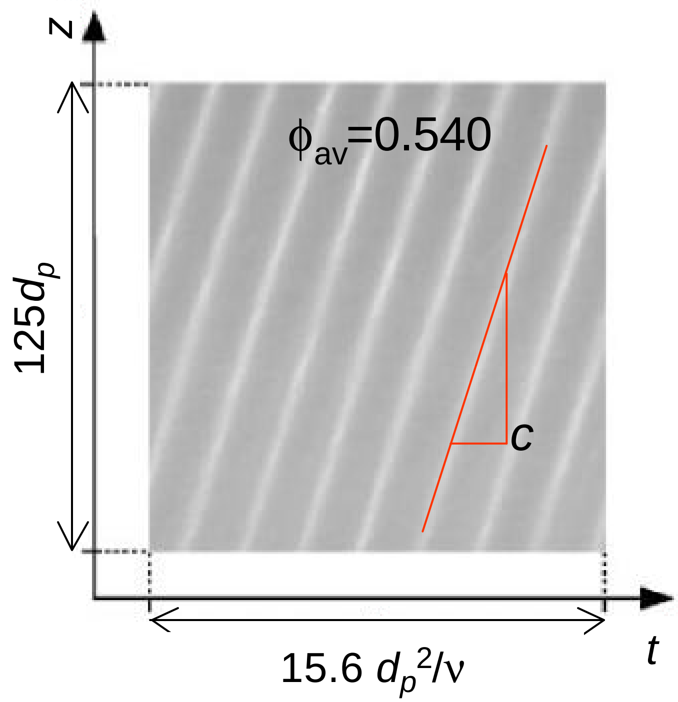
# One step simpler: 1D (narrow) beds

typically 1 mm glass beads in water



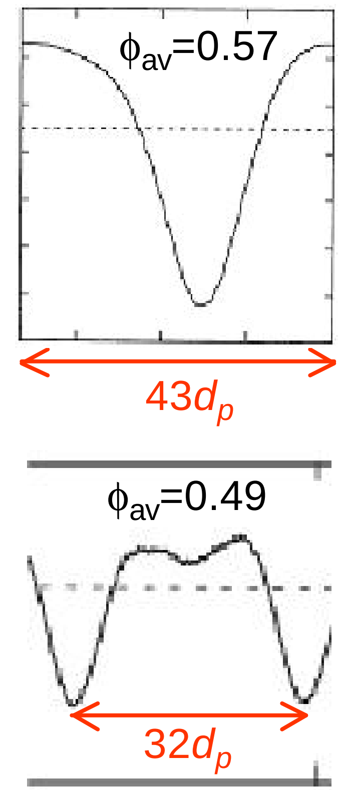



Experimental result*: planar wave instability



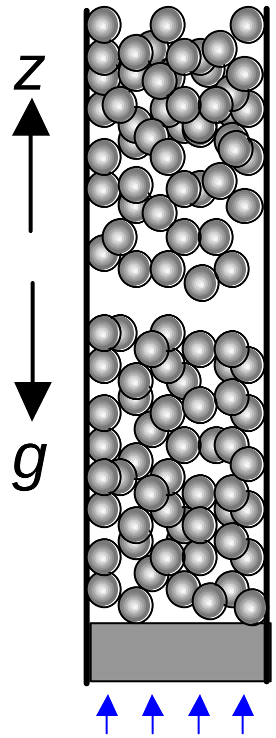

space-time plot of $\phi_s$

$\phi_s$-wave shapes

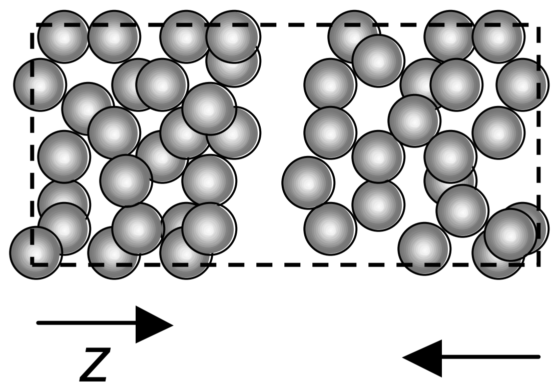



* Duru et al., *JFM* **452** (2002)

# Modeling step




fully periodic 3D box
$20d_p \text{x} 6d_p \text{x} 6d_p$




$$\mathbf{f_p} = -(\rho_s - \rho_{mixture}) g \mathbf{e_z}$$

$$\mathbf{f_f} = (\rho_{mixture} - \rho_f) g \mathbf{e_z}$$

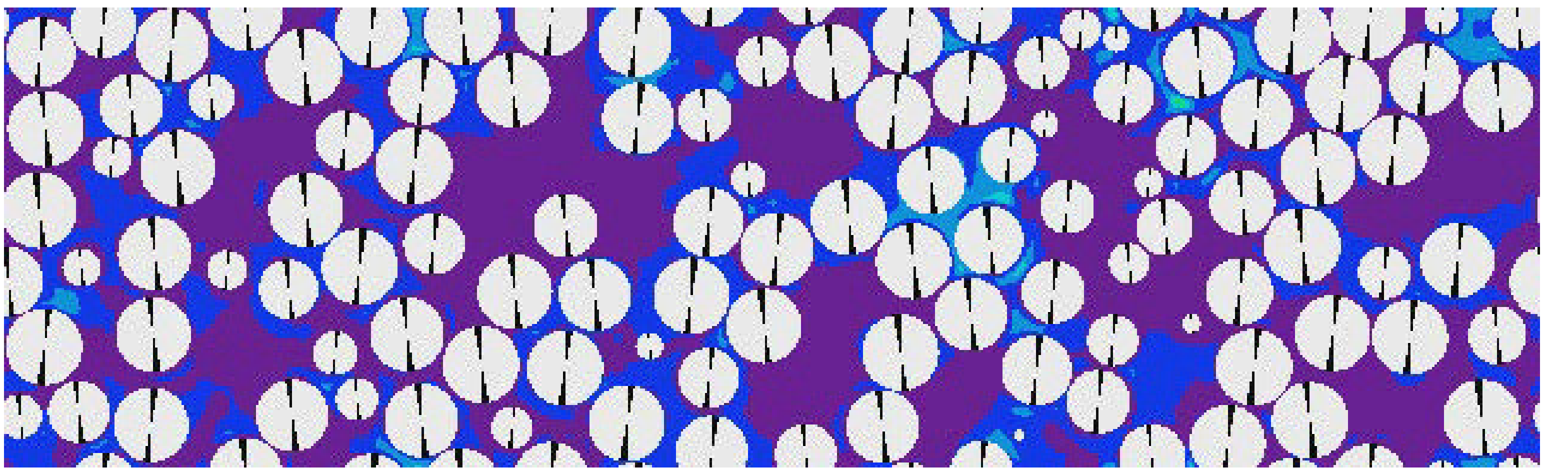
$$\rho_{mixture} = \phi_s \rho_s + (1 - \phi_s) \rho_f$$

# Simulation of L-S fluidization

$\phi_{av}$=0.505 $\frac{\rho_s}{\rho_f} = 4.1$ Cross-section from fully periodic, 3D domain

$20d_p$

$6d_p$

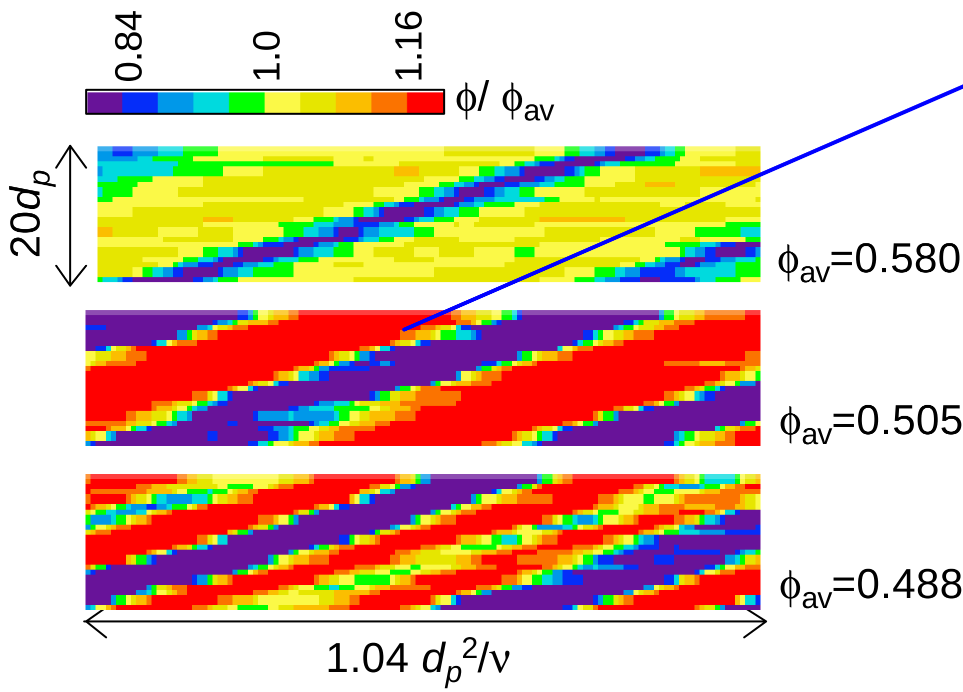


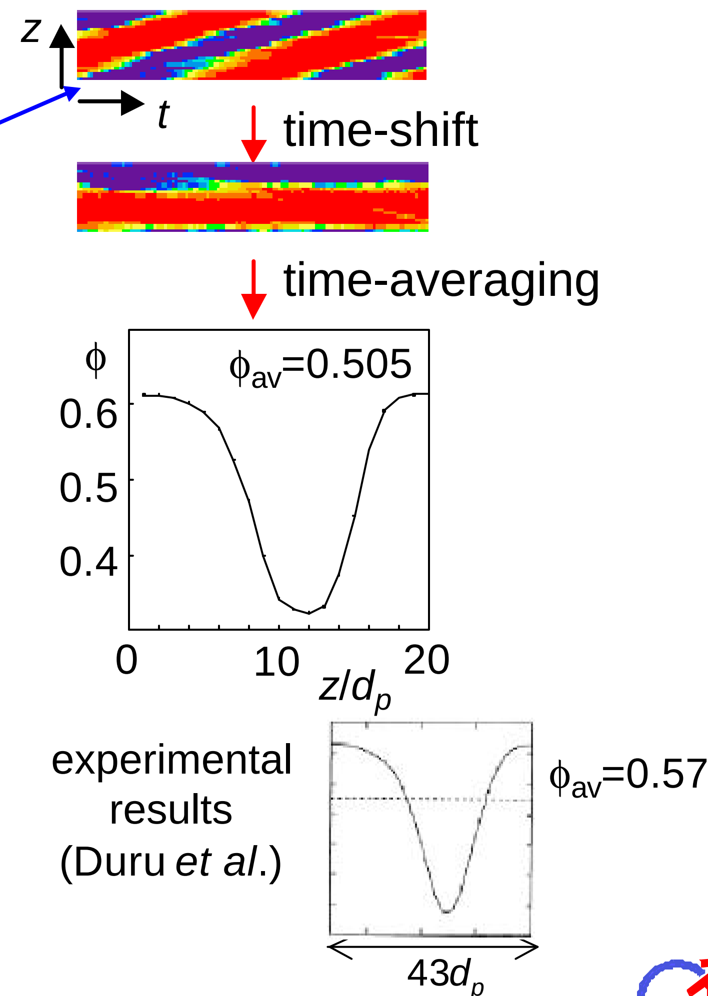
body force on fluid

gravity

# Wave-speed and wave-shape

## Simulated space-time plots




## Wave speeds

$$\left(\frac{c\,d_p}{\nu}\right)_{exp} = 29 \pm 1 \qquad \left(\frac{c\,d_p}{\nu}\right)_{num} = 33 \pm 2$$

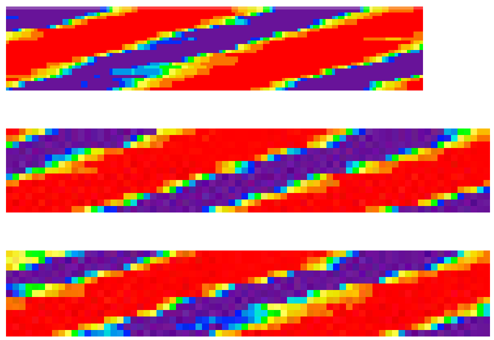
# Collision parameters ($\phi_{av}$=0.505)

frictionless collisions ($\mu_f$=0.0)

$e$=1.0

$e$=0.9

$e$=0.8

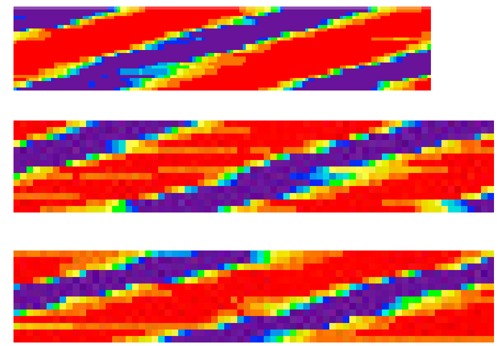


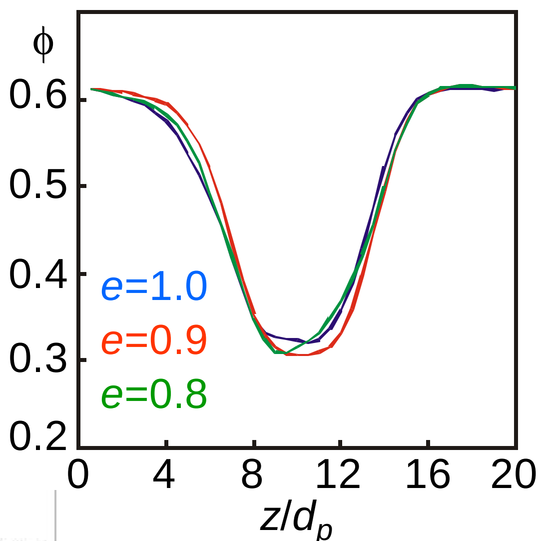
elastic collisions ($e$=1.0)

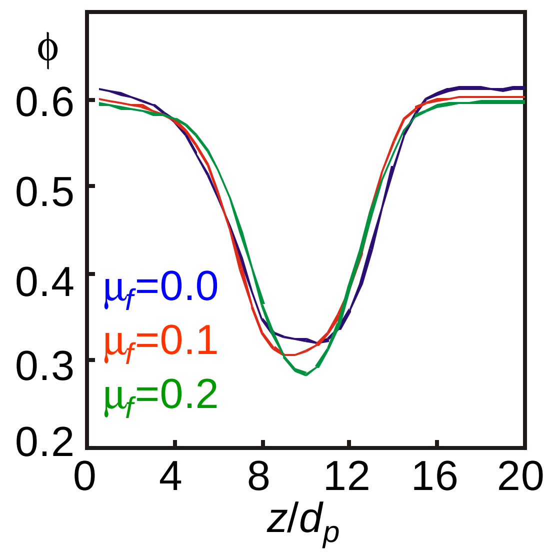
$\mu_f$=0.0

$\mu_f$=0.1

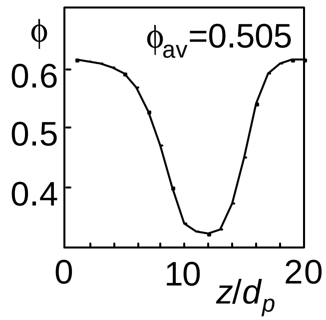
$\mu_f$=0.2





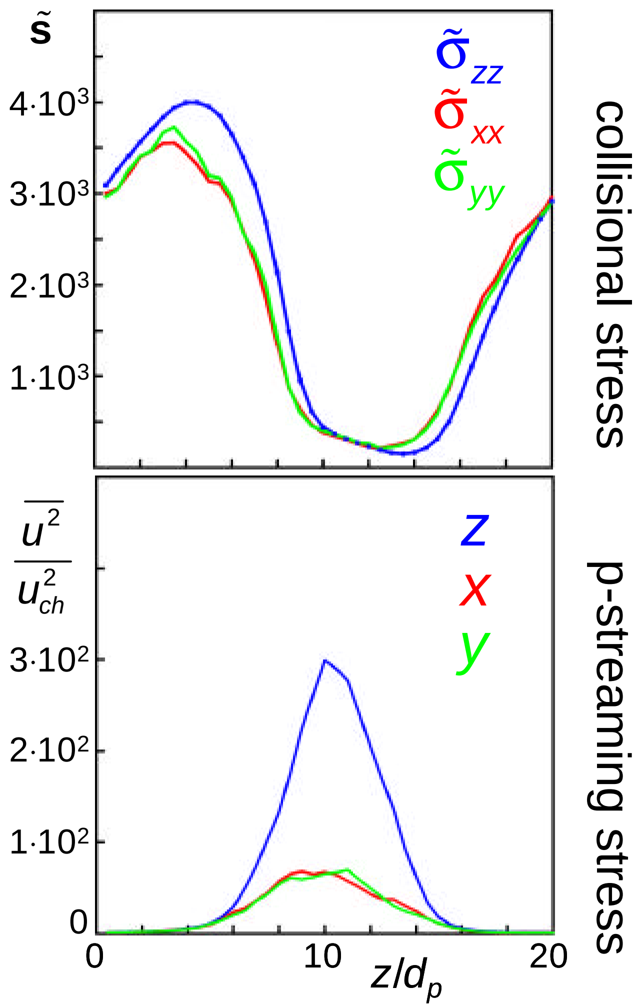
# Momentum transfer (stress)



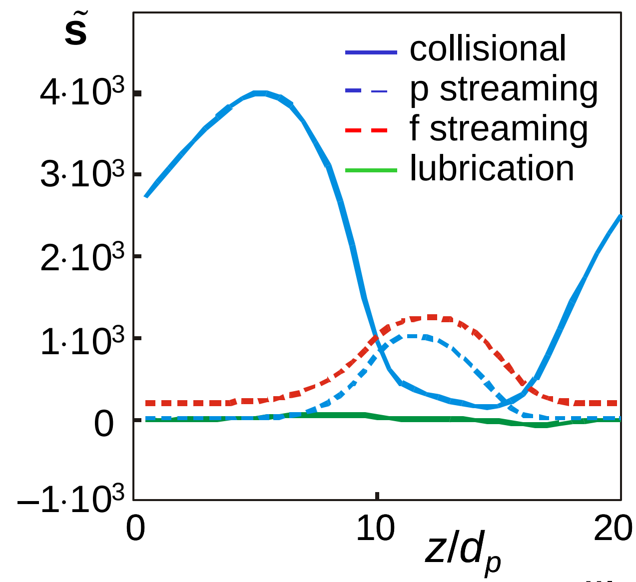

$$\tilde{\mathbf{s}} = \frac{\mathbf{s}}{\rho_f u_{ch}^2}$$

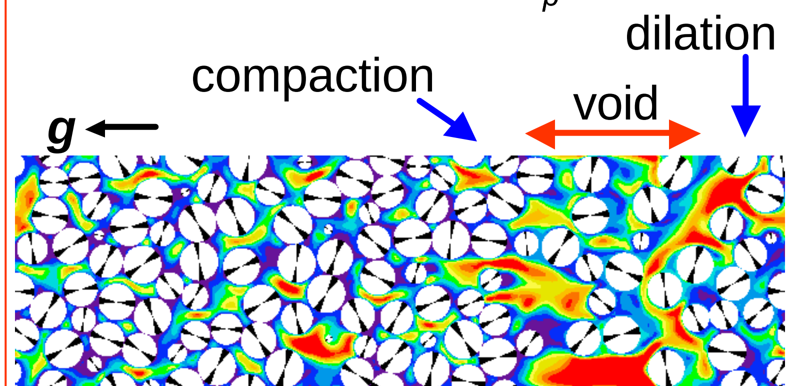
$$u_{ch} = \frac{\nu}{d_p}$$

## (an)isotropy



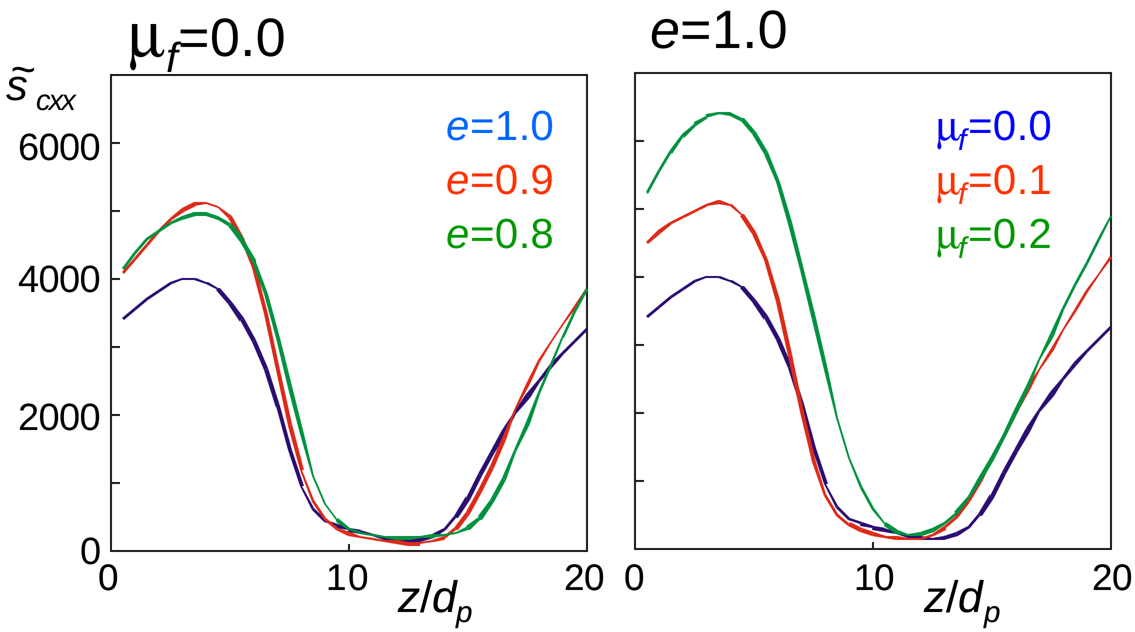

## relative magnitude of *zz* stresses



# Momentum transfer (stress) (2)

influence of collision parameters on $\sigma_{czz}$




$$\tilde{\mathbf{s}} = \frac{\mathbf{s}}{\rho_f u_{ch}^2}$$

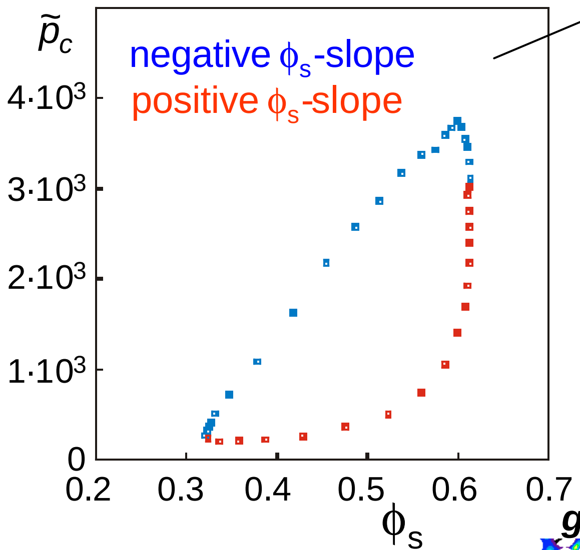
$$u_{ch} = \frac{\nu}{d_p}$$

# Collisional pressure ($\phi_{av}$=0.505)

negative $\phi_s$-slope: compaction
positive $\phi_s$-slope: dilation

$$p_c = \frac{1}{3}\left(\sigma_{c,xx} + \sigma_{c,yy} + \sigma_{c,zz}\right)$$

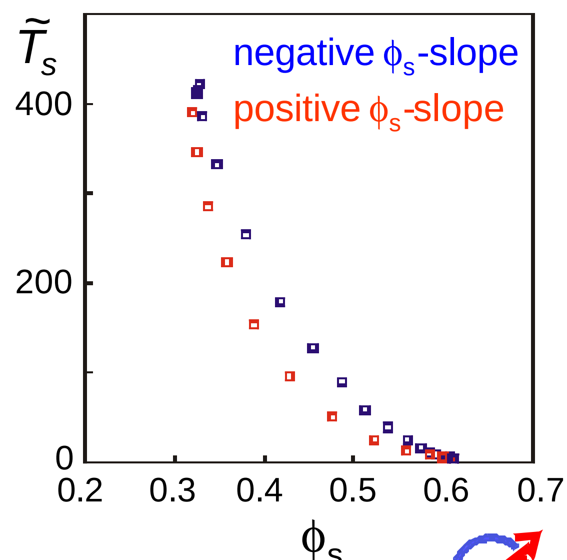
$$\tilde{p}_c = \frac{p_c}{\rho_f u_{ch}^2}$$

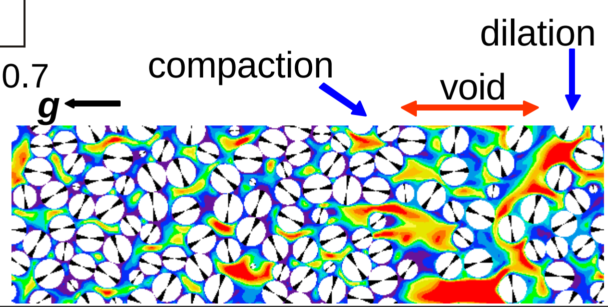



$p_c$ not a unique function of $\phi_s$

*result of granular temperature?*
*(cannot be the only reason)*

$$\tilde{T}_s = \frac{T_s}{u_{ch}^2}$$



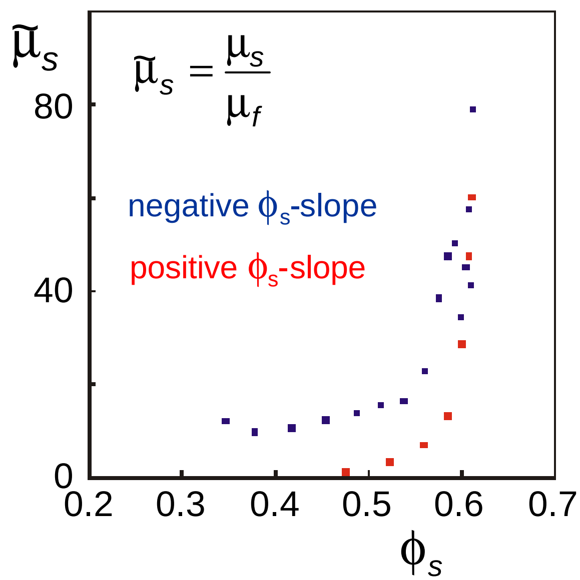



# Solids-phase viscosity ($\phi_{av}$=0.505)

$$\tau_{zz} = \frac{4}{3}\mu_s \frac{du_{z,p}}{dz}$$

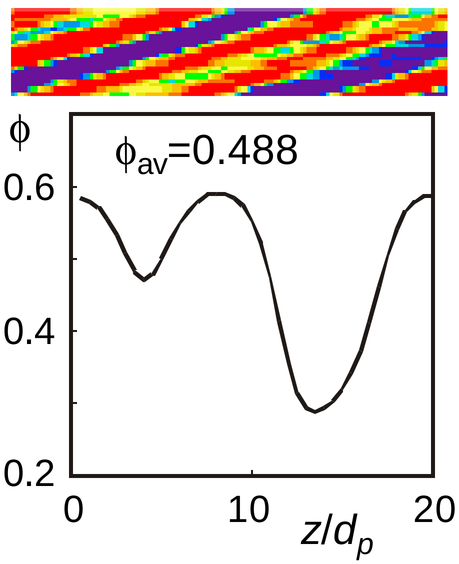
$\tau_{zz}$: deviatoric solids-phase stress

(sum of collisional and p-streaming stress)

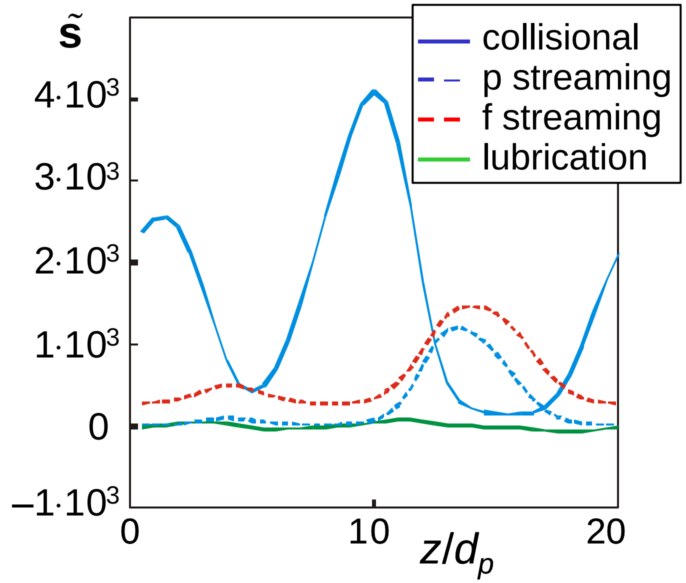


# $\phi_{av}$=0.488

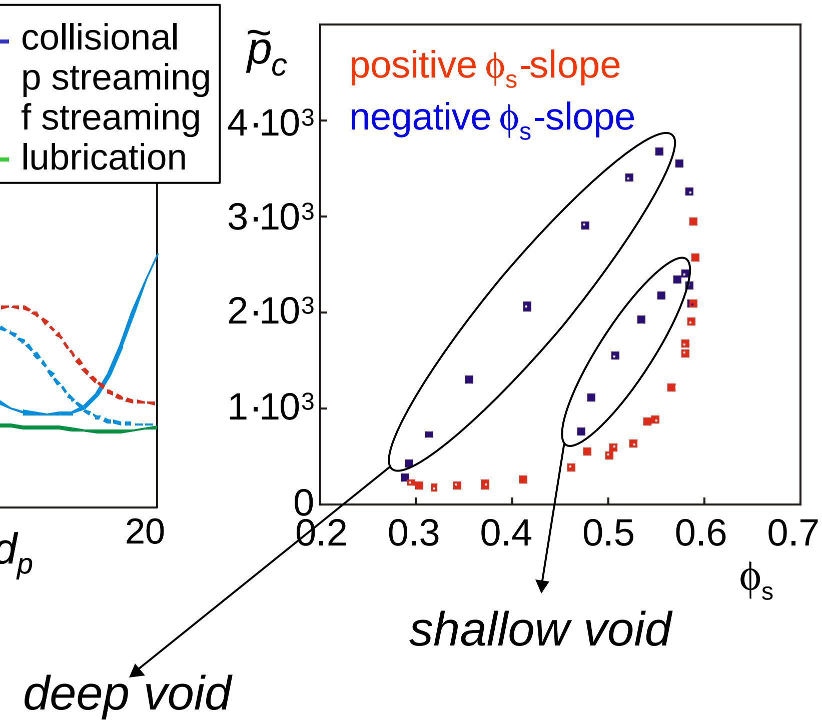
## double hump wave

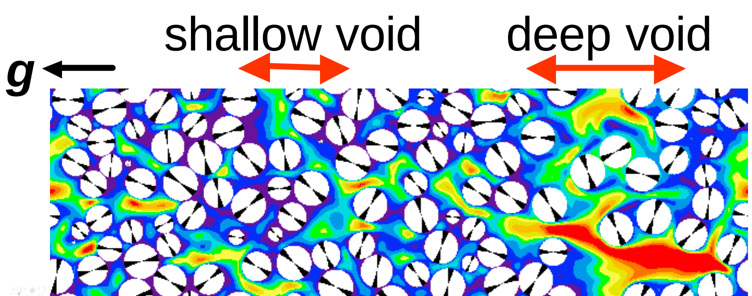



## relative magnitude of stresses




## collisional pressure




$$\tilde{p}_c = \frac{p_c}{\rho_f u_{ch}^2}$$

# Role of bulk viscosity

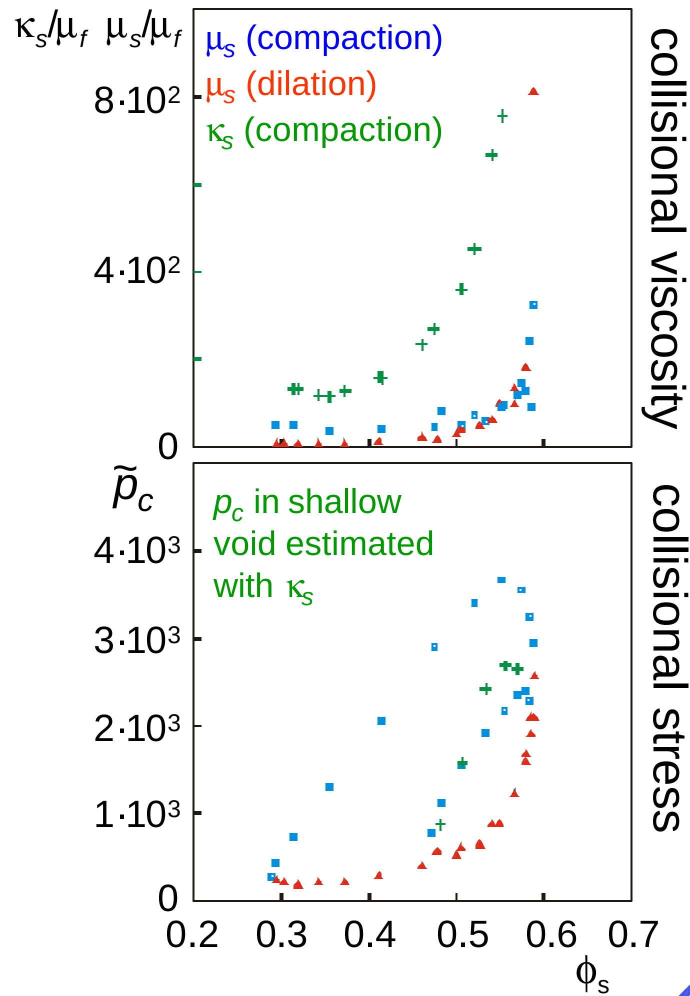
collisional pressure under compaction much higher than under dilation

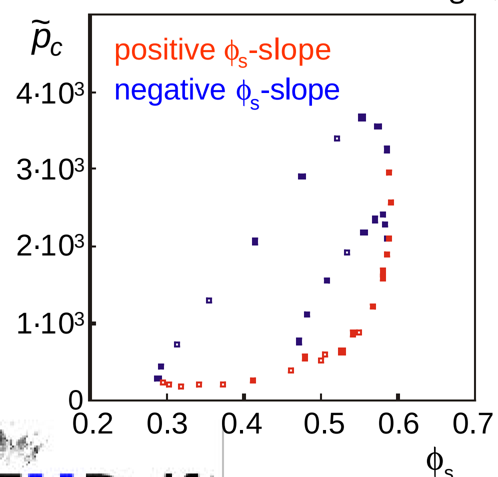
interpret this in terms of the solids-phase *bulk viscosity*

$$p_c = p_s - \kappa_s \frac{\partial u_{pz}}{\partial z}$$

$\kappa_s$ *from compaction in deep void*

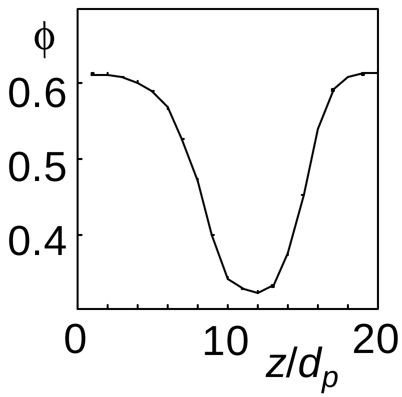
*"baseline" solids pressure, estimate* $p_s(\phi)$ *from dilation*

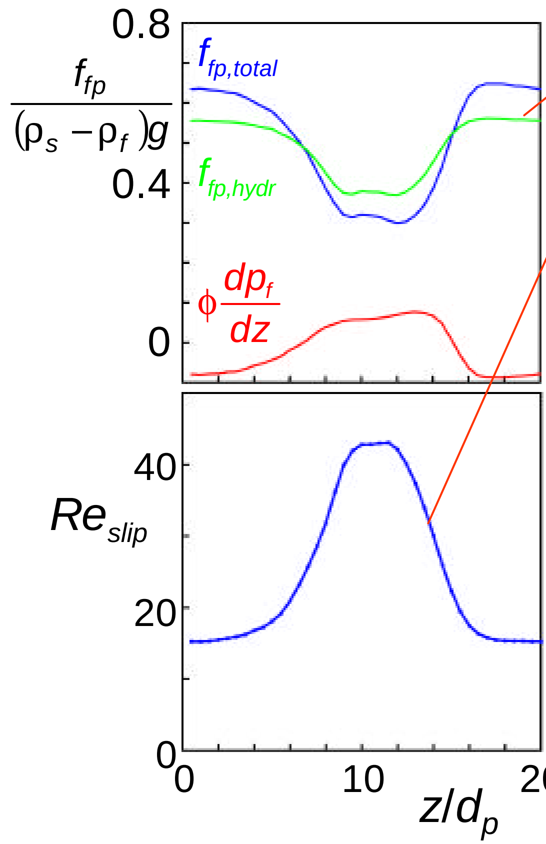


# Hydrodynamic forces

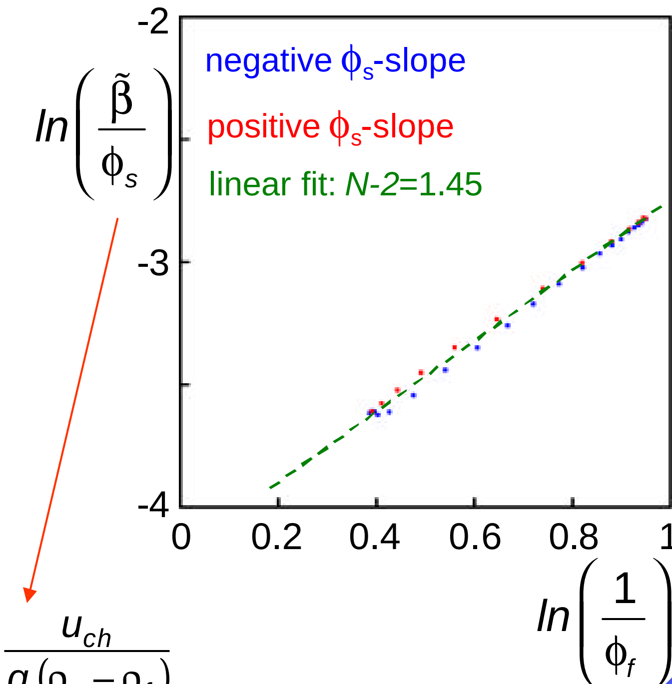
$\phi_{av}=0.505$



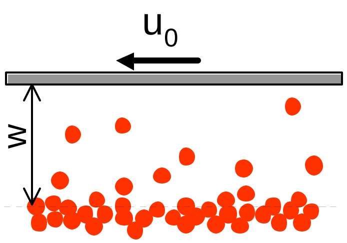

$$\beta = \frac{f_{fp,hydr}}{u_{slip}} \qquad v_{sed} = v_{p\infty}(1-\phi_s)^N$$

$$\beta \frac{\nu}{\phi_s(\rho_s-\rho_f)d_p g} \propto \phi_f^{-(N-2)}\left(\frac{\nu}{d_p u_{p\infty}}\right)$$




$$\tilde{\beta} = \beta\frac{u_{ch}}{g(\rho_s-\rho_f)}$$

# Sheared granular beds

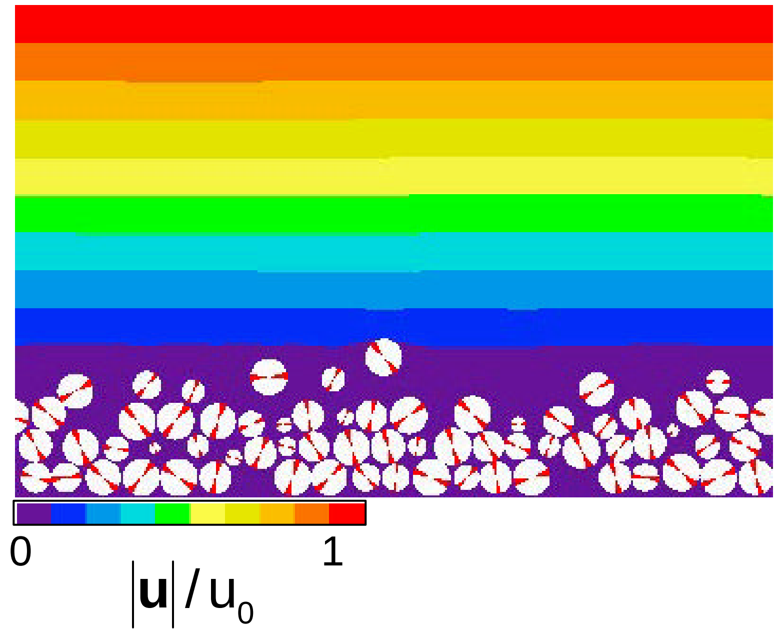



$$Re_{part} = \frac{d_p^{\,2}\,\bar{\dot\gamma}}{\nu} = 0.2 \quad \left(\bar{\dot\gamma} \equiv \frac{u_0}{w}\right) \quad \frac{\rho_s}{\rho_f} = 1.14$$

Experiment (Charru *et al*.)

http://www.imft.fr/recherche/interface/english/theme7/op_2.html

Simulation



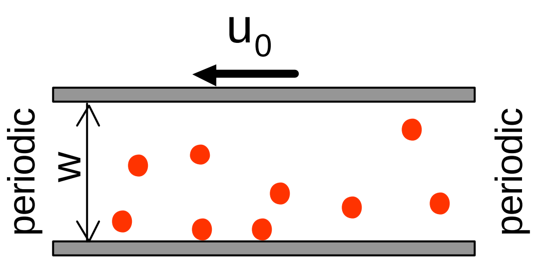
# Increase $Re_{part}$: resuspenion in turbulent flow

sedimentation and resuspension:

*description of particle-wall interaction is critical*

$$Re_{part} = \frac{d_p^{\,2}\,\bar{\dot{\gamma}}}{\nu} \quad \left( \bar{\dot{\gamma}} \equiv \frac{u_0}{w} \right)$$

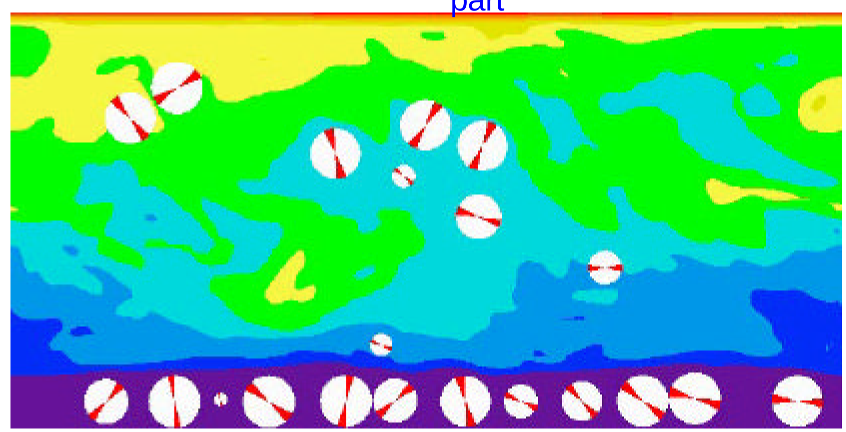
$$\frac{\rho_{part}}{\rho_{fluid}} = 2 \qquad \phi_{V,solid} = 10\%$$





$Re_{part}$=625

$Re_{part}$=1250 → 2500



# Sheared suspensions

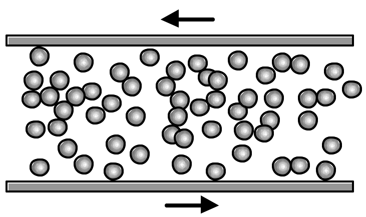
## Momentum transfer in granular-fluid flows

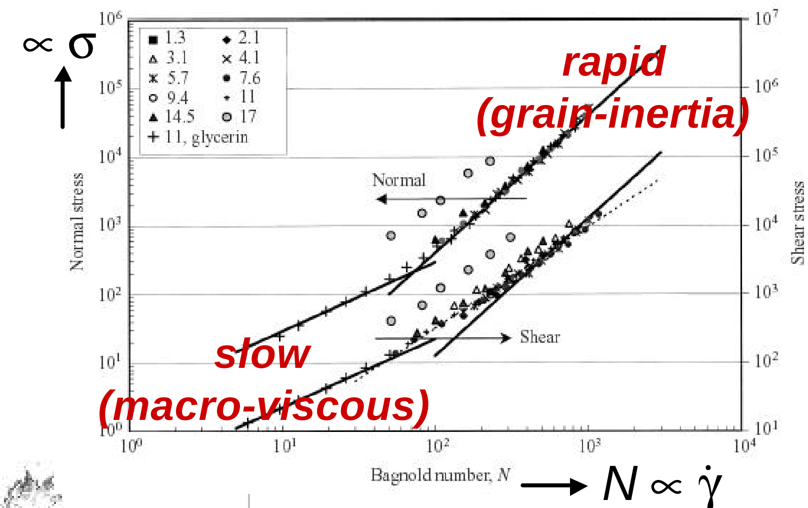
The elementary picture in simple shear due to Bagnold (1954):

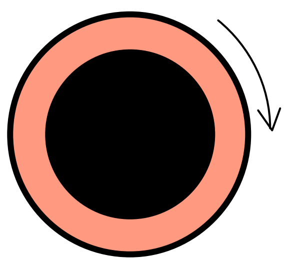
“slow” flows: viscous effects dominate: $\sigma \propto \dot{\gamma}\nu_{fluid}$

“rapid” flows: collisional stress dominates:

$$\sigma \propto \dot{\gamma}\nu_{eff} \rightarrow \nu_{eff} \propto \ell^2\dot{\gamma} \rightarrow \sigma \propto \dot{\gamma}^2$$





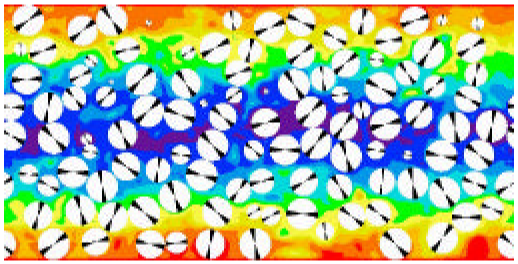



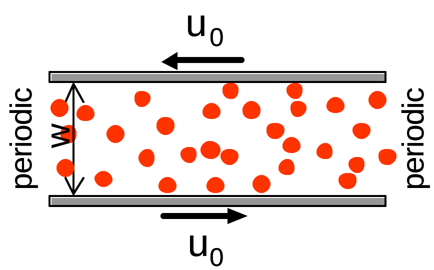
Bagnold’s experiment: Couette device

measurement of shear and normal stress

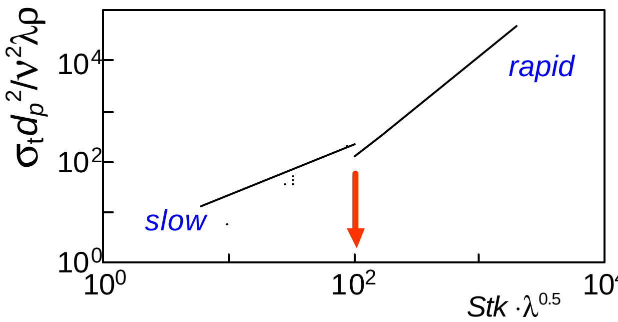
# Qualitative observations

$\phi_{solid}$=0.46

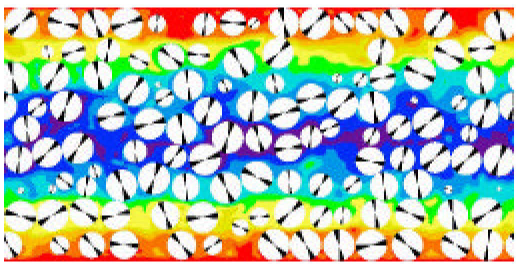




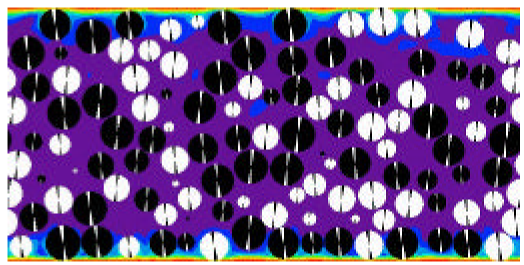

$N = St \cdot \lambda^{0.5} \approx 100$




$\phi_{solid}$=0.50

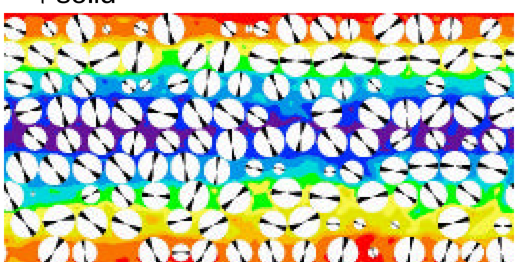


$\phi_{solid}$=0.50



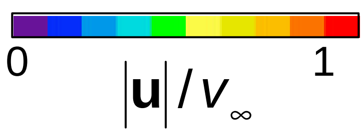
these are cross-sections of 3D simulations: spherical particles move in en out of the plane

$\phi_{solid}$=0.55



bi-disperse system

$d_{p,black}$=1.2$d_{p,white}$

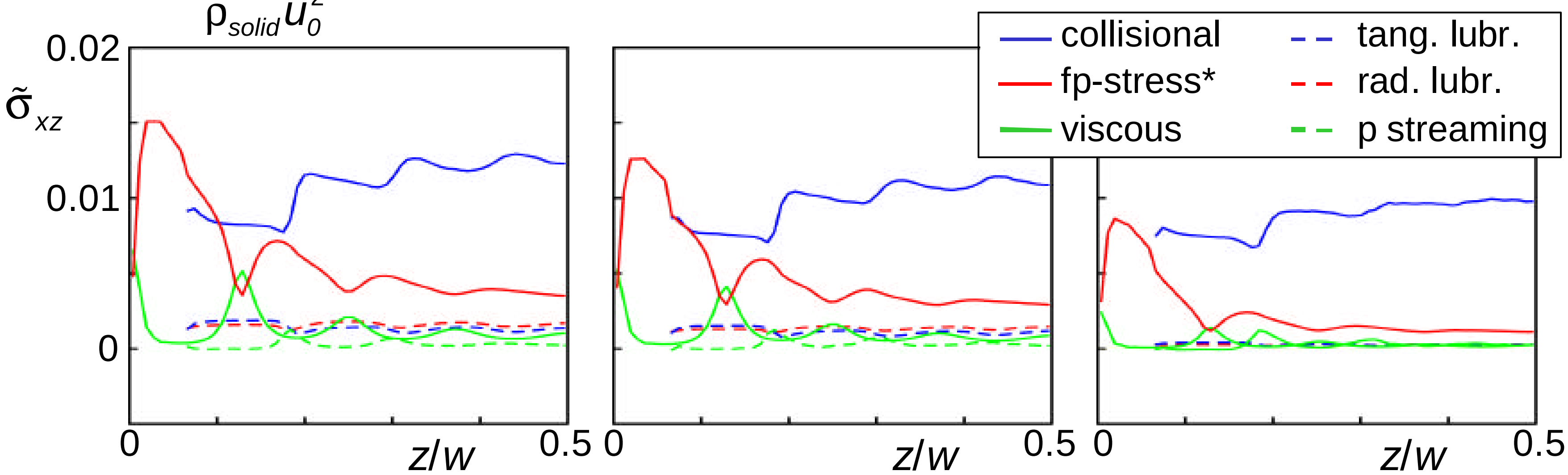



Shear-induced ordering
- consequences for stresses
- polydispersity opposes ordering

monodisperse systems

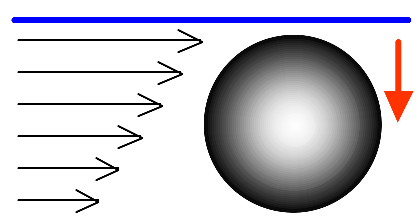
# Shear stresses

$\tilde{s} = \frac{s}{\rho_{solid} u_0^2}$ $\phi_{av}$=0.46 $\rho\overline{uw}$ (fluid streaming stress): very small




*St*=45 $\rho_s/\rho_f$=1.2 *St*=56 $\rho_s/\rho_f$=1.5 *St*=180 $\rho_s/\rho_f$=1.2

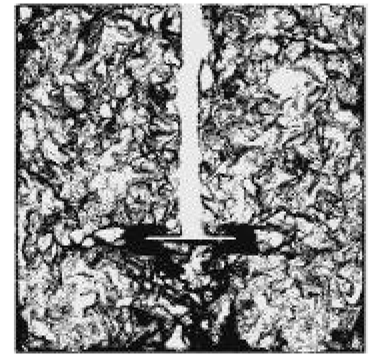
In Bagnold's grain-inertia scaling the dimensionless stresses would be independent of *St*



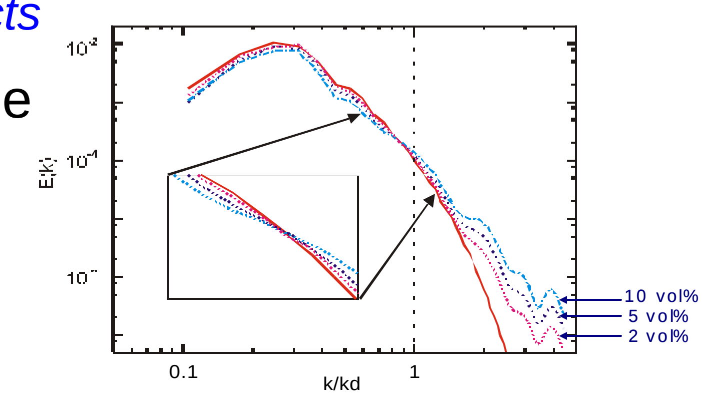
* ***fp-stress***:
momentum transfer in *z*-direction due to traction at particle surface: *finite-size effect*

* Jackson, *Chem. Eng. Sc*. **52** (1997)

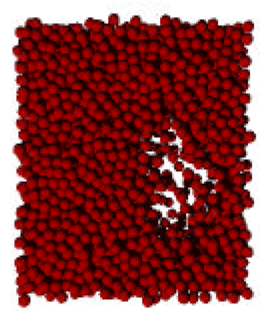
# Summary

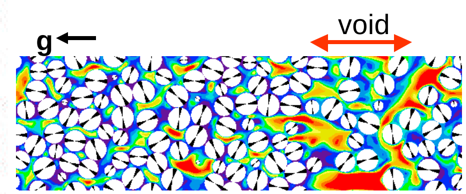


- Point particle approach in turbulent liquid-solid systems: possibilities and limitations
    - detailed information on particle motion and collisions in complex flows
    - the physics of two-way coupling and turbulent scales versus particle size $\rightarrow$ *finite size effects*



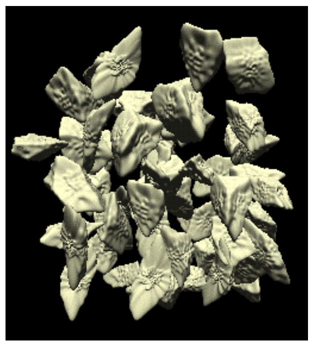

- Lattice-Boltzmann-based methodology for the *dynamics* of (dense) suspensions
- Turbulent suspensions
    - turbulence modification
    - collision dynamics: primary and secondary collisions
- Dense suspensions, fluidization
    - collisional stress dominates in the bulk
    - dilation and compaction behave differently in terms of stresses

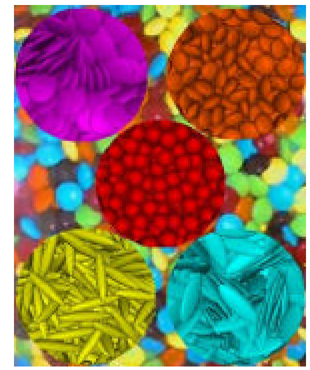




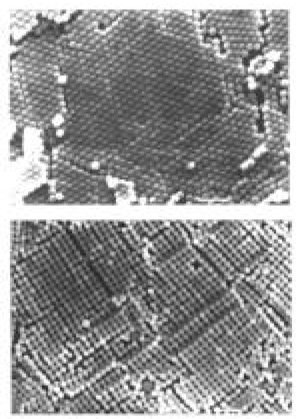
# Simulation tool for complex fluids

## Non-spherical particles
fibrous materials
concrete (high Stokes numbers)





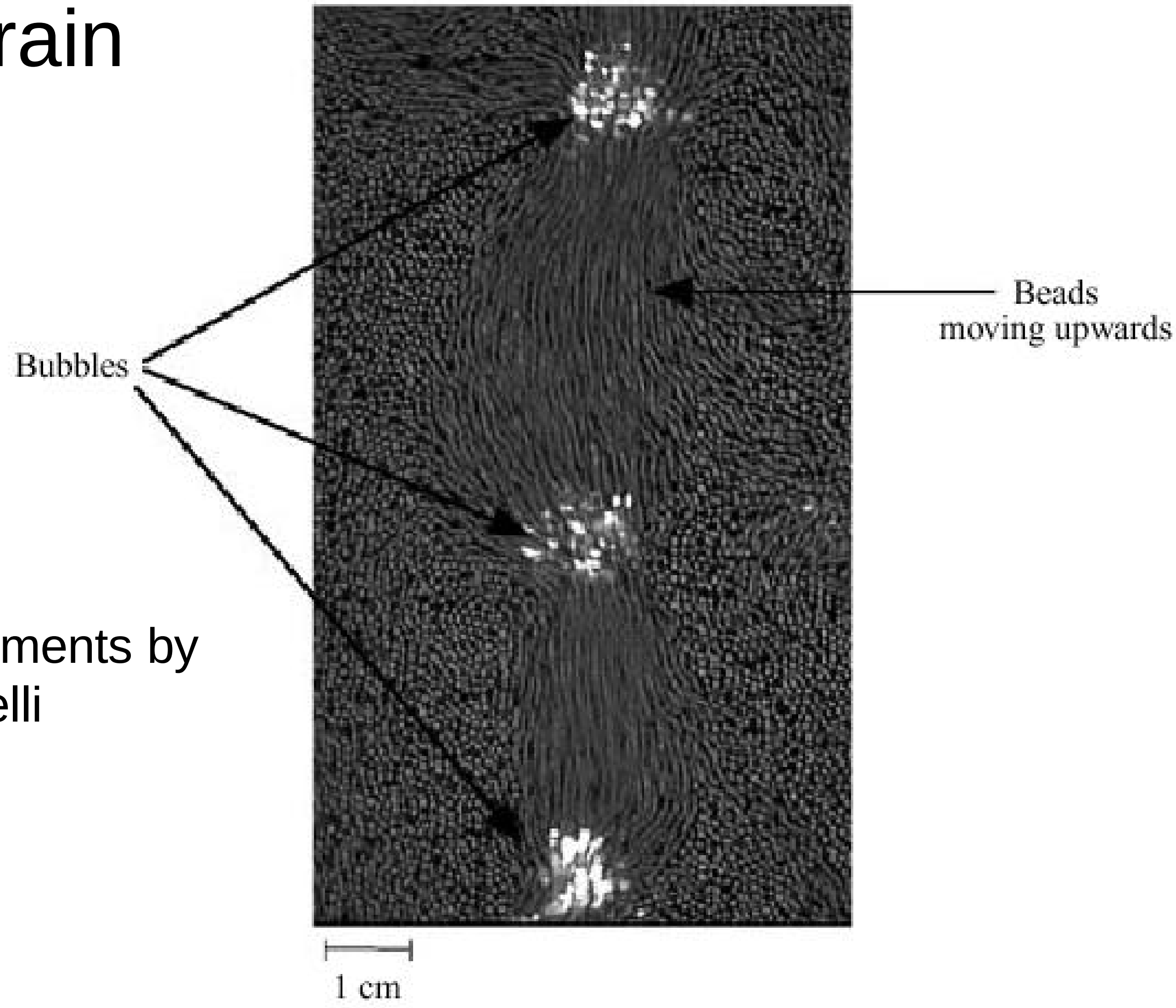
Donev et al., Physical Rev. Lett. 92 (2004)



## Colloidal systems
self organization

## Reservoir engineering
oil recovery by displacement with water

# Bubble train




from the experiments by
Duru & Guazzelli
*JFM* **470** (2002)

# Levels in process simulation

↑ *finer levels*

Track every molecule in physical and composition space

Continuum approach: solve the transport equations including appropriate boundary conditions

Disparity of scales (turbulence: $Re^{-3/4}$, particles,...): generalize the continuum approach to coarser levels

Compartmental approaches, population balances,...

↓ *coarser levels*

derive closure from DNS

LES, RANS, Euler/Euler, KTGM, micro-mixing models,...

**the interactions we call multi-scale modeling**

# 3D view